Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

29 April 2024

[redacted]

Tēnā koe [redacted]

Official Information Act Request

Thank you for the following request under the Official Information Act 1982 (OIA) on 9 April 2024:

> *I have so many students requesting to see the schedules from the 2023 exams in Level 1, but they have been removed from the NZQA website.*
>
> *These are highly valuable for the students for learning from errors and these will be very useful resources for teachers.*
>
> *As the students have a digital copy of their exam, there is no security reason for keeping them restricted. And the schedules can be guessed by comparing multiple students exams, that is not technically secure either. It would be very useful and time saving if these could be put back on the website.*

Response

On 16 April 2024 we sent you an email which:

- informed you that NZQA is in the process of removing resources from its website for the now-expired Level 1 standards to enable teachers and students to only access resources on our website that are valid and current. 2023 assessment schedules for the piloted Level 1 standards are available on our website and can be searched for in the resource search database. https://www.nzqa.govt.nz/ncea/assessment/search.do?query; and
- enquired whether you still require the 2023 assessment schedules for the now-expired Mathematics Level 1 standards (91027, 91028, 91031 and 91037). If you did not require these, we could consider this OIA request closed.

You responded by email on the same day that you still require the schedules for the four expired external exams from 2023 and, if possible, the corresponding examination papers.

In response to your request, please refer to the attached combined PDFs of the assessment schedules and examination papers for standards 91027, 91028, 91031, and 91037 from the 2023 academic year:

- *Mathematics and Statistics Level 1 – assessment schedules, and*
- *Mathematics and Statistics Level 1 – examination papers.*

Your response may be published on our website after five working days. Your name and contact details will be removed before publication.

If you require further assistance or believe we have misinterpreted your request, please contact ministerials@nzqa.govt.nz.

You have the right to seek an investigation or review by the Ombudsman of this decision under section 28(3) of the Official Information Act 1982. Details of how to make a complaint can be found at www.ombudsman.parliament.nz. You can also telephone 0800 802 502 or write to the Ombudsman at PO Box 10152, Wellington, 6143

Nāku nā

Dr Grant Klinkum
Pouwhakahaere/Chief Executive

To be completed by candidate and school

Name: _____

NSN | | | | | | | | | | |

School Code | | | | |

1

SUPERVISOR'S USE ONLY

DAY 1: TUESDAY

NZQA | Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

COMMON ASSESSMENT TASK

Level 1 Mathematics and Statistics 2023

91027 Apply algebraic procedures in solving problems

Tuesday 12 September 2023
Credits: Four

You should attempt ALL the questions in this booklet. Show ALL working.

Calculators may NOT be used.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. 'Guess and check' and 'correct answer only' methods do not demonstrate relational thinking and will limit the grade for that part of the question to a maximum of Achievement. 'Guess and check' and 'correct answer only' may only be used a maximum of one time in the paper and will not be used as evidence of solving a problem. A candidate cannot gain Achievement in this standard without solving at least one problem using algebra.

Answers must be given in their simplest algebraic form.

Where a question is given in words, you are expected to show the equation that you used to solve the problem.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

ASSESSOR'S USE ONLY	Achievement Criteria	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Apply algebraic procedures in solving problems.	Apply algebraic procedures, using relational thinking, in solving problems.	Apply algebraic procedures, using extended abstract thinking, in solving problems.
		Overall level of performance

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Find the value of d, given that $d = 2c^2 - 9\,(2c - 2) + 5$ and $c = -3$.

(b) Using factorisation, simplify as far as possible:

$$\frac{(4x^2 + 3x - 1)(x - 1)}{(x^2 - 1)(4x - 1)}$$

(c) Solve the following equation: $20x^2 + 20x = (2x + 5)^2$

(d) At a certain point on a **1600 kilometre** road trip, one-third of the distance already travelled is equal to one-fifth of the distance remaining.

How many more kilometres of the trip are there remaining?

ASSESSOR'S USE ONLY

(e) The number of small squares used in the n^{th} shape of a pattern is given by $n^2 - 2n + 5$.

Show that the difference between the number of small squares used in two consecutive odd-numbered shapes is always divisible by 4.

QUESTION TWO

ASSESSOR'S USE ONLY

(a) The area of the rectangle shown in the diagram below is $40x^2 + 11x - 2$.

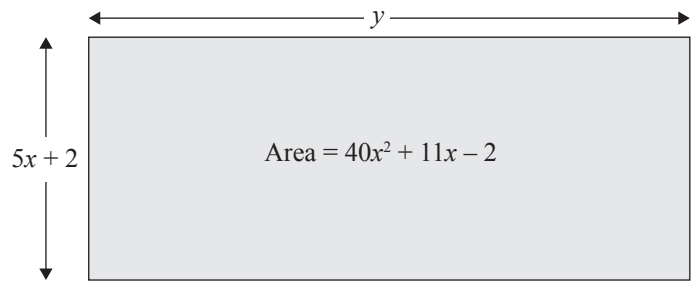
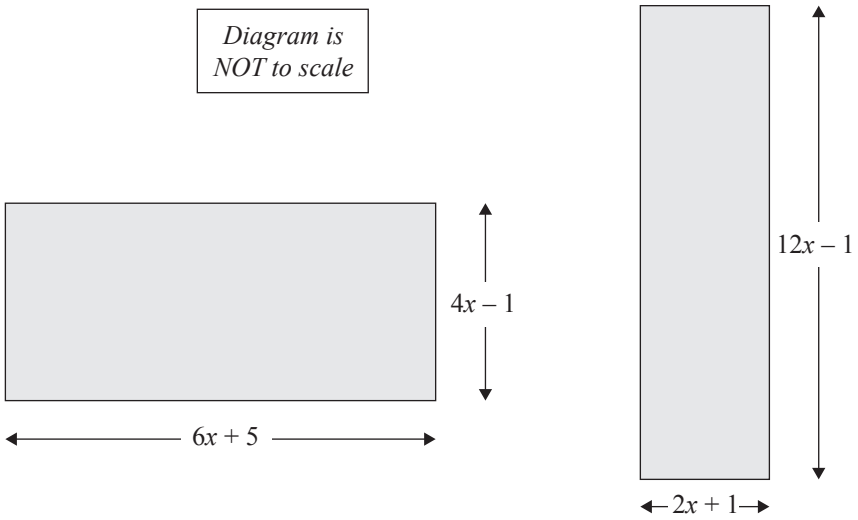


Diagram is NOT to scale

What is the length of the side y, giving your answer in terms of x.

(b) The **areas** of the two rectangles, shown below, are equal to each other.

Diagram is NOT to scale



Find the value of x.

ASSESSOR'S USE ONLY

(c) Solve the following inequality:

$$\frac{4x+1}{5}-\frac{3x-4}{2}\geq 5$$

(d) If $\frac{4w}{5}=\frac{v(w+3)}{4}$, give the equation for w in terms of v.

ASSESSOR'S USE ONLY

(e) The plan of a garden is shown in the diagram below. All measurements are in metres.

The shaded area in the diagram is 8 m².

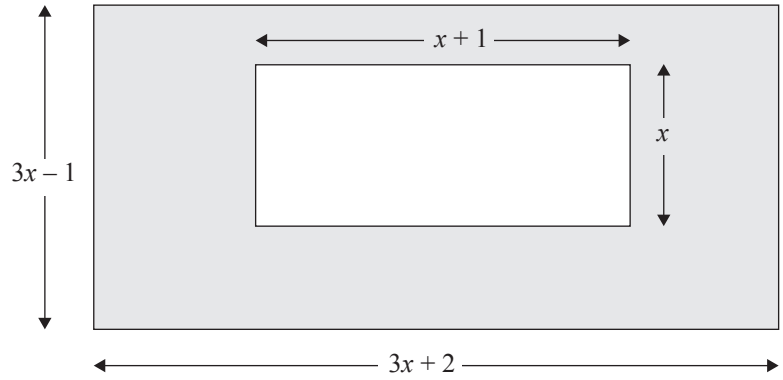


Diagram is NOT to scale

Find the value of x.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) A new playground at a school is being designed in the shape of a regular hexagon, as shown in the diagram below.

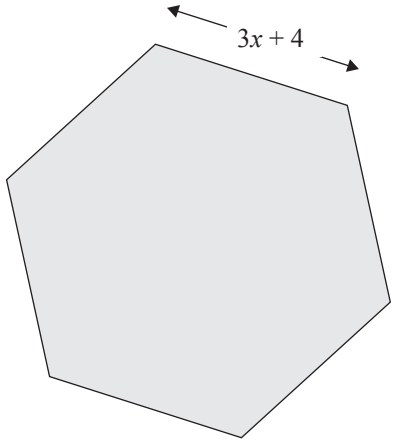


Diagram is NOT to scale

Given that the **perimeter** of the playground is 60 metres, find the value of x.

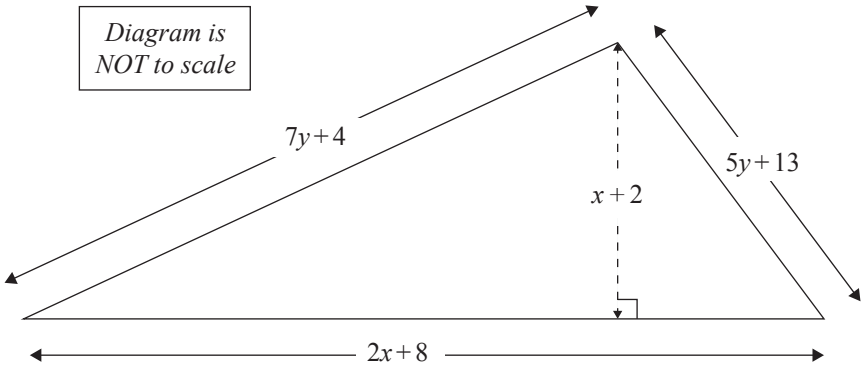
(b) Solve the equation:

$$\frac{2x}{2x-3} - \frac{x+4}{x+2} = 0$$

Question Three continues on the next page.

ASSESSOR'S USE ONLY

(c) The **area** of the triangle drawn below is 24 cm².



Find the value of x.

- Area of a triangle = ½ × base × height
- All measurements are in cm.

(d) Solve the equation $4^{x-2} \times 2^{x+1} = 32^{x}$

(e) A bank robber drops their bag containing some stolen $20 and $50 notes.

The total **number** of notes in the bag is 40.

The total **value** of the bank notes is $1700.

Calculate the **value** of the $20 notes and the **value** of the $50 notes that were stolen.

ASSESSOR'S USE ONLY

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

91027

To be completed by candidate and school

Name: _____

NSN | | | | | | | | | | |

School Code | | | | |

1

SUPERVISOR'S USE ONLY

DAY 2: THURSDAY

NZQA

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

COMMON ASSESSMENT TASK

Level 1 Mathematics and Statistics 2023

91027 Apply algebraic procedures in solving problems

Thursday 14 September 2023
Credits: Four

You should attempt ALL the questions in this booklet. Show ALL working.

Calculators may NOT be used.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. 'Guess and check' and 'correct answer only' methods do not demonstrate relational thinking and will limit the grade for that part of the question to a maximum of Achievement. 'Guess and check' and 'correct answer only' may only be used a maximum of one time in the paper and will not be used as evidence of solving a problem. A candidate cannot gain Achievement in this standard without solving at least one problem using algebra.

Answers must be given in their simplest algebraic form.

Where a question is given in words, you are expected to show the equation that you used to solve the problem.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

ASSESSOR'S USE ONLY	Achievement Criteria	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Apply algebraic procedures in solving problems.	Apply algebraic procedures, using relational thinking, in solving problems.	Apply algebraic procedures, using extended abstract thinking, in solving problems.
		Overall level of performance

QUESTION ONE

ASSESSOR'S USE ONLY

(a) The area of the rectangle shown in the diagram below is $42x^2 + 11x - 3$.

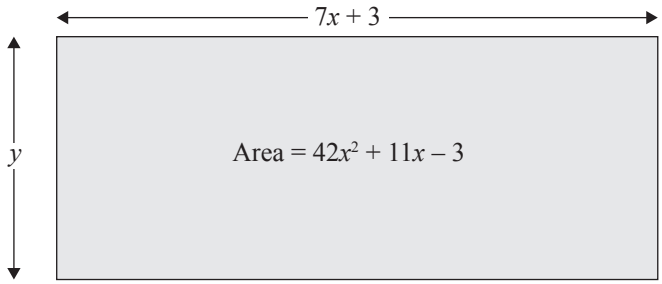


Diagram is NOT to scale

What is the length of the side y, giving your answer in terms of x.

(b) The **areas** of the two rectangles, shown below, are equal to each other.

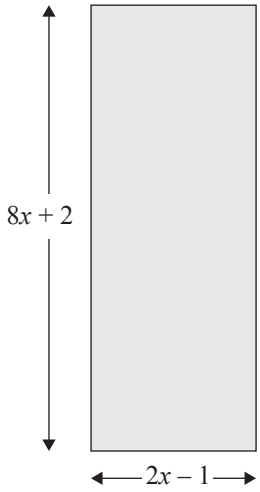
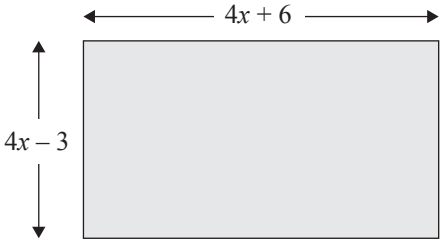


Diagram is NOT to scale



Find the value of x.

ASSESSOR'S USE ONLY

(c) Solve the following inequality:

$$\frac{3x+2}{4}-\frac{4x-1}{3}\geq 2$$

(d) If $\frac{5g}{6}=\frac{h(g+4)}{5}$, give the equation for g in terms of h.

ASSESSOR'S USE ONLY

(e) The plan of a garden is shown in the diagram below. All measurements are in metres.

The shaded area in the diagram is 6 m².

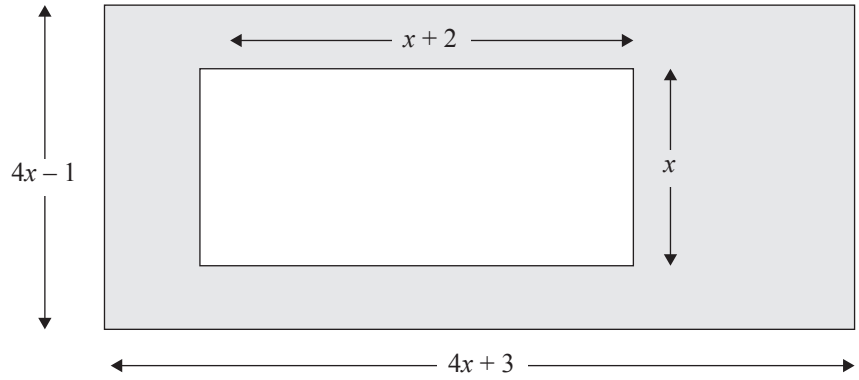


Diagram is NOT to scale

Find the value of x.

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Find the value of w, given that $w = 3v^2 - 8(3v - 2) + 6$ and $v = -3$.

(b) Using factorisation, simplify as far as possible:

$$\frac{(5x^2 - 9x - 2)(x + 2)}{(x^2 - 4)(5x + 1)}$$

(c) Solve the following equation $18x^2 + 24x = (3x + 4)^2$

ASSESSOR'S USE ONLY

(d) At a certain point on a **1200 kilometre** road trip, one-quarter of the distance already travelled is equal to one-sixth of the distance remaining.

How many more kilometres of the trip are there remaining?

(e) The number of small squares used in the n^{th} shape of a pattern is given by $n^2 - 3n + 6$.

Show that the difference between the number of small squares used in two consecutive odd-numbered shapes is always divisible by 2.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) A new playground at a school is being designed in the shape of a regular octagon, as shown in the diagram below.

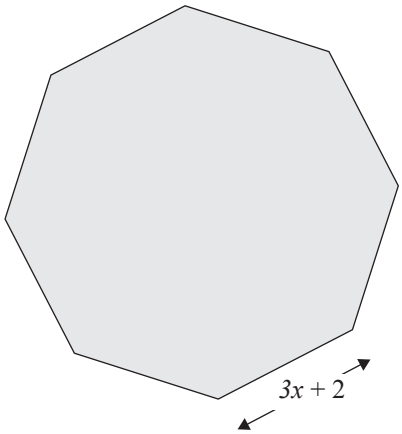


Diagram is NOT to scale

Given that the **perimeter** of the playground is 88 metres, find the value of x.

(b) Solve the equation:

$$\frac{4x}{4x-3} - \frac{x+6}{x+3} = 0$$

Question Three continues on the next page.

ASSESSOR'S USE ONLY

(c) The **area** of the triangle drawn below is 32 cm².

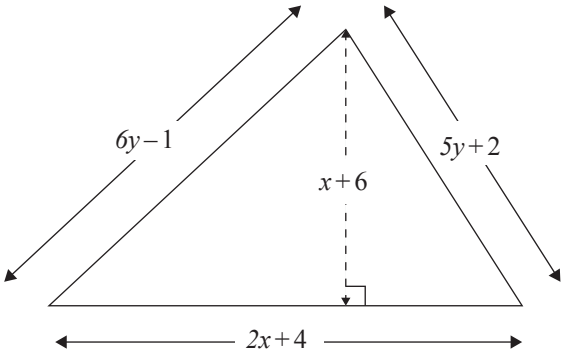


Diagram is NOT to scale

Find the value of x.

- Area of a triangle = ½ × base × height
- All measurements are in cm.

(d) Solve the equation $2^{5x-2} \times 4^{x+2} = 16^{x}$

(e) A farmer has a shed containing sacks of potatoes; some sacks weigh 20 kg and other sacks weigh 50 kg.

The total **number** of sacks in the shed is 60.

The total **weight** of all the sacks of potatoes is 1500 kg.

Calculate the total **weight** of all of the 20 kg sacks, and the total **weight** of all of the 50 kg sacks in the shed.

ASSESSOR'S USE ONLY

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

91027

SUPERVISOR'S USE ONLY

1

91028

910280

Draw a cross through the box (☒) if you have NOT written in this booklet

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NZQA | Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 1 Mathematics and Statistics 2023

91028 Investigate relationships between tables, equations and graphs

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Investigate relationships between tables, equations and graphs.	Investigate relationships between tables, equations and graphs, using relational thinking.	Investigate relationships between tables, equations and graphs, using extended abstract thinking.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–24 in the correct order and that none of these pages is blank.

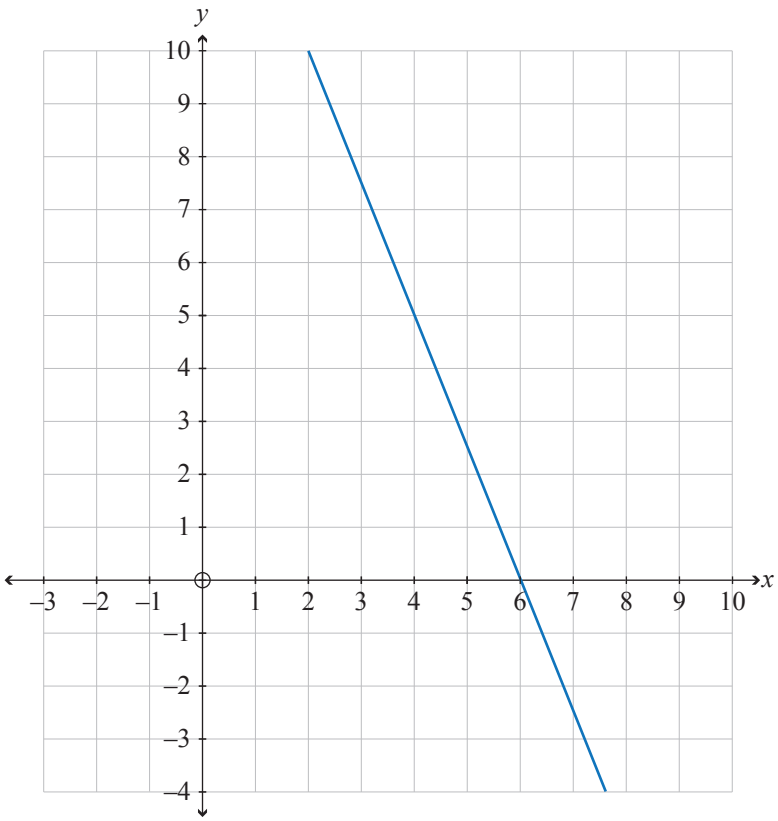
Do not write in any cross-hatched area (DO NOT WRITE IN THIS AREA DO NOT WRITE). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

This page has been deliberately left blank.
The assessment starts on the following page.

QUESTION ONE

(a) (i) Give the equation of the graph shown below.



Equation is: _____

(ii) Find the equation of the new line if the graph shown above is reflected in the y-axis, and also shifted 10 units vertically upwards.

(b) A city council plans to create a fenced grassed area in Fantail Reserve.

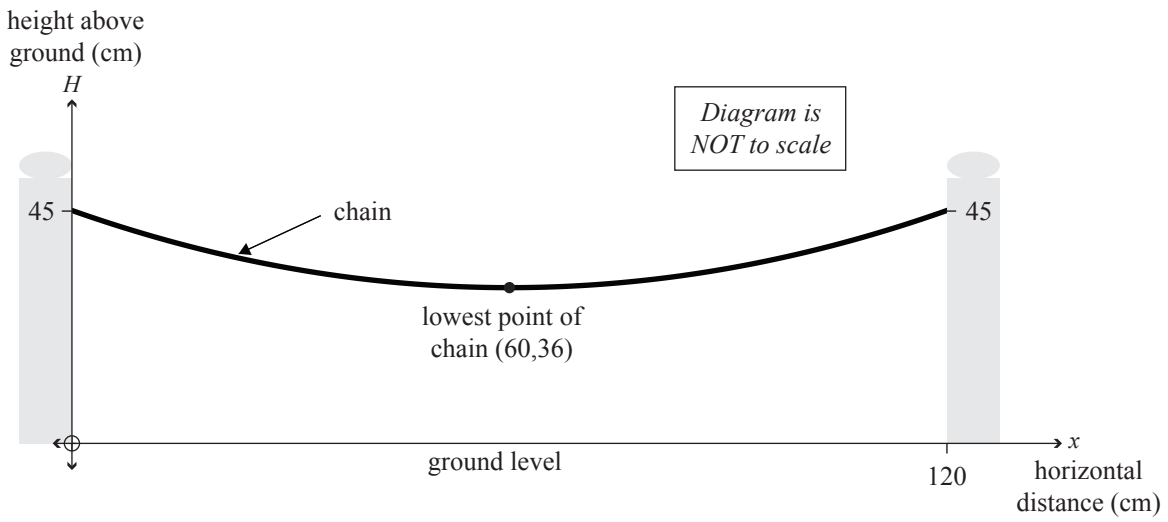
They decide to install some posts with a chain fence between them, as shown right.

The distance between the inside of each of the two posts is 120 cm.

The chain is attached to each post 45 cm above the horizontal ground, and hangs symmetrically.

The lowest point on the fence is 36 cm above the ground.

One section of the fencing is shown in the diagram below.



(i) Find an equation that would model the height of the chain between the two posts, as shown in the diagram above, where x is the horizontal distance from the inside of the left-hand post and H is the vertical height of the chain above the ground.

The x-axis will be at ground level, and the H-axis will be in line with the inside of the left-hand post.

Justify your answer with full and clear working.

(ii) After reviewing the design, the council decides that the lowest point of the chain should actually be lower than the design shown. The posts cannot be moved or changed, as they have already been installed, but where the chain fixes onto the post could be changed if required.

Suggest at least one way in which the original equation of the chain fence design could be altered to make the chain hang lower, but with the chain being in the same shape as the original chain.

Describe how your suggested change would affect the shape of the chain fence AND provide the equation of your alternative design.

(c) One side of the reserve has trees along it, and this side will not be fenced.

The council has a budget to purchase only 240 metres of material for the total of the other three sides of the rectangular grassed area, as shown in the diagram below.

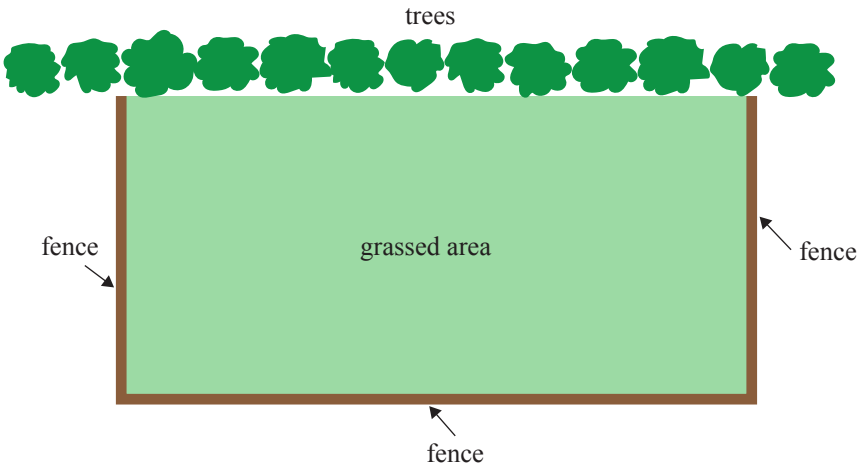
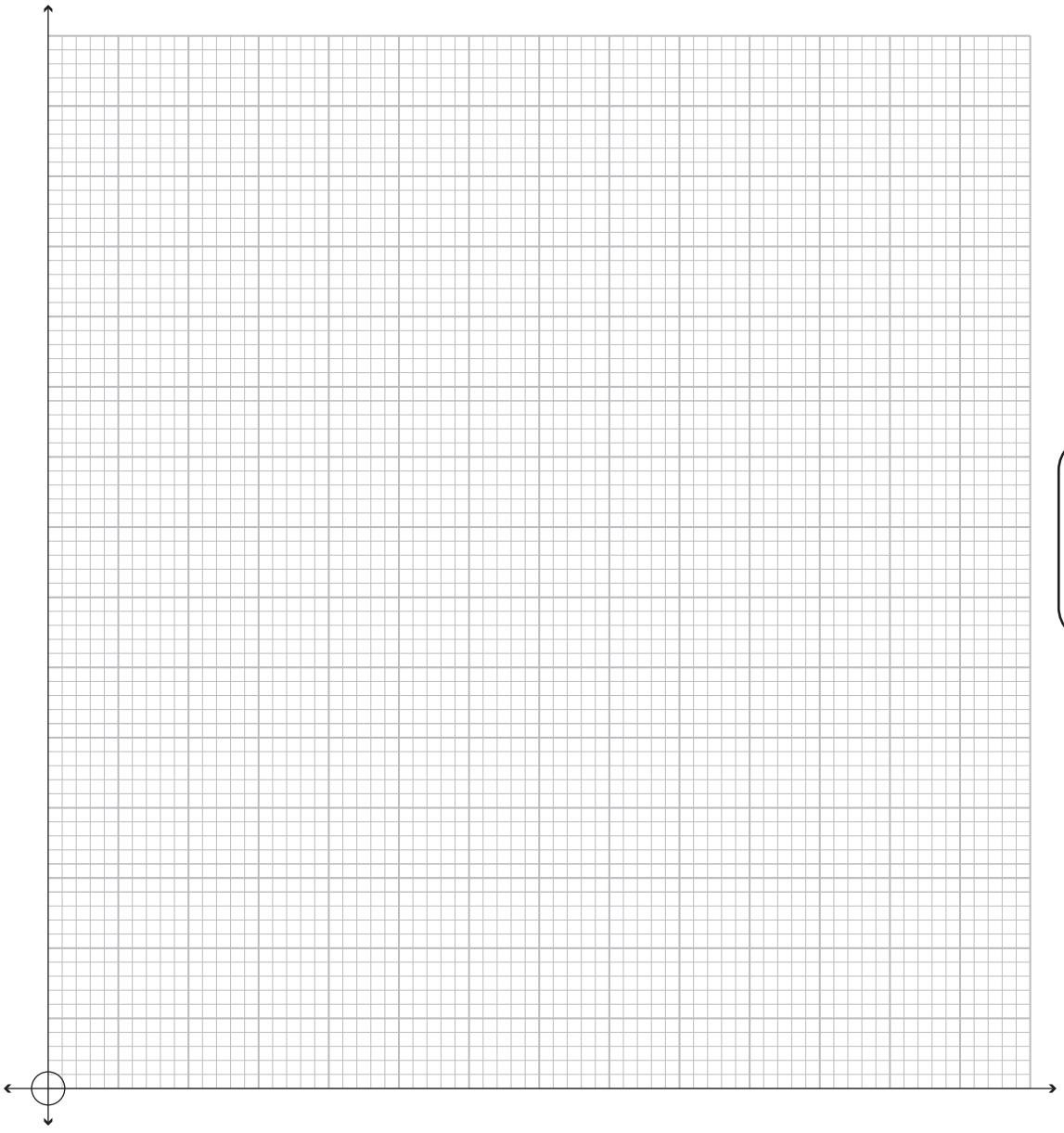


Diagram is NOT to scale

Use tables, equations, AND graphs to investigate the relationship between the length and the width of the grassed reserve and the area enclosed by the fences.

Provide at least THREE different comments that follow from your investigation.

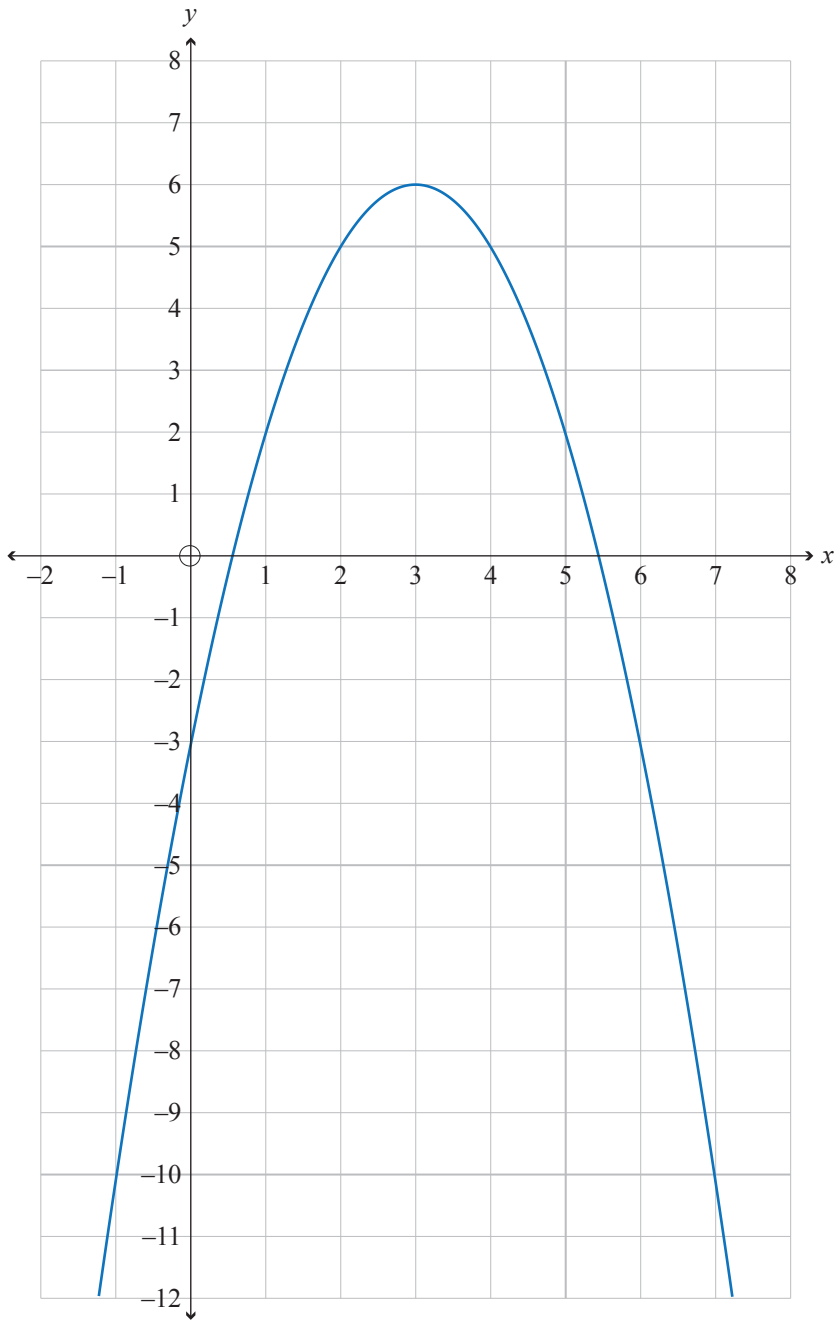
Justify your comments with full details.



If you need to redraw your graphs, use the grid on page 17.

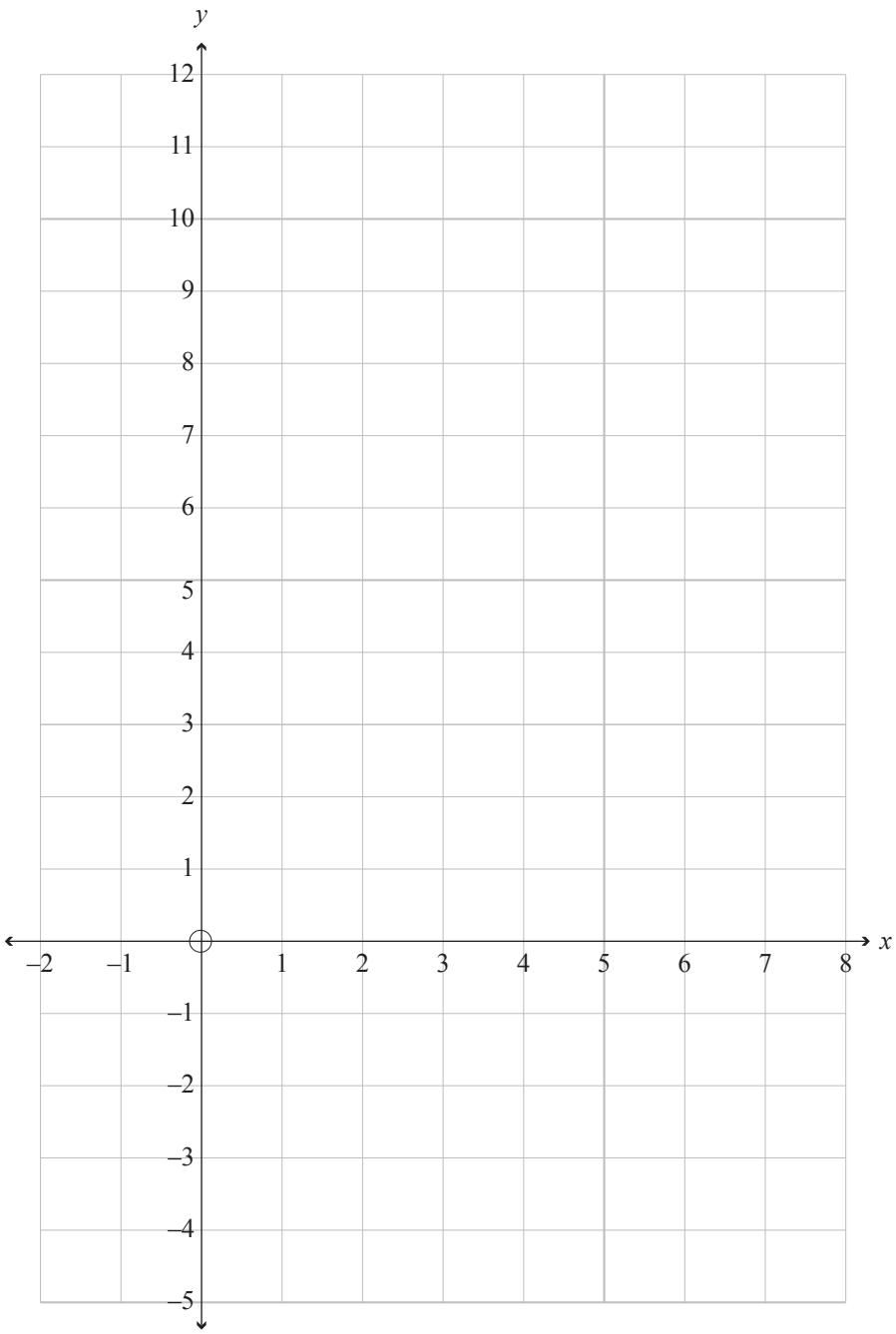
QUESTION TWO

(a) Give the equation of the graph shown below.



Equation is: _____

(b) Using the set of axes provided below, draw the two graphs $y = 2^{x-3}$ and $y = 2x - 3$.
Using your graphs, solve the equation $2^{x-3} = 2x - 3$.



If you need to redraw your graph, use the grid on page 18.

(c) Adam is looking at the number of flowers on a particular tree in his garden.

He counts the number of flowers on each level of branch on the tree. Branch level 1 is the branch closest to the ground, branch level 2 is the next one up, and so on.

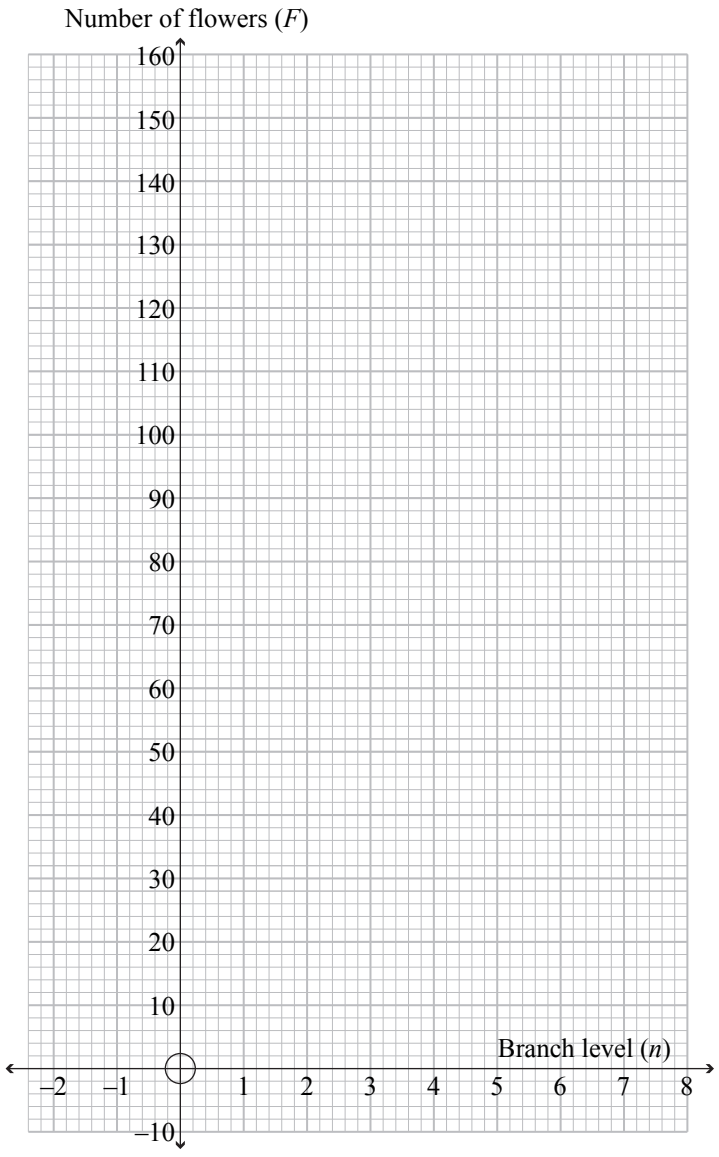
The table below lists the results from his research.

Branch level (n)	Number of flowers on that branch (F)
1	6
2	16
3	30
4	48
5	70
6	
7	
8	

https://www.southernliving.com/garden/trees/cherry-blossom-tree

(i) Using the axes below, draw the graph that best represents the relationship between "Branch level" (n) and "Number of flowers on that branch" (F).

Show the results as far as $n = 8$.



If you need to redraw your graph, use the grid on page 19.

(ii) Find an equation that represents the "Number of flowers" on any given branch.

Justify your answer with clear working, **including an appropriate domain**.

(iii) Adam counts the number of flowers on each level of branch on a different tree.

The table below lists the results from his research on this second tree.

Branch level (n)	**Number of flowers on that branch (F)**
1	1
2	4
3	16
4	64
5	256
6	
7	
8	

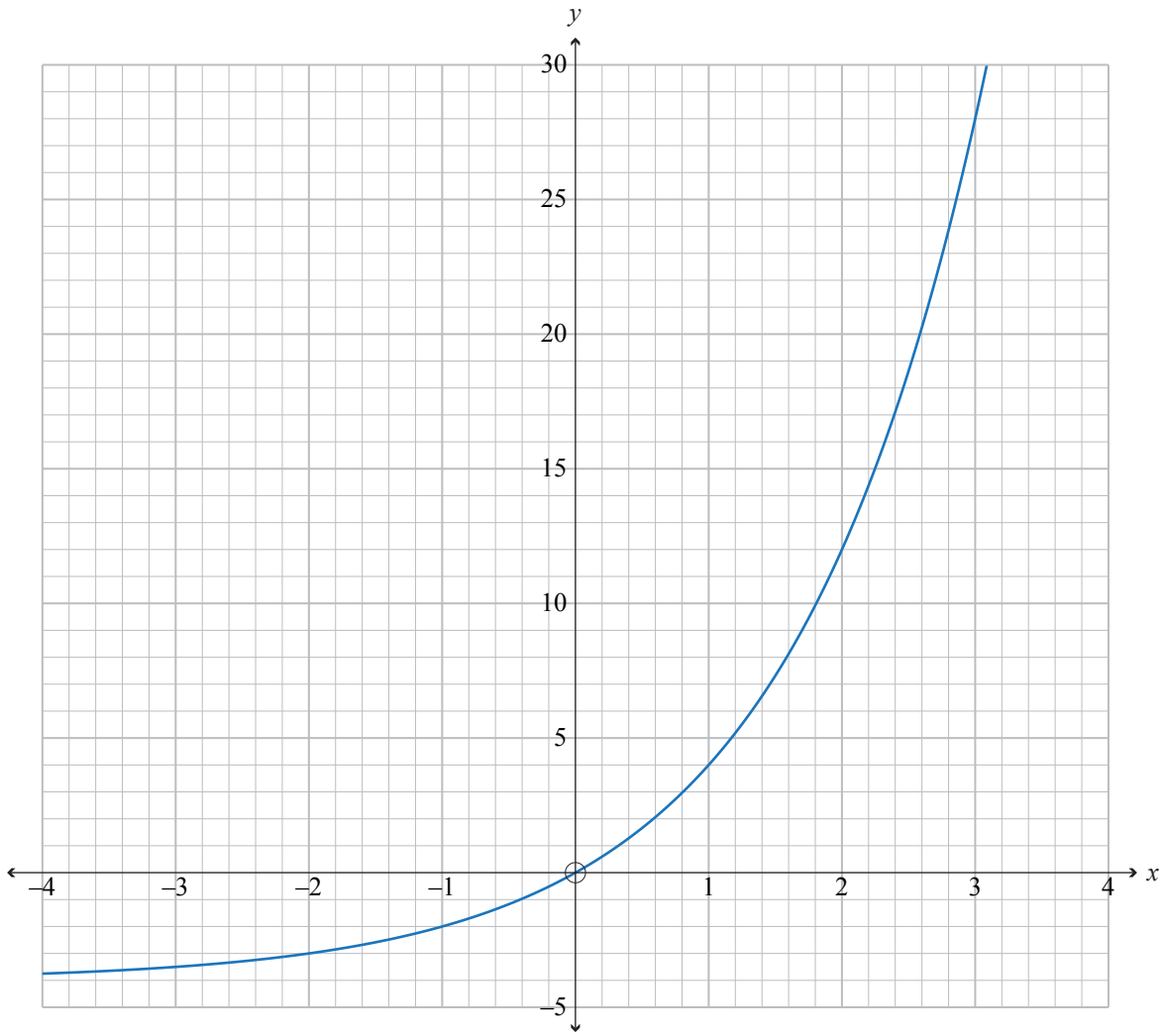
Find an equation that represents the "Number of flowers" on any given branch on this second tree. Justify your answer.

(iv) Adam claims that this formula will help him predict the number of flowers on trees in New Zealand.

Comment on this claim giving at least TWO different statements to support your reasoning.

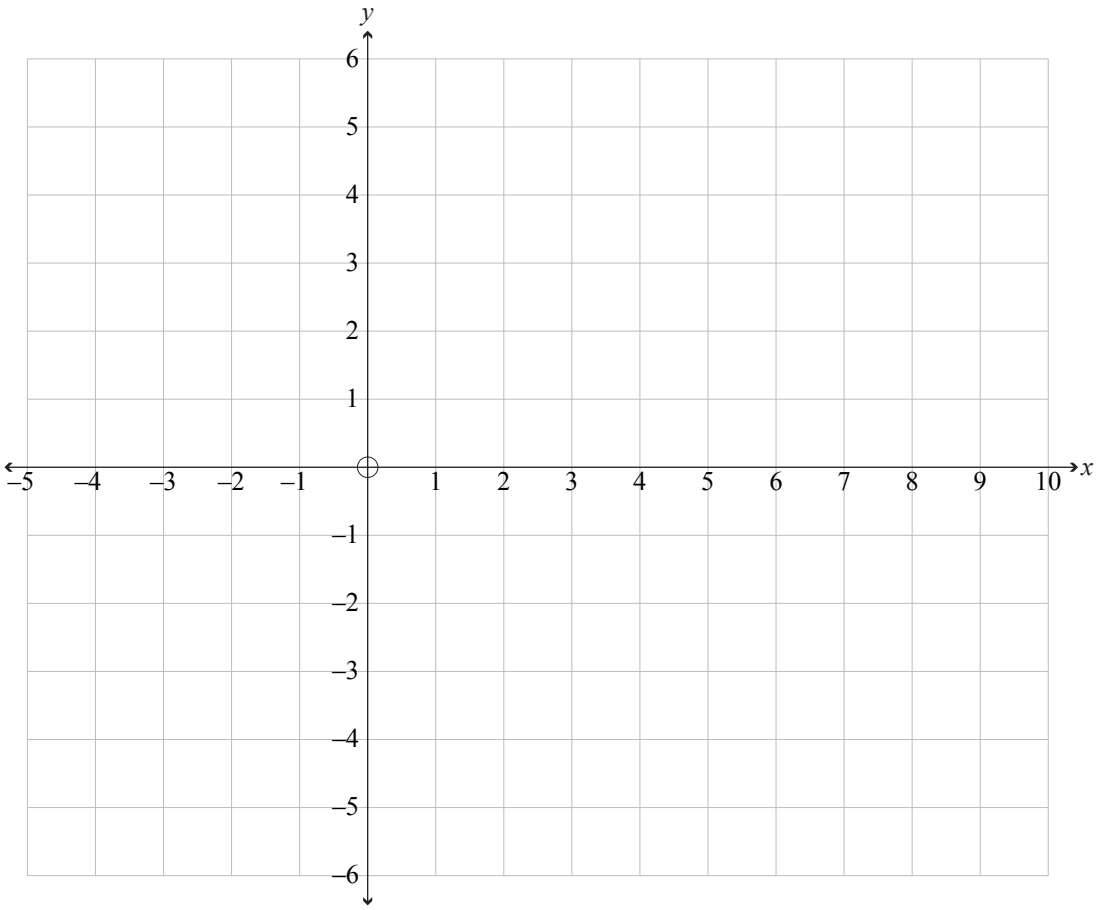
QUESTION THREE

(a) (i) Give the equation of the graph shown below.



Equation is: _____

(ii) On the axes below, draw the graph of $3y - 2x + 6 = 0$.



If you need to redraw your graph, use the grid on page 20.

(b) Bronwyn plans to invest **$1000** using her bank. The bank offers three different types of savings plans, and she needs to decide which one to choose.

She wants to compare how her savings will increase over the next 6 years.

The details of each savings plan are listed below.

Bronwyn does not plan to take out any money during the 6 years.

Note: All graphs in this question can be considered to be continuous.

Savings Plan A: Invest $1000 at the start, and then receive the same regular payment of **$250** at the end of each year.

Savings Plan B: The equation for Savings Plan B is modelled by the formula

$$S = 60t^2 + 10t + 1000$$

where t represents the number of years since Bronwyn started her savings plan,
and S represents the total amount ($) in Bronwyn's account.

Savings Plan C: The details of Savings Plan C are shown in the table below, which can be modelled by an exponential equation of the form $S = p \times q^t$.

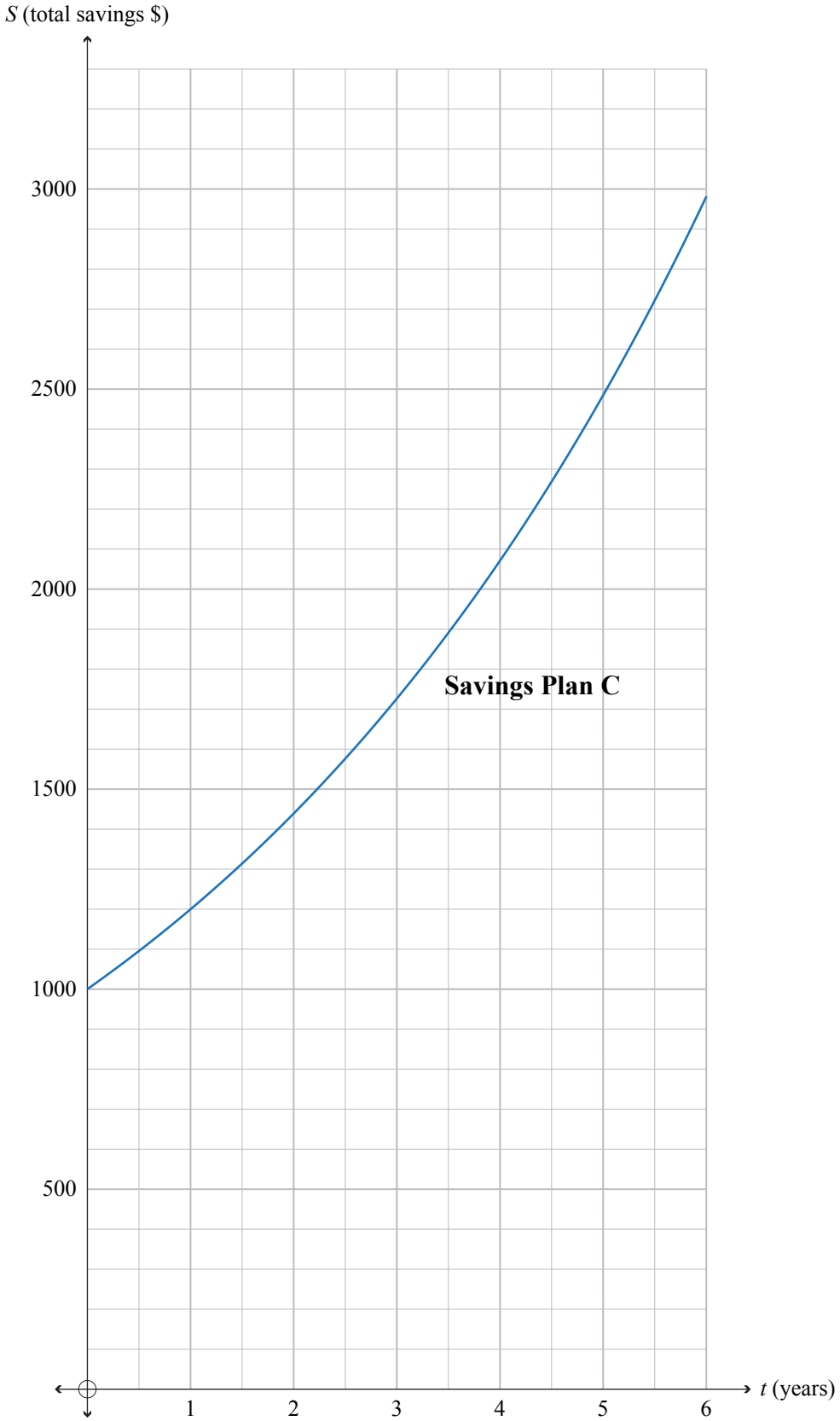
where p and q are numbers to be found,
t represents the number of years since Bronwyn started her savings plan,
and S represents the total amount ($) in Bronwyn's account.

Note: The graph of Savings Plan C has already been drawn for you on page 15.

End of year (t)	**Total savings amount (S) ($)**
0 (at the start of saving)	1000.00
1	1200.00
2	1440.00
3	1728.00
4	2073.60
5	2488.32
6	2985.98

(i) Write the equation of **Savings Plan C**.

(ii) Draw the graphs that model **Savings Plan A** and **Savings Plan B** below.
(The graph of **Savings Plan C** has been drawn for you.)



If you need to redraw your graphs, use the grid on page 21.

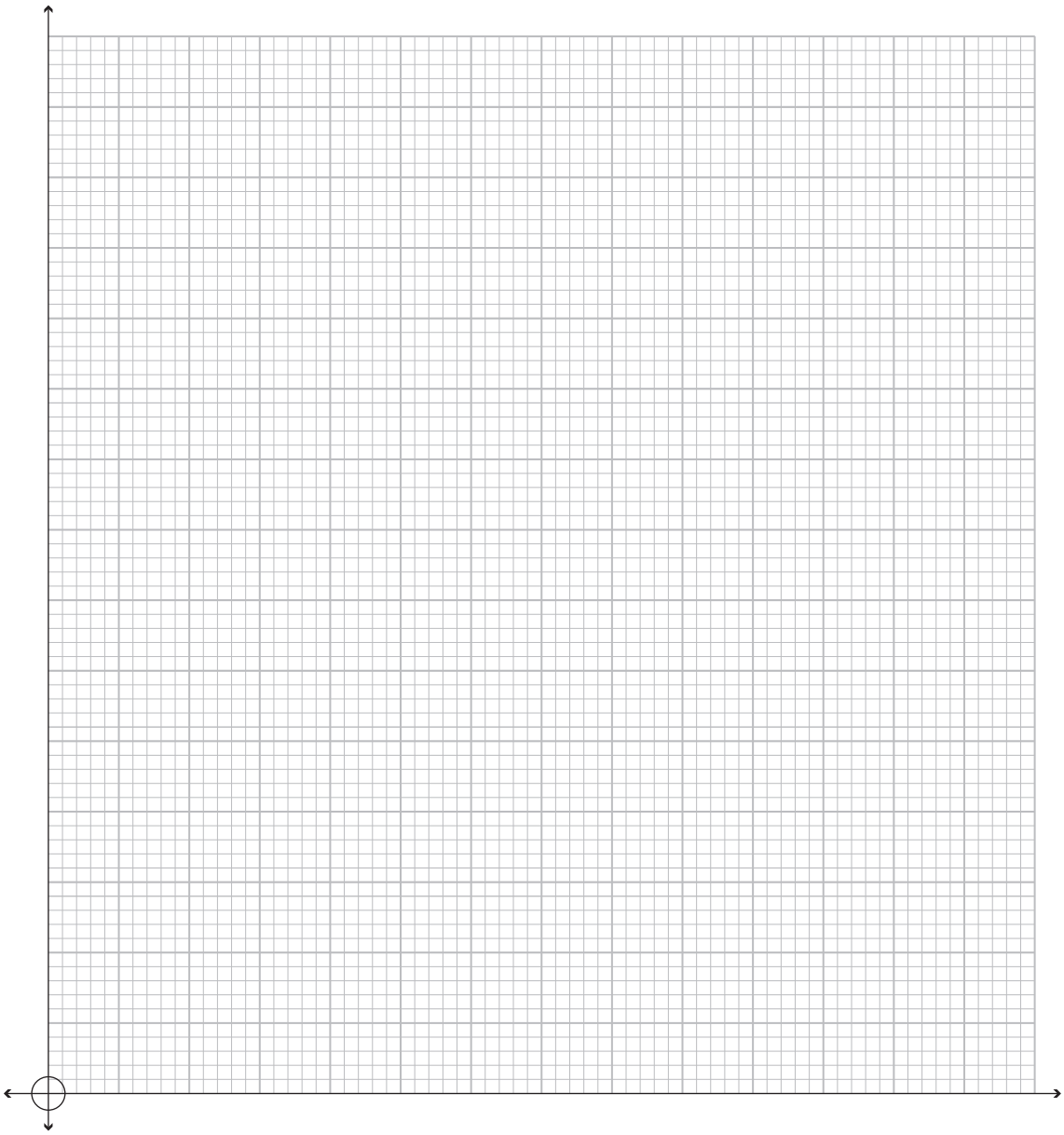
(iii) Using tables and the graphs in part (ii) on page 15, give a detailed comparison between the three savings plans at various stages during the first six years.

Provide at least THREE comments that follow from your comparison.

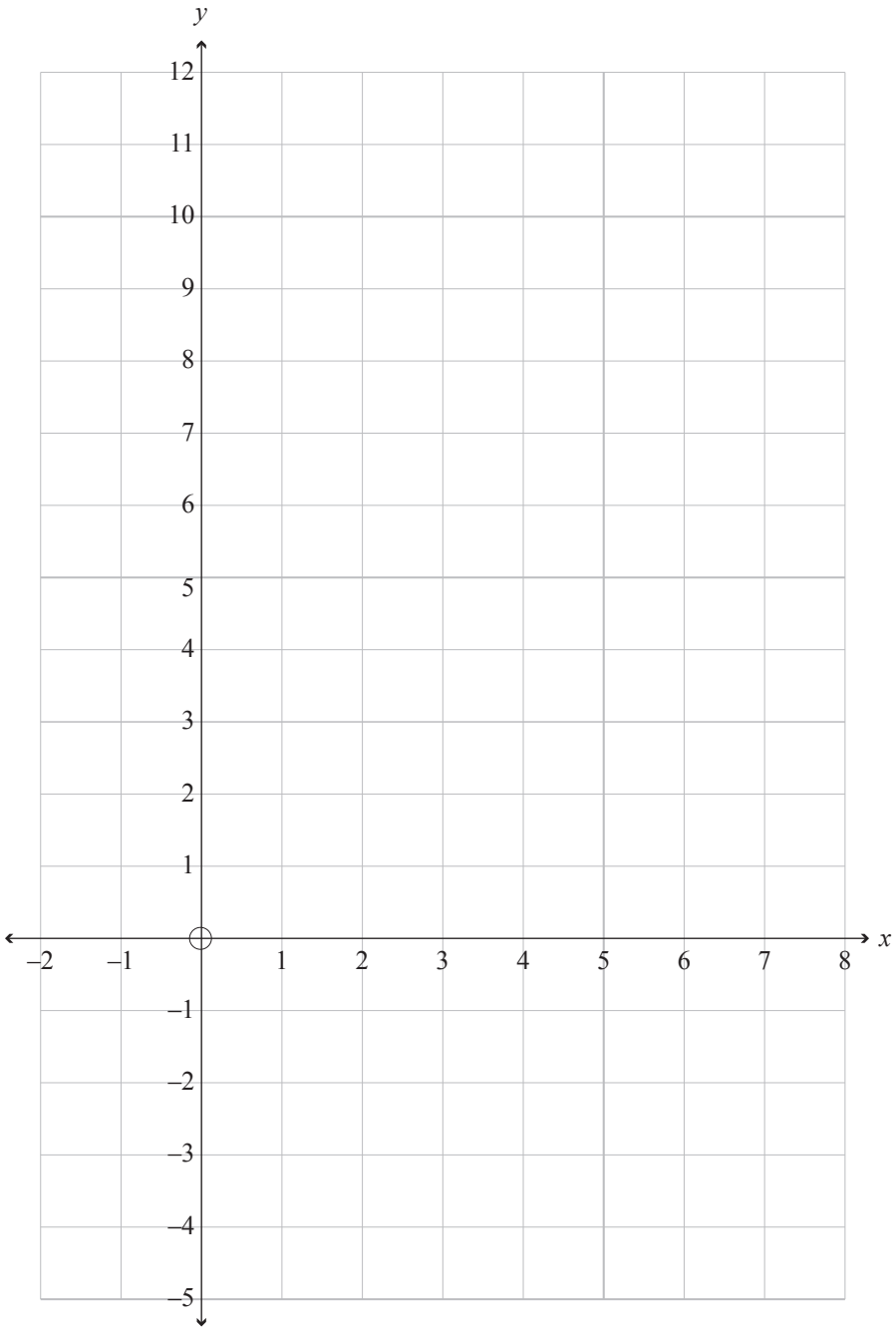
End of Year (t)	Total savings Savings Plan A	Total savings Savings Plan B	Total savings Savings Plan C	
0			1000.00	
1			1200.00	
2			1440.00	
3			1728.00	
4			2073.60	
5			2488.32	
6			2985.98	

SPARE DIAGRAMS

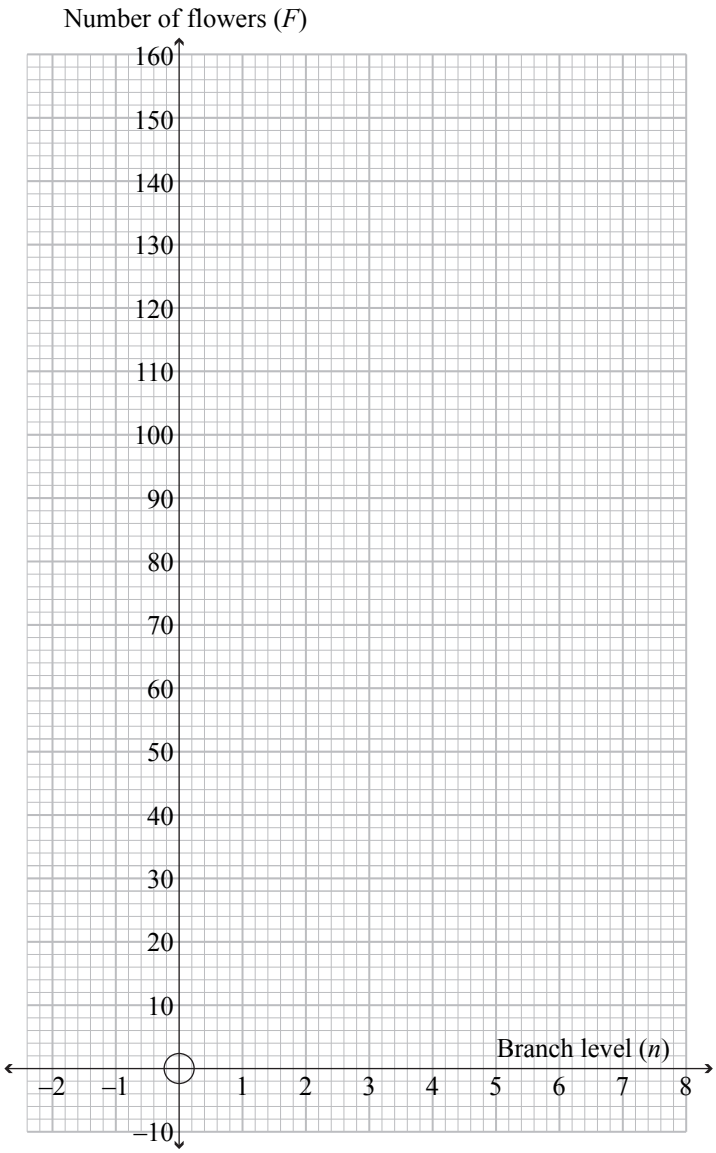
If you need to redraw your graphs for Question One (c), use the grid below. Make sure it is clear which answer you want marked.



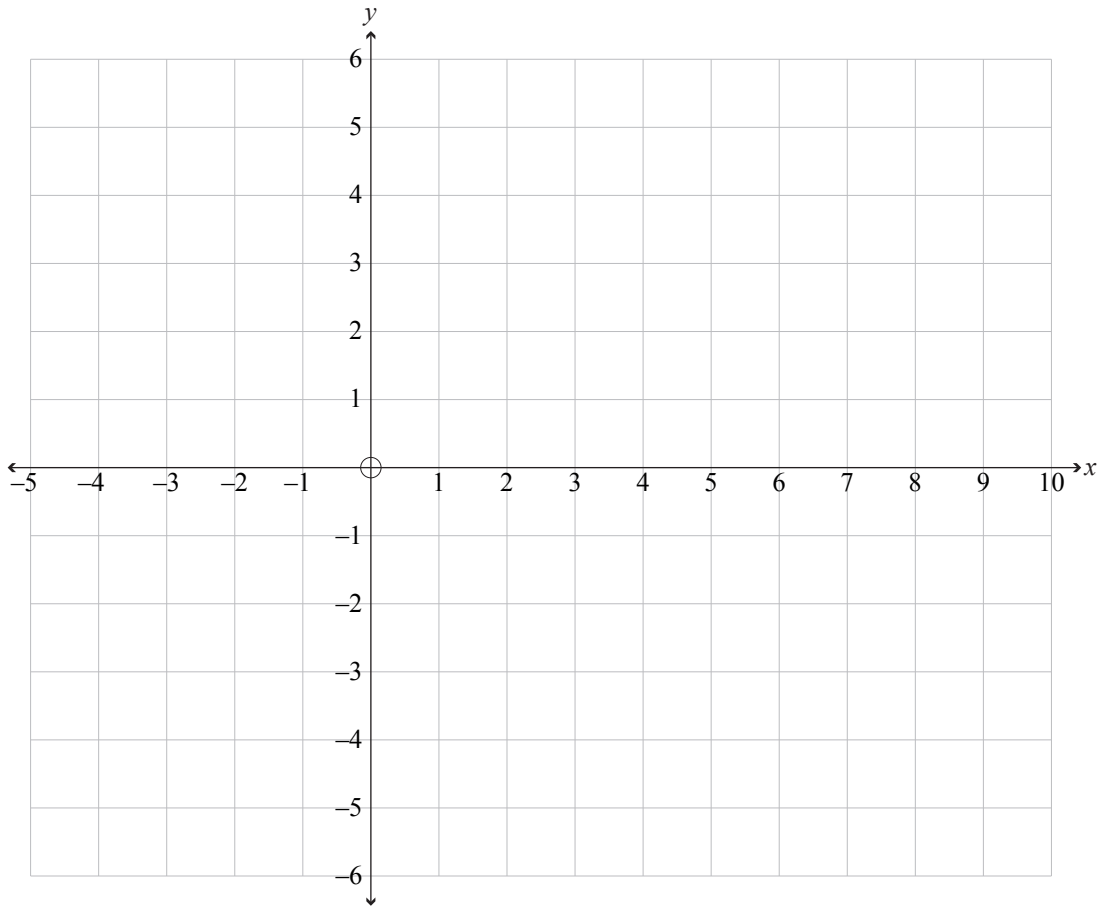
If you need to redraw your response to Question Two (b), use the diagram below. Make sure it is clear which answer you want marked.



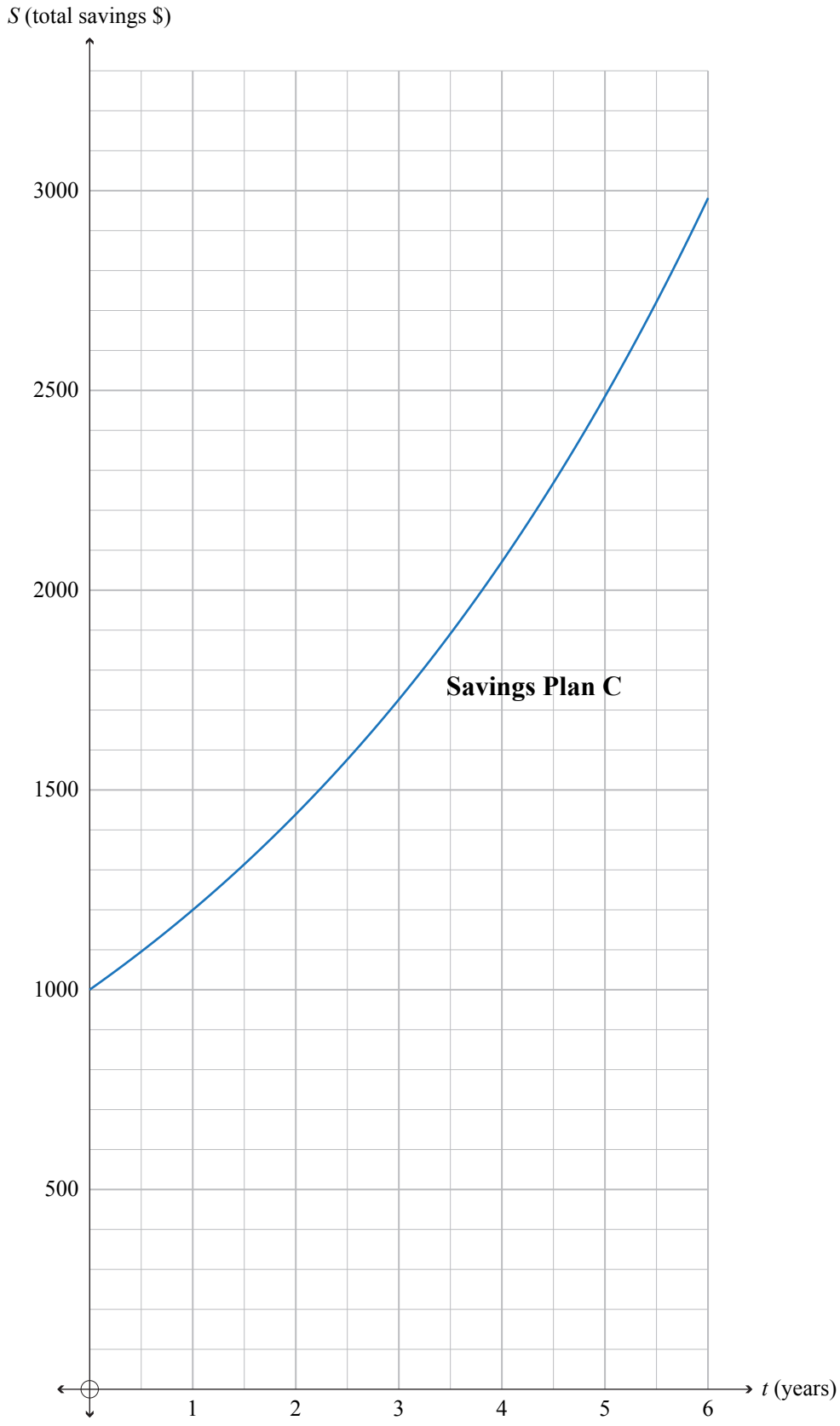
If you need to redraw your response to Question Two (c)(i), use the diagram below. Make sure it is clear which answer you want marked.



If you need to redraw your response to Question Three (a)(ii), use the diagram below. Make sure it is clear which answer you want marked.



If you need to redraw your response to Question Three (b)(ii), use the diagram below. Make sure it is clear which answer you want marked.



Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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Write the question number(s) if applicable.

QUESTION NUMBER

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

SUPERVISOR'S USE ONLY

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Draw a cross through the box (☒) if you have NOT written in this booklet

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New Zealand Qualifications Authority

Level 1 Mathematics and Statistics 2023

91031 Apply geometric reasoning in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply geometric reasoning in solving problems.	Apply geometric reasoning, using relational thinking, in solving problems.	Apply geometric reasoning, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

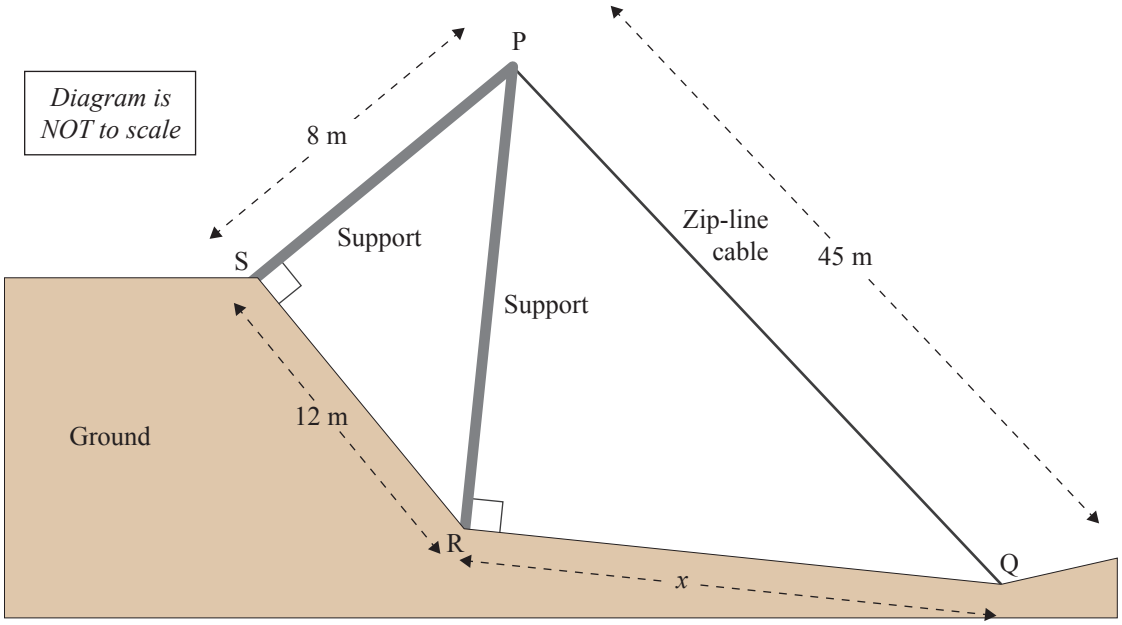
Do not write in any cross-hatched area (DO NOT WRITE IN THIS AREA DO NOT WRITE). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) A zip-line is being built at a theme park that will attract tourists to the area.
Towers and cables are being designed that will support the zip-line cable.
The diagram below represents one of the possible designs.

Angle PSR = angle PRQ = 90°, SR = 12 metres, PS = 8 metres, PQ = 45 metres.



Calculate the length, x, from R to Q.

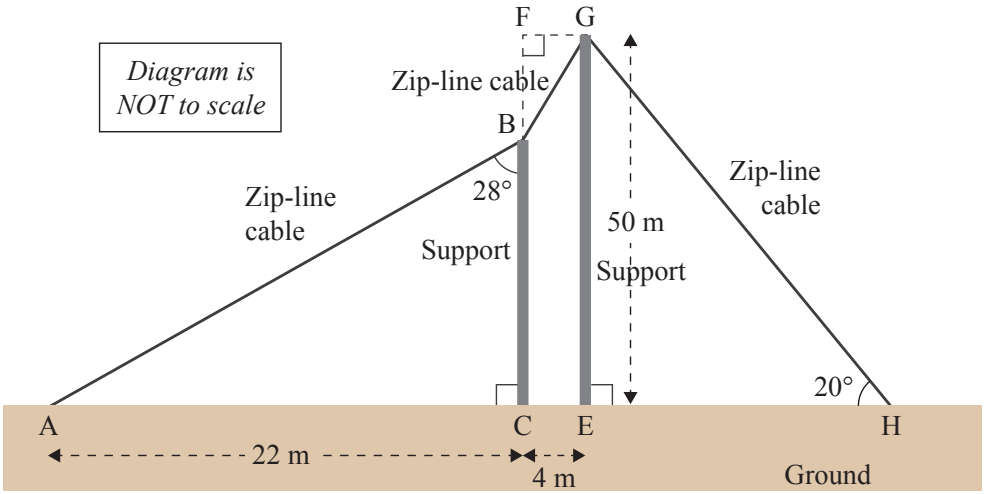
Show your working clearly.

(b) The diagram below represents another possible design for a zip-line.

The zip-line cable is attached joining A to B, then B to G, and then G to H.

Angle ABC = 28°, angle EHG = 20°, angle FCA = angle GEH = angle CFG = 90°.
AC = 22 metres, CE = 4 metres, EG = 50 metres. AH is on horizontal ground.

An advertisement claims "more than 200 metres of zip-line fun".



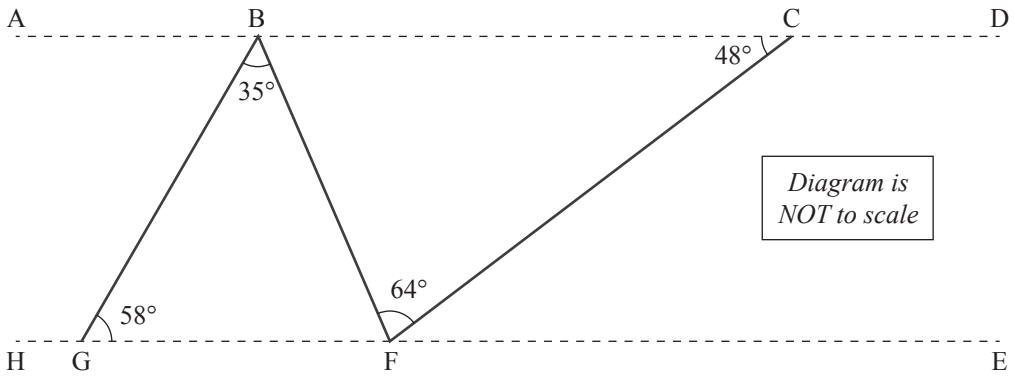
Show whether the advertisement's claim is true or false.

Show your working clearly.

(c) The diagram below represents another zip-line design.

The designers claim that the straight lines ABCD and HGFE are parallel to each other.

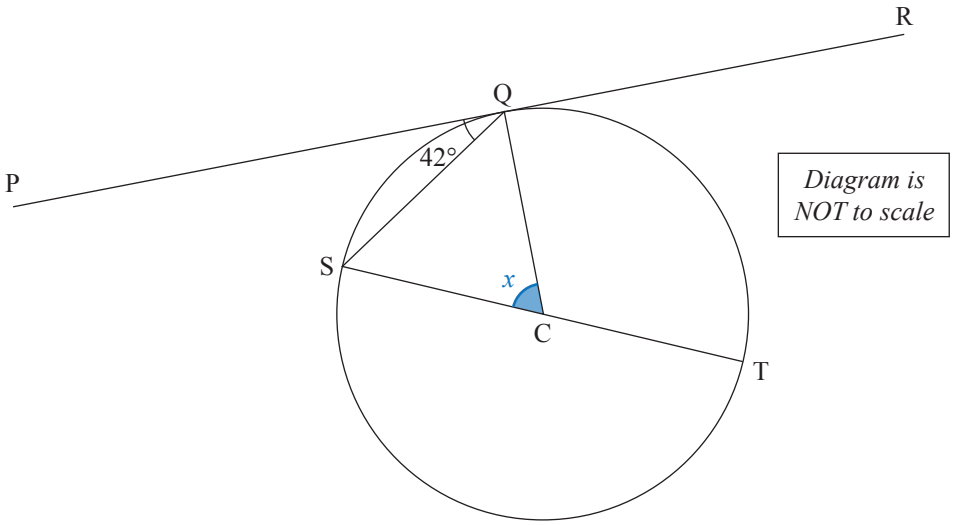
Angle GBF = 35°, angle BGF = 58°, angle BFC = 64°, angle BCF = 48°.



Show whether the designers' claim is true.

Justify your answer with clear geometric reasoning.

(d) The points Q, S, and T all lie on the circumference of a circle, with centre C.
The straight line PQR is a tangent to the circle at Q. SCT is a diameter of the circle.
Angle PQS = 42°.

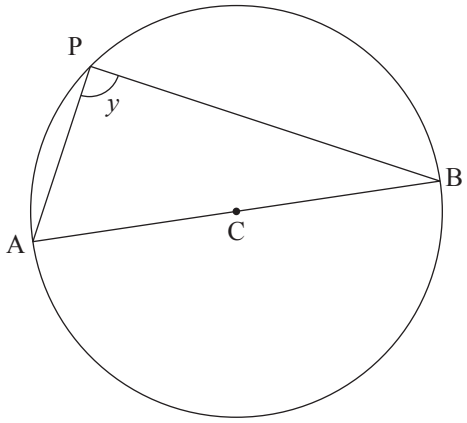


Find the size, x, of angle QCS.

Justify your answer.

(e) The points A, B, and P all lie on the circumference of a circle, centre C.

A student says that the size of angle APB, y, will always be 90°, whatever the size of the circle or the position of P on the circumference.



Prove that the student is correct.

Justify your answer with clear geometric reasoning.

QUESTION TWO

(a) The points P, Q, R, and S all lie on the circumference of a circle, centre C.
Angle PTQ = 110°, angle QSR = 40°.

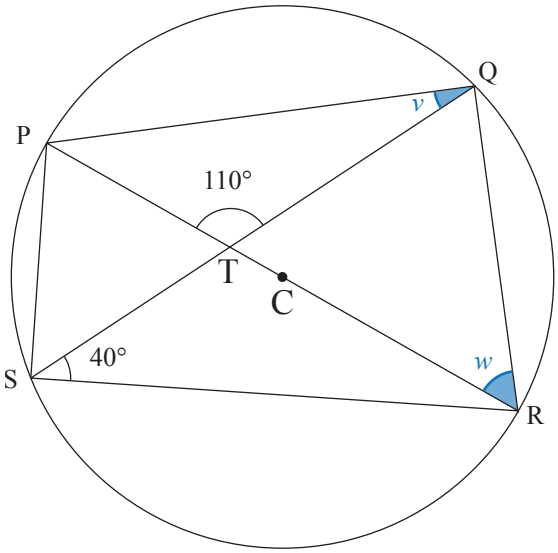


Diagram is NOT to scale

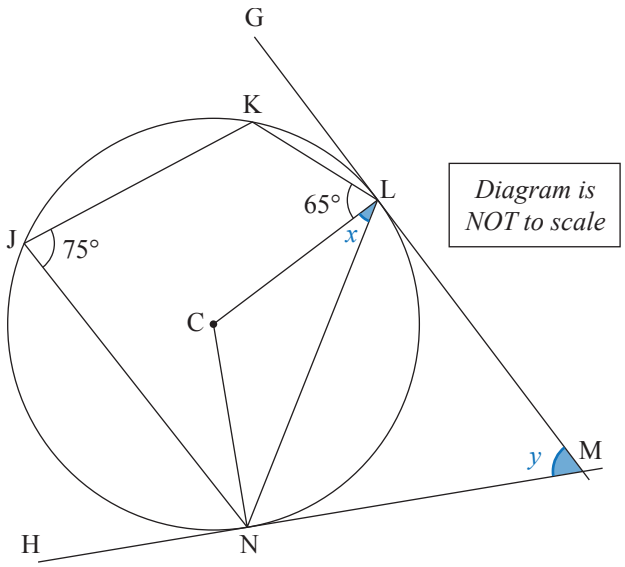
(i) Find the size, *v*, of angle PQS.

Justify your answer.

(ii) Find the size, *w*, of angle PRQ.

Justify your answer with clear geometric reasoning.

(b) The points J, K, L, and N all lie on the circumference of a circle, centre C.
Straight lines GLM and HNM are both tangents to the circle at L and N respectively.
Angle KLC = 65°, angle KJN = 75°.



(i) Show that the size, x, of angle CLN is 40°.

Show your working clearly.

(ii) Find the size, y, of angle LMN.

Justify your answer with clear geometric reasoning.

(c) Angle AEB = angle ADC = 90°. ABC and AED are both straight lines.
AB = 53 cm, BE = 45 cm, CD = p cm.

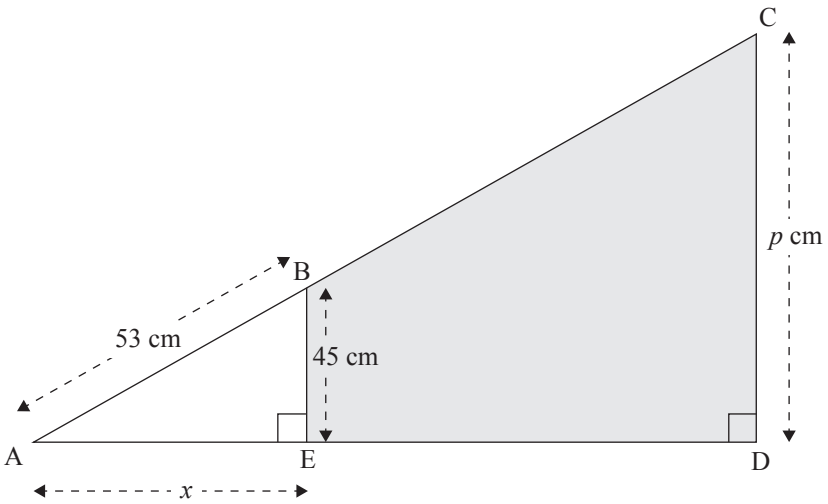


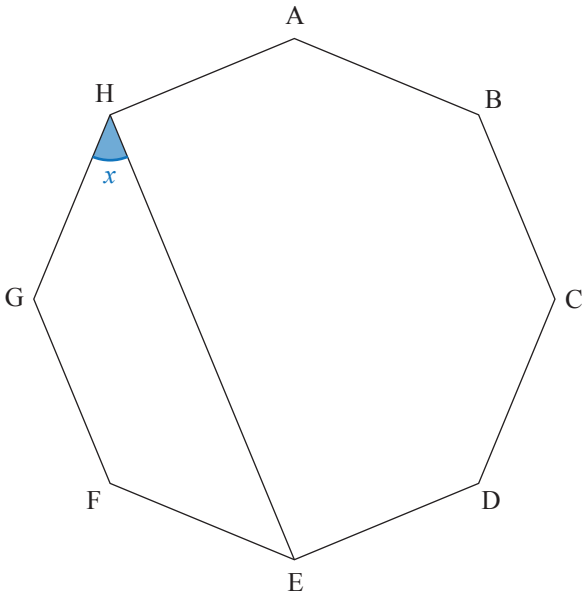
Diagram is NOT to scale

(i) Show that the distance, x, from A to E is 28 cm.
Show your working clearly.

(ii) Find the perimeter of the shaded quadrilateral BCDE, giving your answer in terms of p.
Justify your answer with clear geometric reasoning and working.
Show all appropriate working as a fraction or as a decimal correct to 4 decimal places.

QUESTION THREE

(a) The diagram below shows a regular octagon.



Find the size, x, of angle GHE.

Justify your answer.

(b) Angle QSR = 32°, angle SRP = 90°, PQ = 6.8 cm, SQ = 25 cm.

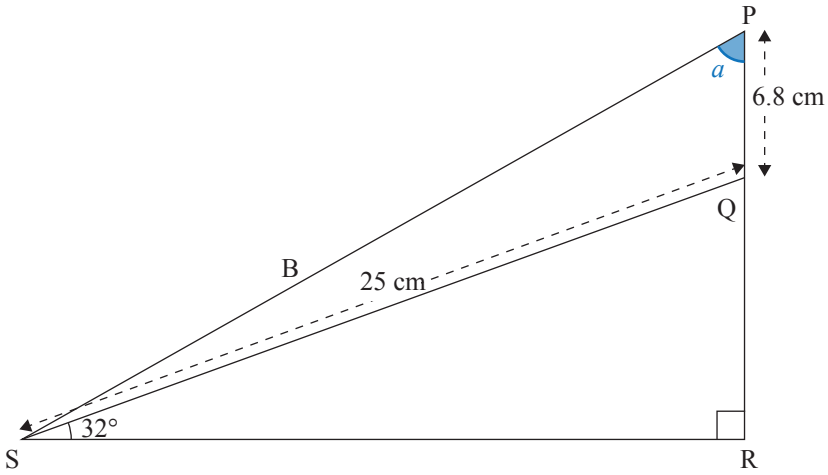


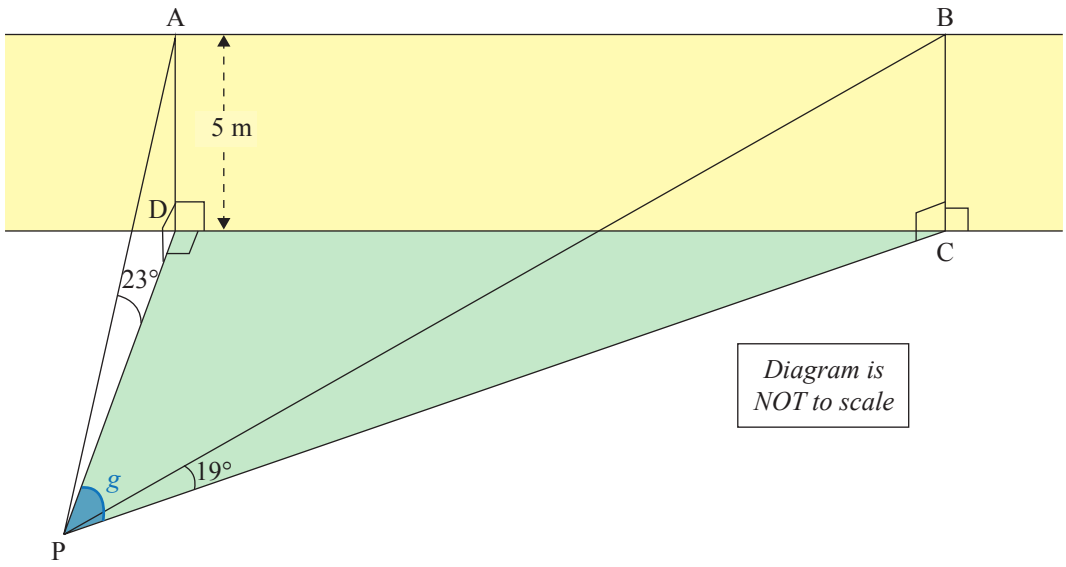
Diagram is NOT to scale

Find the size, *a*, of angle SPR.

Show your working clearly.

(c) The diagram below represents a vertical yellow wall, ABCD, built on horizontal flat ground, CDP.

Angle PDA = Angle PCB = Angle PDC = Angle ADC = Angle BCD = 90°.
Angle APD = 23°, Angle BPC = 19°. AD = 5 metres. The line AB is parallel to the line DC.



(i) Petra is stood directly in front of the wall, at the point P.

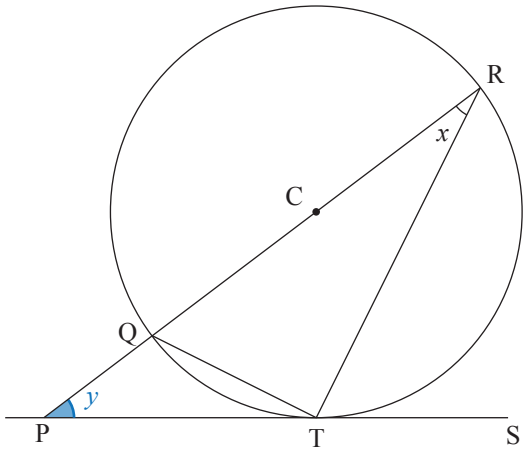
Calculate the shortest distance, PD, from Petra to the wall.

Show your working clearly.

(ii) Angle *g* is the angle on the ground between the lines PD and PC.

Find the size, *g*, of angle DPC.

(d) The points Q, R, and T all lie on the circumference of a circle, with centre C.
The straight line PTS is a tangent to the circle, at T. Angle QRT = x.



Find the size, y, of angle RPT, giving your answer in terms of x.

Justify your answer with clear geometric reasoning.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

SUPERVISOR'S USE ONLY

1

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Draw a cross through the box (☒) if you have NOT written in this booklet

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New Zealand Qualifications Authority

Level 1 Mathematics and Statistics 2023

91037 Demonstrate understanding of chance and data

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Demonstrate understanding of chance and data.	Demonstrate understanding of chance and data, justifying statements and findings.	Demonstrate understanding of chance and data, showing statistical insight.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

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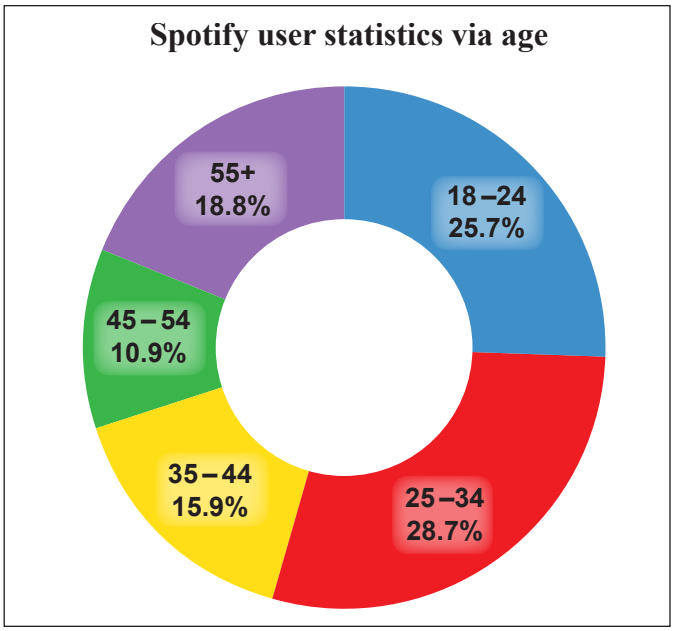
QUESTION ONE

Spotify is one of the largest online music streaming providers, which continues to grow in popularity around the world.

The diagram below shows the ages of the 587 million Spotify users in 2021.



(a) (i) What is the probability that a randomly selected Spotify user is aged 45 or over?

(ii) In terms of gender, 54% of Spotify users identify as male and 46% identify as female.

Assuming that these percentages are true for all age groups, what is the probability that a randomly selected Spotify user is a female aged between 25 and 34 years old?

Show clearly the calculations that give your probability value.

(iii) Niko conducted a survey among his family members and his work colleagues.

Niko surveyed 81 people and the results showed that 63 of them use Spotify.

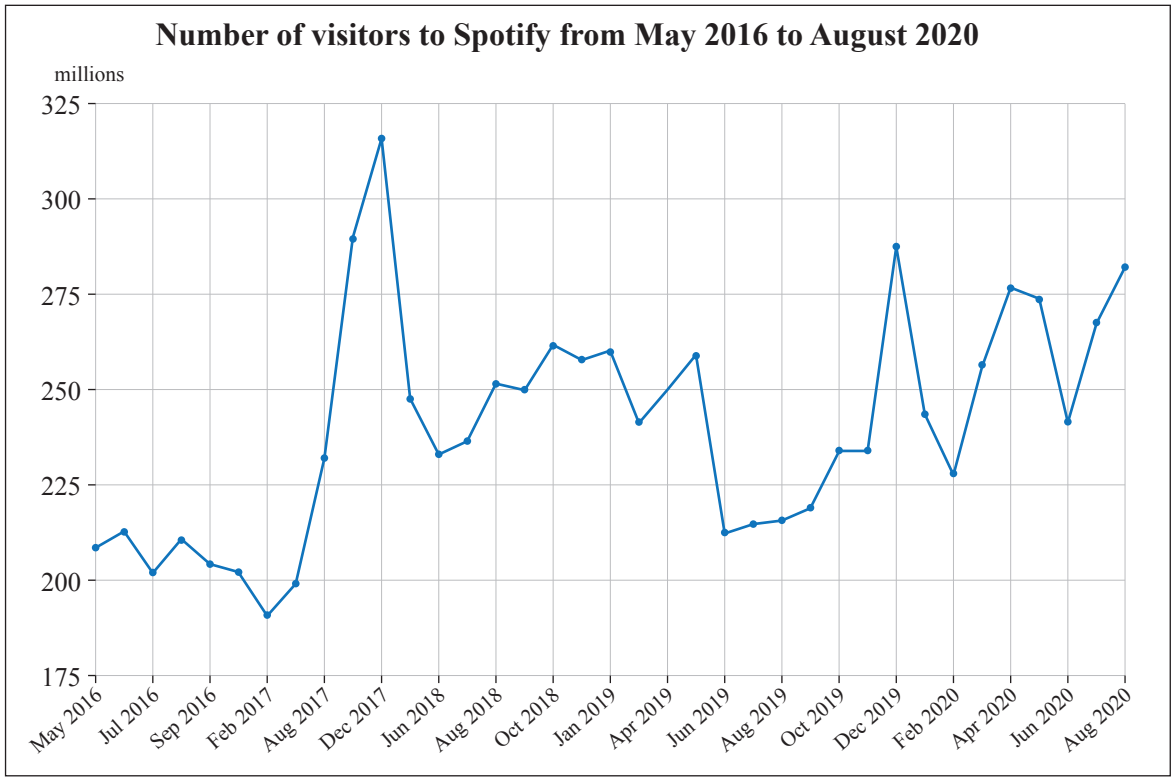
His results indicated that 8 Spotify users out of these 63 Spotify users were aged 55 or over.

Based on his results, and by comparing his results to the diagram on page 2, Niko claims that this result of 8 Spotify users in the age category of 55 years or over is nothing strange, and within what could be expected.

Comment on Niko's claim.

Justify your answer using statistical reasons.

(b) The graph below shows the number of users who visited Spotify from May 2016 through to August 2020.



Adapted from: https://www.statista.com/statistics/244989/number-of-unique-us-visitors-to-spotifycom/

(i) When was the least number of users visiting Spotify recorded in the time period shown in this graph?

Provide evidence from the graph to justify your answer.

(ii) Discuss and describe any trends and unusual features that you notice in the graph above.

Provide evidence from the graph to back up your statements.

Justify your answer using statistical reasons.

In your answer, describe at least THREE key features.

(iii) Margo studied the time series graph and decided that "this is a misleading graph".

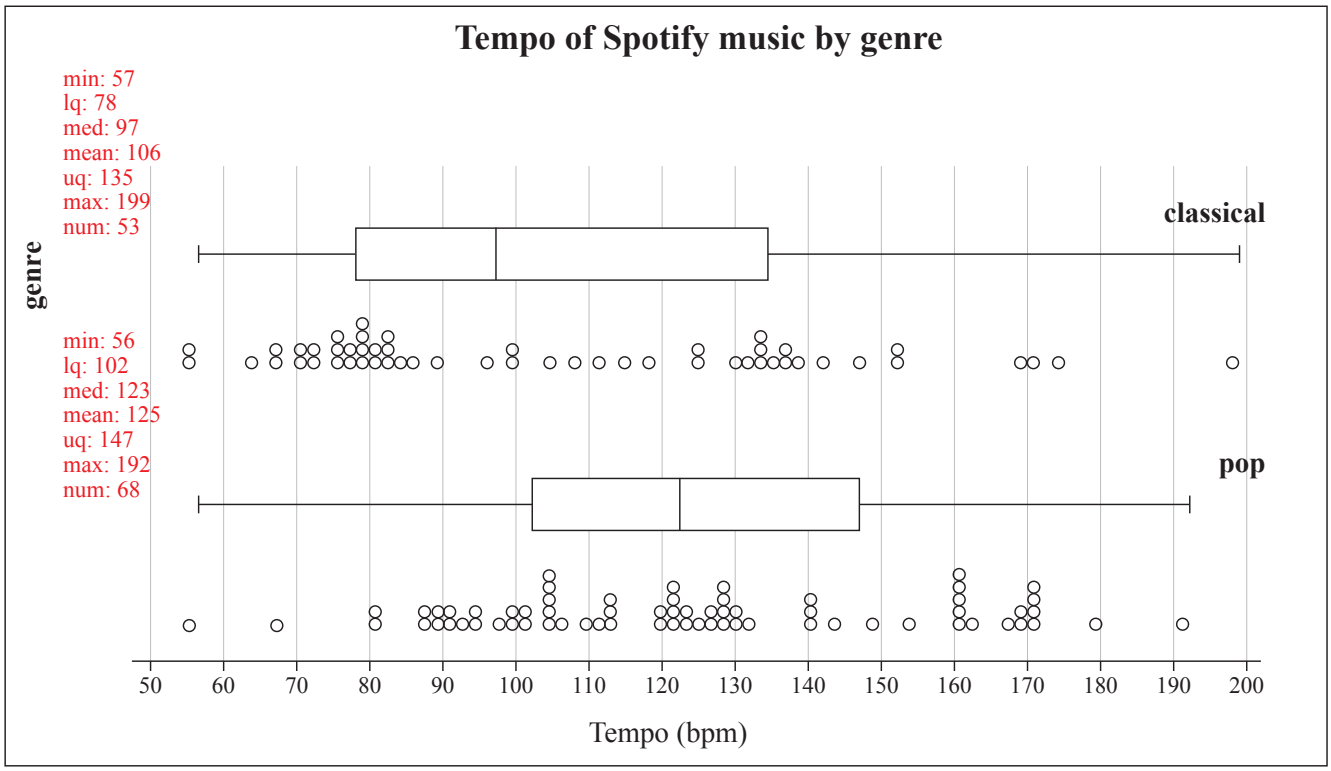
Do you agree?
Justify your answer using the evidence from the graph and statistical reasons.
Explain how any misleading aspects will have an effect.

QUESTION TWO

(a) In musical terminology, tempo is the speed or pace of the piece of music. It is measured as beats per minute (bpm).

Random samples of classical music and pop music are selected from the Spotify collection.

The graph below compares the tempo between the two genres of "classical" and "pop" music.



(i) If two pieces of pop music are randomly selected from the above sample, what is the probability that the tempos of both pieces of music are greater than 147 bpm?

(ii) Compare the distributions of the music tempo shown in the sample data opposite.
Note any similarities and differences considering centre, shift/overlap, shape, spread.
Provide numerical evidence where appropriate.
In your answer, describe at least THREE different key features.

(iii) Thom made a claim that it is not possible to decide that classical music tempo tends to be slower than pop music tempo for all music on Spotify.

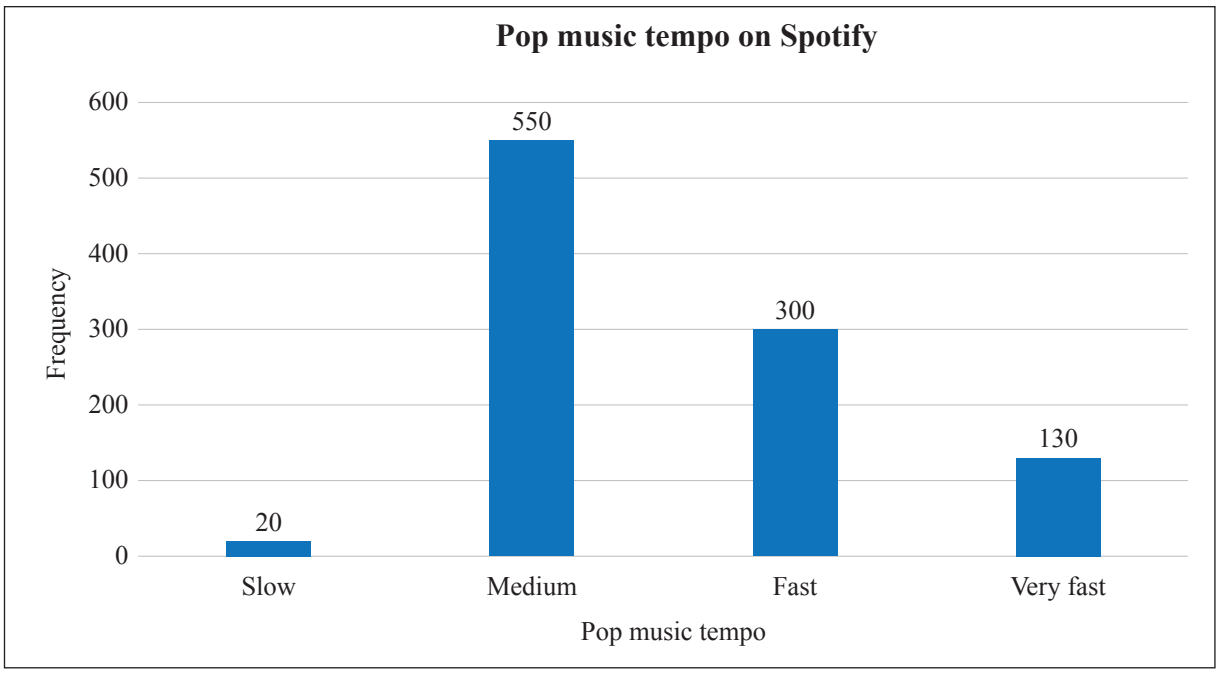
Comment on this claim, based on the sample of music tempo provided (shown in the diagram on page 6).

Justify your answer using statistical reasons, and include numerical evidence where appropriate.

Comment on how confident you are in your answer.

(b) There are four levels of pop music tempo. These are slow, medium, fast, and very fast.

A random sample of **1000 pop songs** selected from Spotify is shown in the graph below.



(i) If a piece of music is randomly selected from this sample, what is the probability that it is NOT a slow tempo or medium tempo piece of music?

(ii) Another sample of 200 pop songs is selected from Spotify.

Describe any similarities and differences that you would expect to see in this new sample compared to the random sample shown.

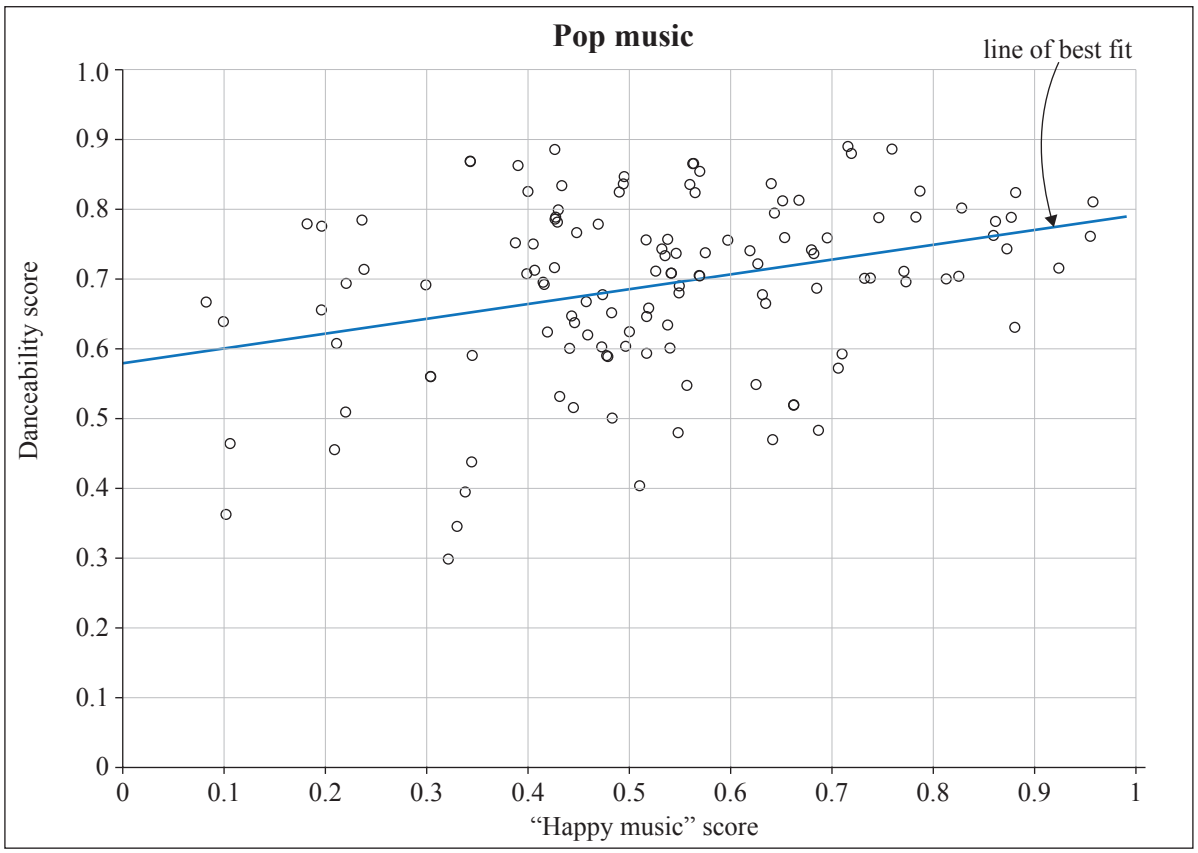
Justify your answer using statistical reasoning.

QUESTION THREE

(a) Most people would agree that dancing makes people feel happy. But does "Happy music" encourage people to dance?

A sample of 120 pieces of pop music was selected from Spotify to study the possible relationship between "Happy music" and "Danceability".

Variable	Description
Danceability	A piece of music is given a "Danceability" rating with a score between 0 and 1. This score gives a numerical value to how much the piece of music encourages the listener to dance. E.g. A "Danceability" rating with a low score, lowest of 0, would mean that the piece of music does not encourage the listener to dance. A "Danceability" rating with a high score, highest of 1, would mean that the piece of music is encouraging the listener to "get on their feet and dance".
"Happy Music"	A piece of music is given a "Happy music" rating with a score between 0 and 1. This score gives a numerical value to the feeling of "happiness" in the piece of music. E.g. A "Happy music" rating with a low score, lowest of 0, would mean that the piece of music is a very sad piece of music. A "Happy music" rating with a high score, highest of 1, would mean that the piece of music is cheerful and fun.



(i) If a piece of music has a "Happy music" score of 0.1, what "Danceability" score would you expect it to have?

Comment on how confident you feel with the accuracy of your answer, with justification.

(ii) Describe and interpret at least TWO different **features** of the relationship shown in the graph above of "Danceability" score versus "Happy music" score.

Justify your answer using statistical reasons.

(iii) Meremere believes that this scatter graph would be very useful to predict the "Danceability" score based on a piece of music's "Happy music" score.

Evaluate Meremere's claim using statistical reasoning, giving at least TWO justified statements, providing numerical evidence where appropriate.

Question Three continues on the next page.

(b) A survey was conducted among **500** randomly selected Spotify users.

They were asked how many hours they listened to Spotify in the past week.

The results are shown in the table below.

Spotify weekly usage	**Less than 2 hours**	**Between 2 and 5 hours**	**More than 5 hours**	**Total**
Free subscription	40	133	27	200
Premium subscription	21	187	92	300
Total	61	320	119	500

(i) One Spotify user was randomly selected from this sample.

What is the probability that it is a premium user who listens for more than 5 hours?

(ii) Based on the data from this survey, Meremere claimed that the free subscription users are more likely to spend less than 2 hours per week listening to Spotify than the premium subscription users.

Comment on Meremere's claim using the information in the table, and providing numerical evidence.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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QUESTION NUMBER

Assessment Schedule – 2023

Mathematics and Statistics: Apply algebraic procedures in solving problems (91027A)

Day 1

Candidates must show algebraic working.
Equivalent methods of solving problems are accepted on condition that the candidate is demonstrating algebraic solutions/thinking at curriculum level 6.
Once a student has made an error, credit can be given from subsequent working which is at curriculum level 6.

Q	Expected Coverage	Grade (generated by correctly demonstrating the procedures listed in EN4) Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent.
ONE (a)	$d = 2\times(-3)^2 - 9\times(2\times-3-2)+5$ $d = 2\times 9 - 9\times -8+5$ $d = 18+72+5$ $d = 95$	For award of u: • correct solution. Accept C.A.O.
(b)	$= \dfrac{(4x-1)(x+1)(x-1)}{(x+1)(x-1)(4x-1)}$ $= 1$	For award of u: ONE of: • numerator or denominator factorised • consistent simplification from their factorisation.
		For award of r: ONE of: • expression fully simplified • numerator and denominator both **expanded** correctly and then cancelled down to 1.

Assessment Schedule – 2023

Mathematics and Statistics: Apply algebraic procedures in solving problems (91027B)

Day 2

Candidates must show algebraic working.
Equivalent methods of solving problems are accepted on condition that the candidate is demonstrating algebraic solutions/thinking at curriculum level 6.
Once a student has made an error, credit can be given from subsequent working which is at curriculum level 6.

Q	Expected Coverage	Grade (generated by correctly demonstrating the procedures listed in EN4) Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent.
ONE (a)	$42x^2+11x-3=(7x+3)(6x-1)$ i.e. $y=6x-1$ Allow C.A.O. Accept $y=\frac{42x^2+11x-3}{7x+3}$	For award of u: • stating that $y = 6x - 1$ or $y = \frac{42x^2+11x-3}{7x+3}$
(b)	$(8x+2)(2x-1)=(4x+6)(4x-3)$ $16x^2-8x+4x-2=16x^2-12x+24x-18$ $16x^2 - 4x - 2 = 16x^2 + 12x - 18$ #1 $-4x-2=12x-18$ $-2+18=12x+4x$ $16=16x$ $x=1$	For award of u: • expansion and simplification of a pair of brackets (LHS or RHS), forming a quadratic expression with three terms #1 For award of r: • equation solved to find $x = 1$.
(c)	$\frac{3(3x+2)-4(4x-1)}{12}\geq 2$ $\frac{9x+6-16x+4}{12}\geq 2$ $\frac{-7x+10}{12} \geq 2$ #1 $-7x+10\geq 24$ $-7x\geq 14$ $x\leq\frac{14}{-7}$ $x\leq -2$ Accept $x \leq \frac{-14}{7}$ or $x \leq -2$ or $-2 \geq x$ as the final answer.	For award of u: ONE of: • correct arrangement for both numerator and denominator (does not need to be expanded or simplified). Accept $9x + 6 - 16x - 4$ for the numerator • consistent solution found (with either $\leq$, $\geq$ or = sign). For award of r: ONE of: • correct linear inequation at #1 • equation solved to find $x = -2$ • inequation solved to find $x \geq -2$ • consistently reverses inequality sign due to mult/div of a negative number. For award of t: • inequation solved to find $x \leq -2$.

<table>
<tr>
<td>(d)</td>
<td>$\frac{5g}{6} = \frac{h(g+4)}{5}$
$25g = 6h(g+4)$ #1
$25g = 6hg + 24h$
$25g - 6hg = 24h$ #2
$g(25-6h) = 24h$ #3
$g = \frac{24h}{25-6h}$ #4
Accept other equivalent solutions.</td>
<td>For award of u:
ONE of:
• cross-multiply #1
• consistently collecting terms involving g and terms not involving g on different sides of the equation #2
• consistently factorising the pair of terms involving g #3
• consistently rearranging by dividing by the bracket #4.

For award of r:
TWO of:
• cross-multiply #1
• consistently collecting terms involving g and terms not involving g on different sides of the equation #2
• consistently factorising the pair of terms involving g #3
• consistently rearranging by dividing by the bracket #4.
OR
• correctly states h in terms of g.

For award of t:
• correct rearrangement.</td>
</tr>
<tr>
<td>(e)</td>
<td>$(4x-1)(4x+3) - x(x+2) = 6$
$16x^2 + 12x - 4x - 3 - x^2 - 2x - 6 = 0$
$15x^2 + 6x - 9 = 0$
$5x^2 + 2x - 3 = 0$
$(5x-3)(x+1) = 0$
Either $5x - 3 = 0$
$x = \frac{3}{5}$
OR
$x + 1 = 0$
$x = -1$ Ignore as not appropriate.
Units not required.</td>
<td>For award of u:
ONE of:
• forming correct expression $(4x - 1)(4x + 3)$ simplified to $16x^2 + 8x - 3$
• forming correct equation for the shaded area
• consistent simplification to a quadratic equation in three terms.

For award of r:
ONE of:
• simplification to a quadratic equation in three terms
• consistent solving of their quadratic equation, with evidence of negative value disregarded.

For award of t:
• correct positive solution found for the question, with evidence of negative value disregarded.</td>
</tr>
</table>

Q	Expected Coverage	Grade (generated by correctly demonstrating the procedures listed in EN4) Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent.
TWO (a)	$w = 3 \times (-3)^2 - 8 \times (3 \times -3 - 2) + 6$ $w = 3 \times 9 - 8 \times -11 + 6$ $w = 27 + 88 + 6$ $w = 121$	For award of u: • correct solution. Accept C.A.O.
(b)	$= \frac{(5x+1)(x-2)(x+2)}{(x+2)(x-2)(5x+1)}$ $= 1$	For award of u: ONE of: • numerator or denominator factorised • consistent simplification from their factorisation.
		For award of r: ONE of: • expression fully simplified • numerator and denominator both **expanded** correctly and then cancelled down to 1.
(c)	$18x^2 + 24x = (3x+4)^2$ $18x^2 + 24x = 9x^2 + 24x + 16$ $9x^2 = 16$ $x^2 = \frac{16}{9}$ $x = \pm\frac{4}{3}$ Accept $x = \pm\sqrt{\frac{16}{9}}$ OR Alternative method: $18x^2 + 24x = (3x+4)^2$ $18x^2 + 24x = 9x^2 + 24x + 16$ $9x^2 - 16 = 0$ $(3x-4)(3x+4) = 0$ $x = \pm\frac{4}{3}$ Allow answer in any form.	For award of u: ONE of: • expansion and simplification of RHS • consistently solves, giving both solutions
		For award of r: • correctly solves for both solutions.

(d)	Let F be the first part of the journey and S be the second part of the journey. $F + S = 1200$ $\Rightarrow S = 1200 - F$ $\frac{1}{4}F = \frac{1}{6}S$ $\Rightarrow 6F = 4S$ $6F = 4(1200 - F)$ $6F = 4800 - 4F$ $10F = 4800$ $F = 480$ Then $S = 1200 - 480 = 720$ km OR Alternative Method: Let x be the distance covered in the first part of the journey; then $1200 - x$ is the distance remaining. $\frac{x}{4} = \frac{1200 - x}{6}$ $6x = 4(1200 - x)$ $6x = 4800 - 4x$ $10x = 4800$ $x = 480$ Remaining distance is $1200 - 480 = 720$ km Units not needed. Allow alternative algebraic methods.	For award of u: ONE of: • forms the equation $\frac{1}{4}F = \frac{1}{6}S$ or $6F = 4S$ • consistent combining of their equations in one variable. For award of r: ONE of: • combining of the equations in one variable • consistent distances found for both parts of the journey. For award of t: • correct distances found for both parts of the journey.

(e)

$\left[(2n+3)^2-3(2n+3)+6\right]-\left[(2n+1)^2-3(2n+1)+6\right]$

$=\left[4n^2+12n+9-6n-9+6\right]-\left[4n^2+4n+1-6n-3+6\right]$

$=\left[4n^2+6n+6\right]-\left[4n^2-2n+4\right]$

$=8n+2$ #1

$=2(4n+1)$

This expression has a factor of 2, it is divisible by 2.

OR

Alternative method:

Assume n is odd – not required to be stated.

$\left[(n+2)^2-3(n+2)+6\right]-\left[n^2-3n+6\right]$

$=\left[n^2+4n+4-3n-6+6\right]-\left[n^2-3n+6\right]$

$=\left[n^2+n+4\right]-\left[n^2-3n+6\right]$

$=4n-2$

$=2(2n-1)$

This expression has a factor of 2, it is divisible by 2.

OR

Alternative method :

Assume n is even – not required to be stated.

$\left[(n+3)^2-3(n+3)+6\right]-\left[(n+1)^2-3(n+1)+6\right]$

$=\left[n^2+6n+9-3n-9+6\right]-\left[n^2+2n+1-3n-3+6\right]$

$=\left[n^2+3n+6\right]-\left[n^2-n+4\right]$

$=4n+2$

$=2(2n+1)$

This expression has a factor of 2, it is divisible by 2.

Accept any order of the differences considered.

Allow alternative algebraic methods.

OR

Allow other algebraic methods used by considering other consecutive odd terms, e.g. $2n-1$ and $2n+1$.

For award of u:

ONE of:

- correct expression for the difference between two successive odd-numbered patterns
- consistent expansion and simplification of the square shaped brackets
- consistently simplifies to an expression without any brackets (equivalent to #1)
- Any valid numerical working (using one example) with a clear justification.

For award of r:

ONE of:

- correctly simplifies to an expression without any brackets #1
- consistently uses algebraic reasoning and explanation to justify clearly that the result is divisible by 2
- Any valid numerical working (using two or more examples) with a clear justification.

For award of t:

- uses algebraic reasoning and explanation to justify clearly that the result is divisible by 2.

Q	Expected Coverage	Grade (generated by correctly demonstrating the procedures listed in EN4) Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent.
THREE (a)	$\text{Perimeter} = 8(3x+2) = 88$ $24x + 16 = 88$ $24x = 72$ $x = \frac{72}{24} = 3$ OR Alternative method: $\text{Perimeter} = 8(3x+2) = 88$ $3x + 2 = 11$ $3x = 9$ $x = \frac{9}{3} = 3$ Allow solution as an unsimplified fraction.	For award of u: • correct solution for the value of x Accept C.A.O.
(b)	$\frac{4x}{4x-3} = \frac{x+6}{x+3}$ $4x(x+3) = (x+6)(4x-3)$ $4x^2 + 12x = 4x^2 - 3x + 24x - 18$ $12x = 21x - 18$ $18 = 9x$ $x = 2$ OR Alternative method: $\frac{4x(x+3) - (x+6)(4x-3)}{(4x-3)(x+3)} = 0$ $\frac{4x^2+12x-(4x^2+21x-18)}{(4x-3)(x+2)} = 0$ $4x^2 + 12x - 4x^2 - 21x + 18 = 0$ $-9x + 18 = 0$ $x = 2$	For award of u: ONE of: • correct arrangement for both numerator and denominator (does not need to be expanded or simplified). Accept $4x^2 + 12x - 4x^2 + 21x - 18$ for the numerator • consistent solution found. For award of r: • correct value for x found.

(c)	$\frac{1}{2} \times (2x + 4)(x + 6) = 32$ $(x + 2)(x + 6) = 32$ $x^2 + 8x + 12 = 32$ $x^2 + 8x - 20 = 0$ $(x + 10)(x - 2) = 0$ Either $x = -10$ Ignore as not appropriate Or $x = 2$ OR Alternative method: $\frac{1}{2} \times (2x+4)(x+6) = 32$ $(2x+4)(x+6) = 64$ $2x^2 + 12x + 4x + 24 - 64 = 0$ $2x^2 + 16x - 40 = 0$ $x^2 + 8x - 20 = 0$ $(x+10)(x-2) = 0$ Either $x = -10$ Ignore as not appropriate. Or $x = 2$	For award of u: ONE of: • form and simplify a quadratic expression for the area of the triangle, with or without the 32 being used • consistent solution found, with evidence of the invalid value disregarded. For award of r: • correct solution found, with evidence of the invalid value disregarded.

<table>
<tr>
<td>(d)</td>
<td>$2^{5x-2} \times (2^2)^{x+2} = (2^4)^x$
$2^{5x-2} \times 2^{2x+4} = 2^{4x}$
$2^{5x-2+2x+4} = 2^{4x}$ #1
$2^{7x+2} = 2^{4x}$
$7x + 2 = 4x$ #2
$3x = -2$
$x = -\frac{2}{3}$</td>
<td>For award of u:
ONE of:
• recognition of powers of 2 on both sides
• LHS or RHS correct at stage #1.

For award of r:
ONE of :
• forming the linear equation #2
• consistently forming an equation and solving for their x-value.

For award of t:
• correct solution found.</td>
</tr>
<tr>
<td>(e)</td>
<td>Let T be the number of 20 kg sacks.
Let F be the number of 50 kg sacks.
$T + F = 60$
$20T + 50F = 1500$

$T + F = 60$
$2T + 5F = 150$

$2T + 2F = 120$
$2T + 5F = 150$
Subtracting gives:
$3F = 30$
$F = 10$ and $T = 50$

Weight of 20 kg sacks will be 50 × 20 = 1000 kg

Weight of 50 kg sacks will be 10 × 50 = 500 kg

or alternative method:

$20(60 - F) + 50F = 1500$
$F = 10$
$T = 50$

Candidate could use x and y as the variables.
Allow alternative algebraic methods.</td>
<td>For award of u:
ONE of:
• forms both equations
• consistent combining of their equations into one variable.

For award of r:
ONE of:
• consistent number of either 20 kg sacks of 50 kg sacks
• correct value for either 20 kg sacks or 50 kg sacks
• correct combining of the equations into one variable
• consistent value for both 20 kg sacks and 50 kg sacks.

For award of t:
ONE of:
• correct number of both 20 kg sacks and 50 kg sacks
• correct value for both 20 kg sacks and 50 kg sacks.</td>
</tr>
</table>

Assessment Schedule – 2023

Mathematics and Statistics: Investigate relationships between tables, equations and graphs (91028)

Evidence

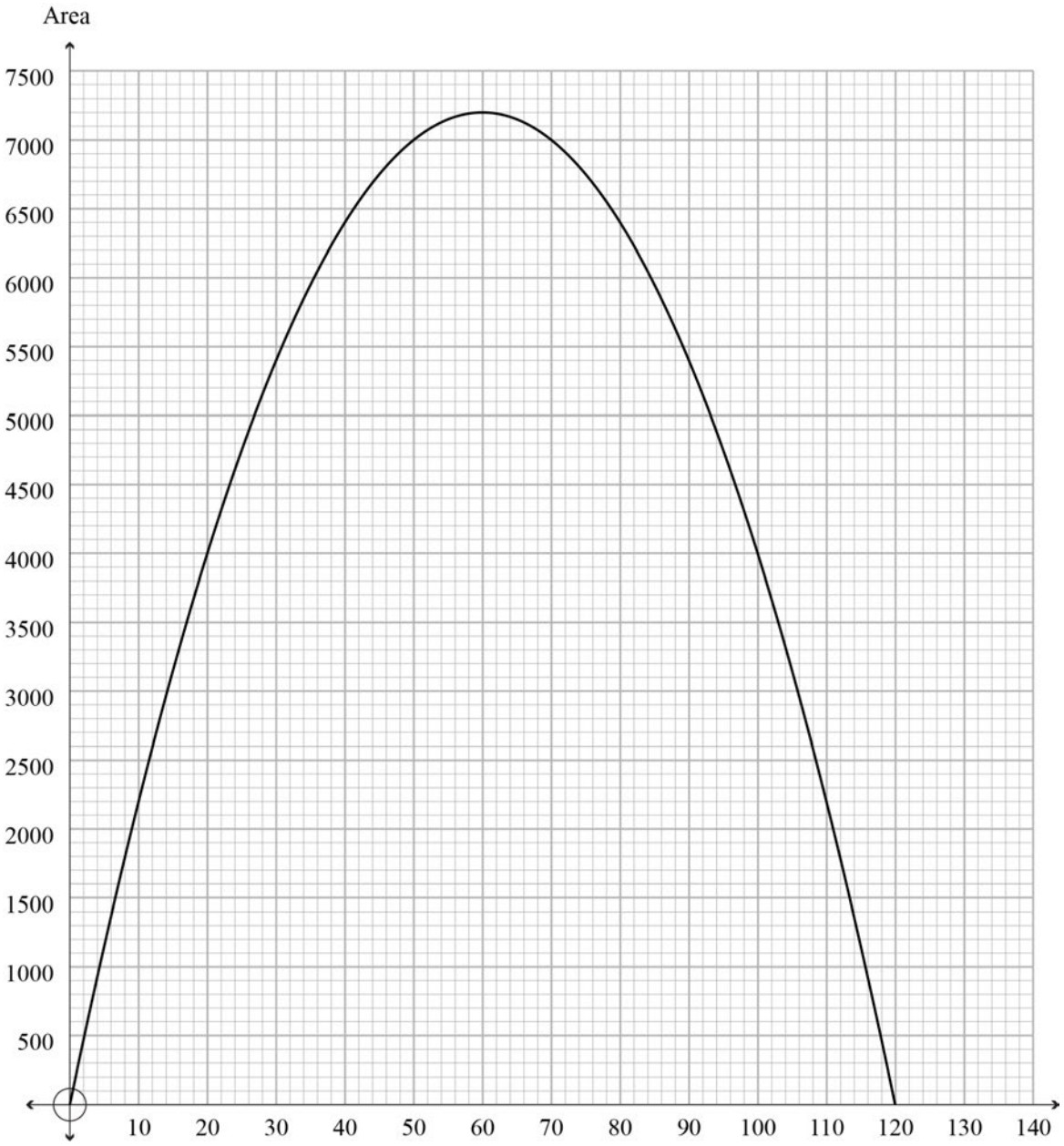
Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
ONE (a)	$y = \frac{-5}{2}x + 15$ Allow alternative forms. Allow C.A.O.	• Correct equation.		
(ii)	$y = \frac{5}{2}x + 15 + 10$ $y = \frac{5}{2}x + 25$ Allow alternative forms. Allow C.A.O.	• Correct equation after one of the transformations. OR Both transformations correct from wrong equation in (a)(i).	• Correct equation after both of the transformations.	
(b)(i)	$H = k(x-60)^2 + 36$ $x = 0, y = 45$ gives $k = \frac{9}{3600} = \frac{1}{400} = 0.0025$ i.e. $H = \frac{1}{400}(x-60)^2 + 36$ OR Alternative formats: $H = \frac{1}{400}x(x-120) + 45$ OR $H = \frac{1}{400}x^2 - \frac{3}{10}x + 45$	• Equation given but with no k-value considered. OR Attempt made to find the value of k in a correct set up of the equation. OR C.A.O.	• Correct equation for H, including full and clear working.	
(ii)	Possible changes are : The whole graph could be shifted downwards. This would represent shifting downwards where the chain fixes onto the post. This would be shown in the equation by reducing the size of the constant at the end. e.g. $H = \frac{1}{400}(x-60)^2 + 20$ This would lower the chain totally by 16 cm. OR other examples where the chain is lowered.	• Valid suggestion of how the equation should be changed. OR Example of equation of new design.	• Valid suggestion of how the equation should be changed with an example equation AND Description of the minimum point of the chain fence, in context .	

(c)	Total perimeter (3 sides). $2x + y = 240$ $y = 240 - 2x$ Area $= x(240 - 2x)$ Allow other versions of this equation, e.g. $y = -2(x - 60)^2 + 7200$. (Allow any correct equation which starts with x + 2y = 240) e.g. $y = -\frac{1}{2}(x - 120)^2 + 7200$ Table produced of the relationship between the two sides of the grassed space and their area with at least 5 correct values. Graph produced relating length of one side and area. Evidence of the use of tables, equations, and graphs to model the area of the grassed space as the lengths of the sides change. Sample comments: • Maximum area is 7200 m^2. • Maximum area is when x = 60 m and y = 120 m. • Graph and area size is symmetrical. • Minimum area is 0 cm^2 (theoretically). • Rate of increase of the area changes for different x-values. • The graph will be a continuous one, as all different x-values are possible, if measurements are taken accurately. • In reality, some of the x-values close to 0 or close to 120 are likely to be inappropriate for the council to design their grassed area with these dimensions.	• Forming equation for area in terms of only one variable. OR Table only with one non-trivial comment. OR Graph only with ONE non-trivial comment. OR Finding maximum area only. OR Table and graph drawn with no comments.	• Evidence of only two aspects of tables, equations, and graphs. AND TWO non-trivial comments.	**E7 / T1** Evidence of table of values. AND Graph drawn. AND Formula for area provided. BUT Only maximum area discussed. OR As evidence for E8 but graph is discrete or of poor quality. **E8 / T2** Evidence of table of values. AND Graph drawn. AND Formula for area provided. AND At least three valid non-trivial comments.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE question attempted towards solution.	1u	2u	3u	2r	3r	1t	2t

Question One

x	$y = 240 - 2x$	$A = x(240 - 2x)$
0	240	0
20	200	4000
40	160	6400
60	120	7200
80	80	6400
100	40	4000
120	0	0

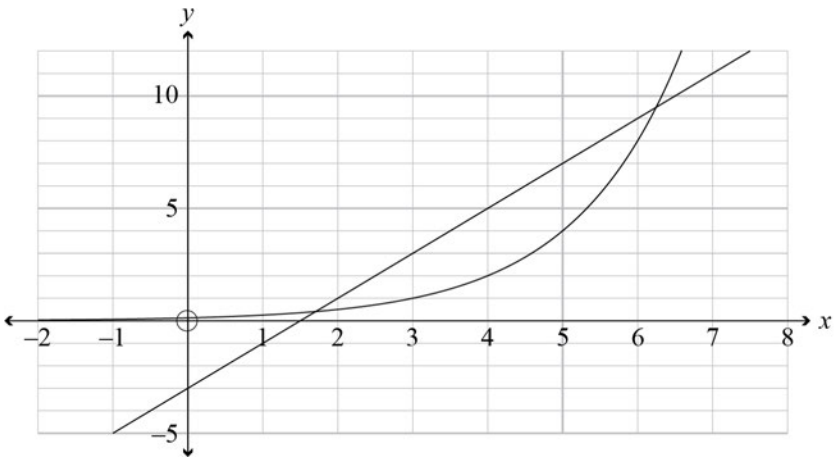

Area
7500
7000
6500
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
10
20
30
40
50
60
70
80
90
100
110
120
130
140

Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
TWO (a)	$y = -(x-3)^2 + 6$ OR $y = -x^2 + 6x - 3$ OR $y = -(x-1)(x-5) + 2$ Allow other equivalent solutions.	• Correct equation.		
(b)	Draw the graph $y = 2^{x-3}$ Draw the graph $y = 2x - 3$ Intersection at $x = 1.7$ and $x = 6.25$ Allow margin of error in the accuracy, consistent with the graph drawing. Allow an error of ±0.2.	• Drawing the graph of $y = 2^{x-3}$ OR Consistent solutions from minor error. OR Only one solution provided. OR CAO	• Two values of x found from the intersection of the two graphs. • Graph must show both intersection points	
(c)(i)	Graph drawn, as discrete points, for $0 < x \le 8$ Allow for minor error in graph	• Graph drawn, as a continuous graph. Do not penalise negative values included.	• Graph drawn as discrete points for $0 \le x \le 8$ OR $0 < x \le 8$	E7 / T1 Correct discrete graph drawn for $0 \le x \le 8$ OR $0 < x \le 8$ AND Correct justified equation, but not with an appropriate domain. OR Correct justified equation, with appropriate domain BUT with a continuous graph E8 / T2 Correct discrete graph drawn for $0 \le x \le 8$ OR $0 < x \le 8$ AND Correct justified equation, with appropriate domain
(ii)	Equation found, with some justification, e.g. second difference of +4 indicated in the table. $F = 2n^2 + 4n$ OR $F = 2n(n+2)$ OR $F = 2(n+1)^2 - 2$ AND Domain for $1 \le x \le 8$ (with integer points) Also allow for x > 0 or equivalent Allow equation given in terms of y and x.	• Recognition that the equation is a quadratic with the coefficient of x^2 as 2. OR Table of first and second differences shown and indication that the equation is a quadratic. OR C.A.O.	• Correct equation, with some valid justification.	
(iii)	Exponential equation of $F = 4^{n-1}$. OR $F = 0.25 \times 4^n$ Allow other equivalent solutions. Allow C.A.O.	• Equation identified as an exponential, with base of 4.	• Correct equation of $F = 4^{n-1}$.	

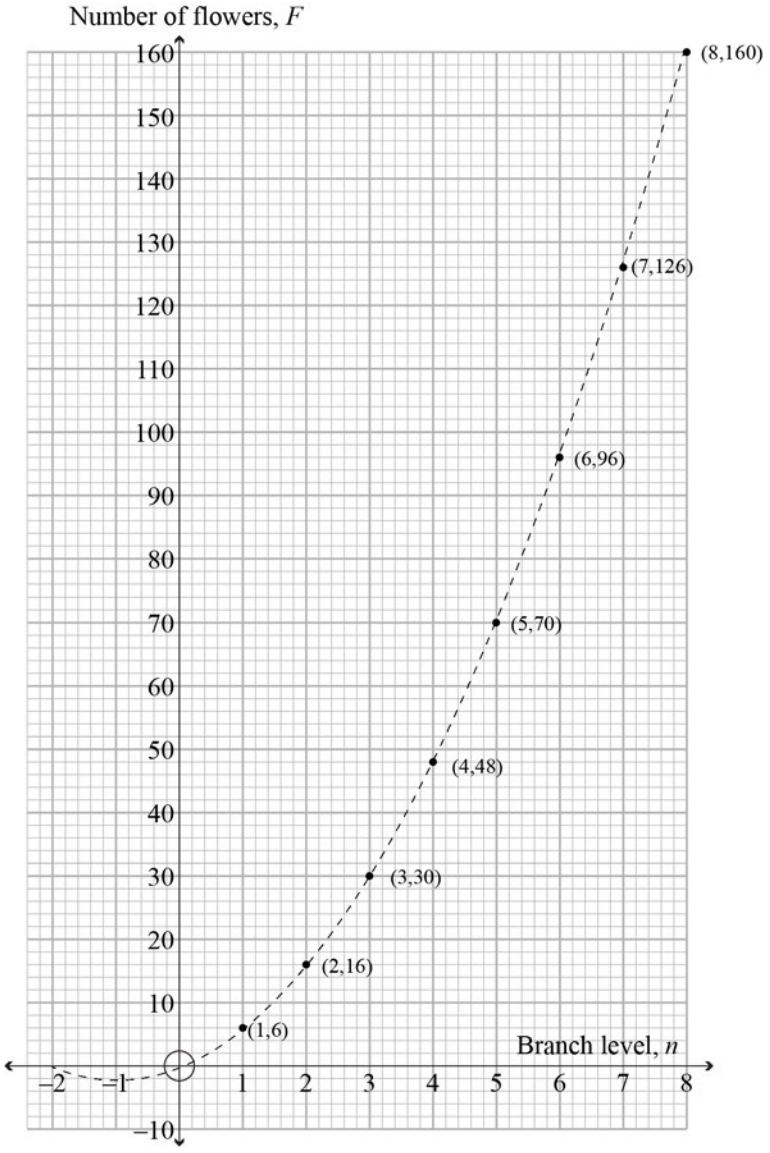
(iv)	Sample comments regarding the false claim are: • No, it is only one set of results from one tree, so cannot be generalised to claim that this will always occur. • No, these results are from only one tree in one particular place, so cannot be generalised to all trees in NZ. • No, different growing conditions in different locations will lead to different results and hence a different formula. • No, the results are only for up to branch 8 flowers. Other-sized trees may not follow the same pattern. • No, trees may be diseased, which would affect the number of flowers on the branches. Allow other valid reasons.	• Recognising that the claim is false. AND With ONE valid comment.		

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE question attempted towards solution.	1u	2u	3u	2r	3r	1t	2t

Question Two (b)



Question Two (c)(i)

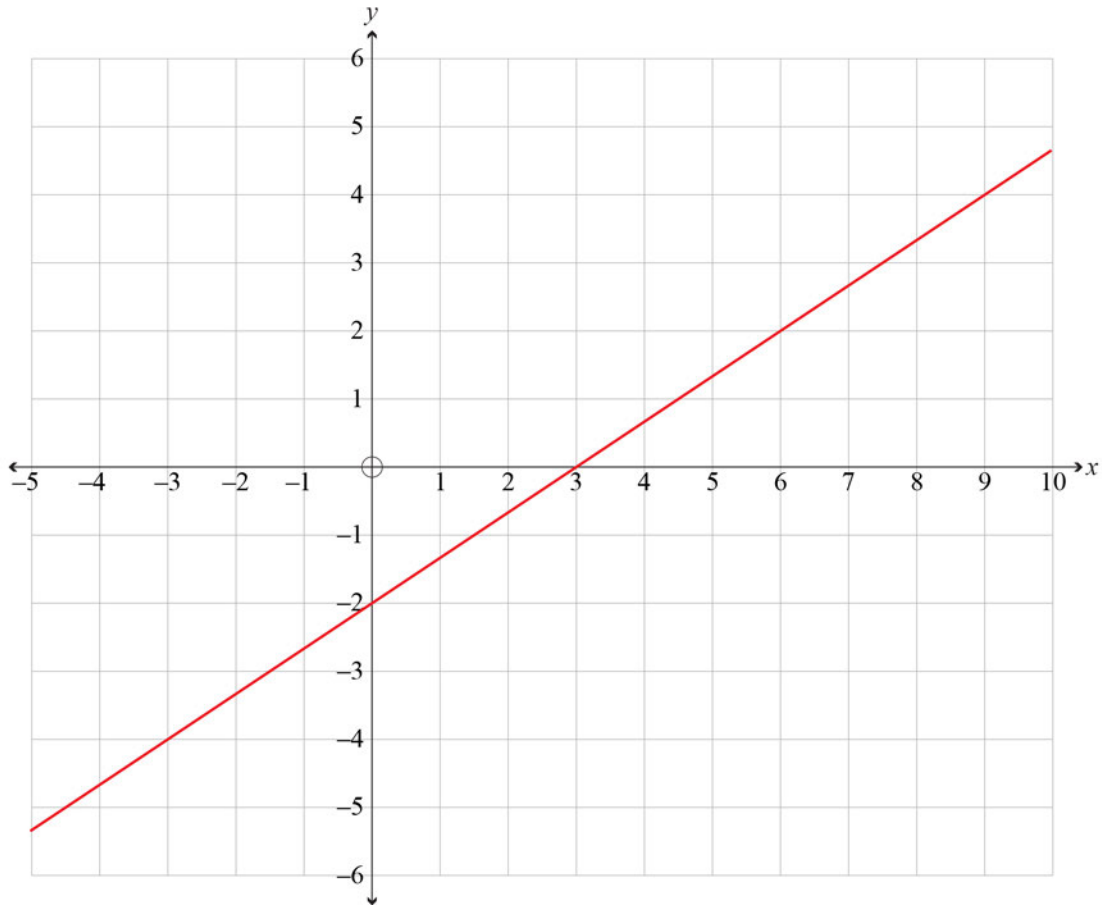


Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
THREE (a)(i)	Correct equation of $y = 2^{x+2} - 4$ OR equivalent.	• Equation identified as an exponential, with base of 2.	• Correct equation.	
(ii)	Straight line graph drawn, showing x-axis intercept at (3,0) AND y-axis intercept at (0,–2).	• Straight line with only one axis-intercept correct.	• Accurate graph drawn.	
(b)(i)	Correct equation of $y = 1000 \times 1.2^x$ OR $S = 1000 \times 1.2^t$	• Included 1000 in the exponential equation. OR Recognised that the base is 1.2.	• Correct equation.	

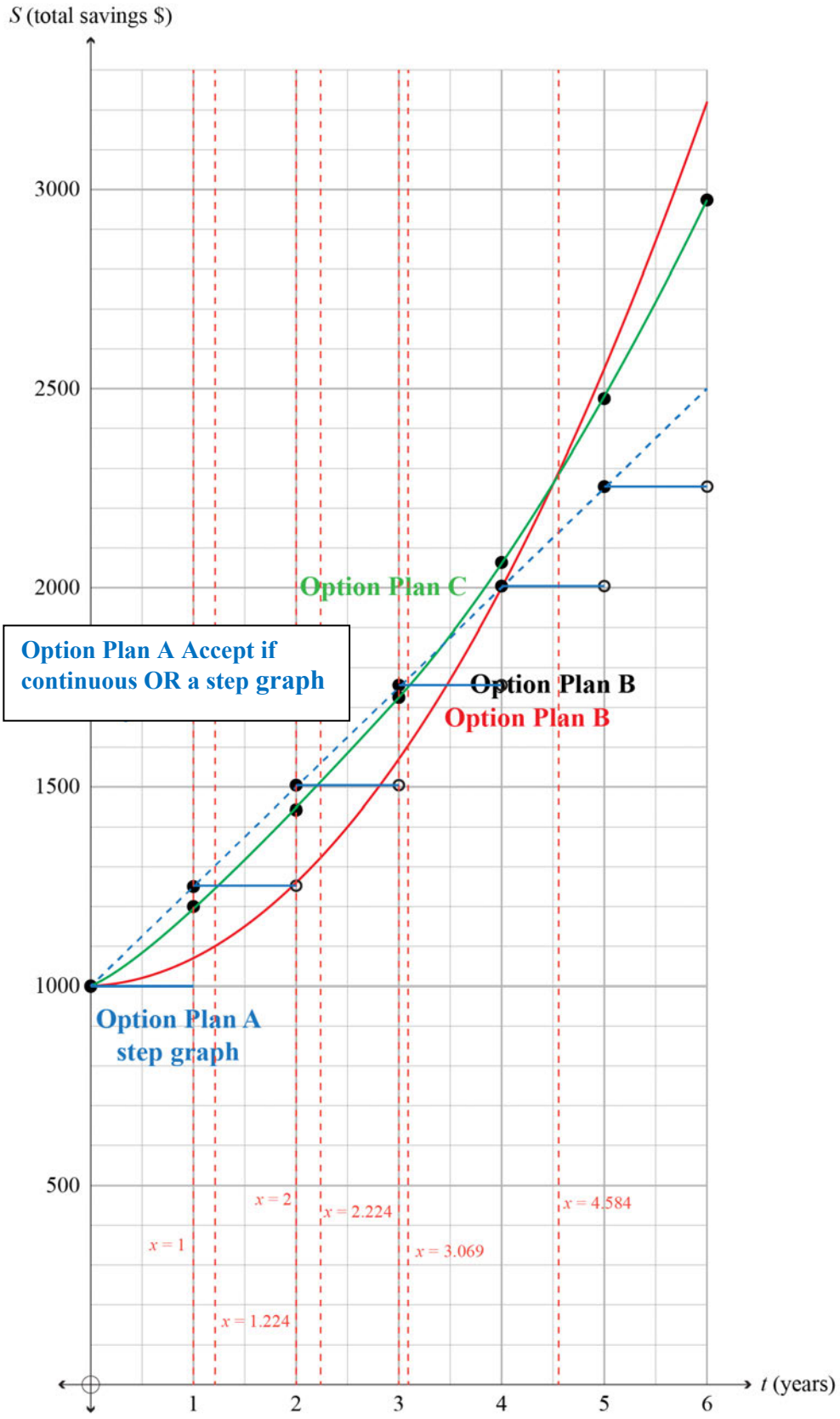
(ii)	Graph drawn for Savings Plan A.	• Continuous graph drawn of $y = 250x + 1000$		**E7/T1** Savings Plan A drawn as a continuous graph OR a correct step-graph AND Equation for savings plan C AND Savings Plan B drawn accurately. AND At least two valid comparisons made before and after the intercept point of $t = 4.6$ years. **E8 / T2** As for E7 AND At least three valid comparisons of the options made both before and after the intercept point of $t = 4.6$ years.
	Graph drawn for Savings Plan B Parabola continuous graph.	• Parabola graph drawn, showing (0,1000) and at least THREE other values drawn, but lacking accuracy.	• Parabola graph drawn correctly	
(iii)	Table of values for Savings Plan A and Savings Plan B produced. Valid comparisons made between the various Option Plans, including the evidence of dates, at least. Examples of possible comparison comments are: • Generally, Savings Plan C will be the best if $t < 4.6$ years (approximately). • Generally, Savings Plan B will be the best if $t > 4.6$ years (approximately). • As the years increase, so Savings Plan B will become better and better compared to the other savings plans. • Occasionally, but for only short time periods, Savings Plan A is the best, just after 1 year and just after 2 years. • Savings Plan A is generally the weakest choice. Other non-trivial valid comparisons acceptable.	• Table for Savings Plan A OR Savings Plan B correct AND ONE valid, non-trivial comparison made.	• Savings Plan A AND Savings Plan B correct in Table AND TWO valid non-trivial comparisons made before and after $t = 4.6$ years. (Can use whole number of years)	

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE question attempted towards solution.	1u	2u	3u	2r	3r	1t	2t

Question Three (a)(ii)



Question Three (b)(ii)



Question Three (b)(iii)

End of Year (t)	Total Savings Option Plan A	Total Savings Option Plan B	Total Savings Option Plan C
0	1000	1000	1000
1	1250	1070	1200
2	1500	1260	1440
3	1750	1570	1728
4	2000	2000	2073.60
5	2250	2550	2488.32
6	2500	3220	2985.98

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 13	14 – 18	19 – 24

Assessment Schedule – 2023

Mathematics and Statistics: Investigate relationships between tables, equations and graphs (91028)

Evidence

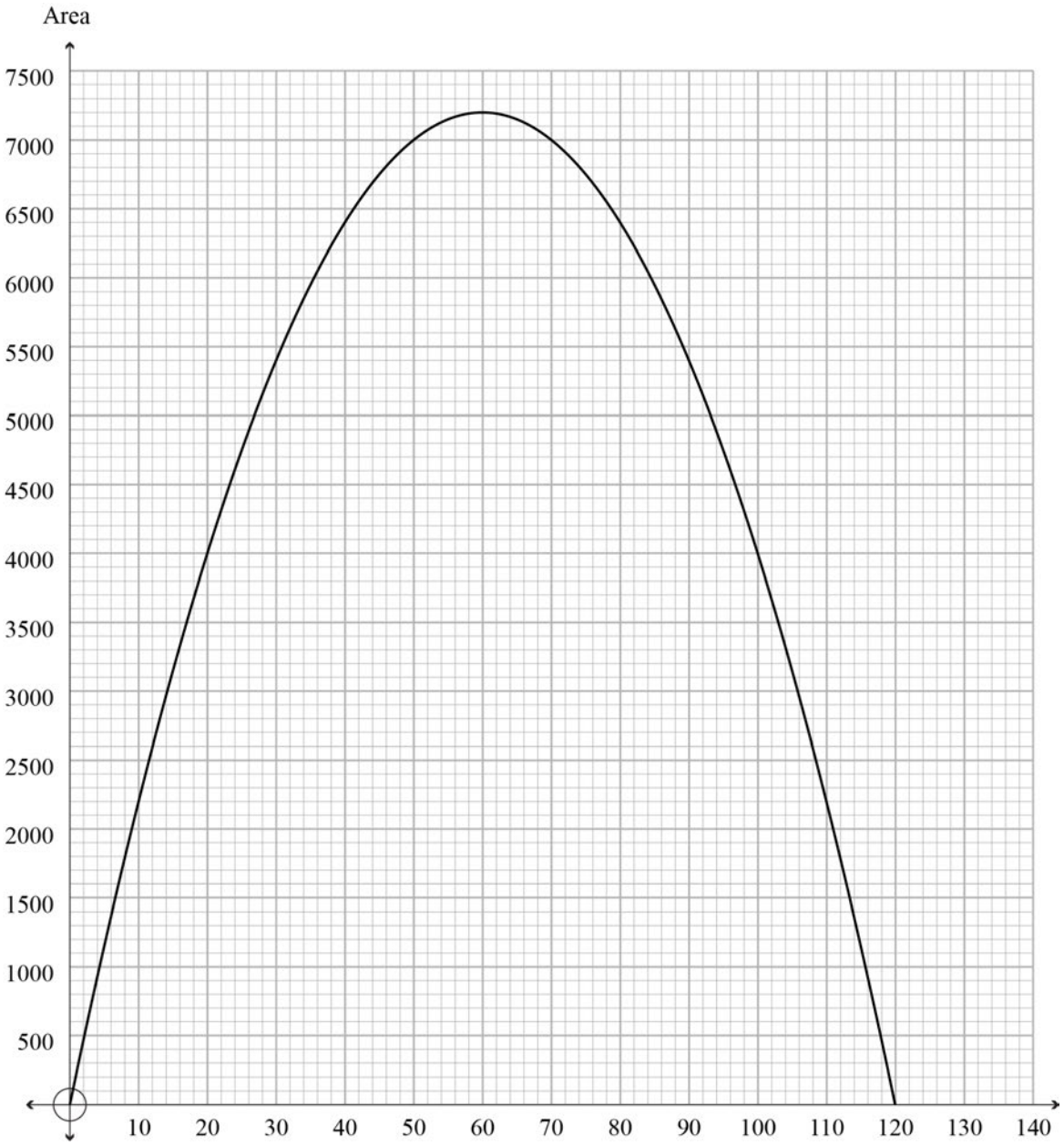
Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
ONE (a)	$y = \frac{-5}{2}x + 15$ Allow alternative forms. Allow C.A.O.	• Correct equation.		
(ii)	$y = \frac{5}{2}x + 15 + 10$ $y = \frac{5}{2}x + 25$ Allow alternative forms. Allow C.A.O.	• Correct equation after one of the transformations. OR Both transformations correct from wrong equation in (a)(i).	• Correct equation after both of the transformations.	
(b)(i)	$H = k(x-60)^2 + 36$ $x = 0, y = 45$ gives $k = \frac{9}{3600} = \frac{1}{400} = 0.0025$ i.e. $H = \frac{1}{400}(x-60)^2 + 36$ OR Alternative formats: $H = \frac{1}{400}x(x-120) + 45$ OR $H = \frac{1}{400}x^2 - \frac{3}{10}x + 45$	• Equation given but with no k-value considered. OR Attempt made to find the value of k in a correct set up of the equation. OR C.A.O.	• Correct equation for H, including full and clear working.	
(ii)	Possible changes are : The whole graph could be shifted downwards. This would represent shifting downwards where the chain fixes onto the post. This would be shown in the equation by reducing the size of the constant at the end. e.g. $H = \frac{1}{400}(x-60)^2 + 20$ This would lower the chain totally by 16 cm. OR other examples where the chain is lowered.	• Valid suggestion of how the equation should be changed. OR Example of equation of new design.	• Valid suggestion of how the equation should be changed with an example equation AND Description of the minimum point of the chain fence, in context .	

(c)	Total perimeter (3 sides). $2x + y = 240$ $y = 240 - 2x$ Area $= x(240 - 2x)$ Allow other versions of this equation, e.g. $y = -2(x - 60)^2 + 7200$. (Allow any correct equation which starts with x + 2y = 240) e.g. $y = -\frac{1}{2}(x - 120)^2 + 7200$ Table produced of the relationship between the two sides of the grassed space and their area with at least 5 correct values. Graph produced relating length of one side and area. Evidence of the use of tables, equations, and graphs to model the area of the grassed space as the lengths of the sides change. Sample comments: • Maximum area is 7200 m^2. • Maximum area is when x = 60 m and y = 120 m. • Graph and area size is symmetrical. • Minimum area is 0 cm^2 (theoretically). • Rate of increase of the area changes for different x-values. • The graph will be a continuous one, as all different x-values are possible, if measurements are taken accurately. • In reality, some of the x-values close to 0 or close to 120 are likely to be inappropriate for the council to design their grassed area with these dimensions.	• Forming equation for area in terms of only one variable. OR Table only with one non-trivial comment. OR Graph only with ONE non-trivial comment. OR Finding maximum area only. OR Table and graph drawn with no comments.	• Evidence of only two aspects of tables, equations, and graphs. AND TWO non-trivial comments.	**E7 / T1** Evidence of table of values. AND Graph drawn. AND Formula for area provided. BUT Only maximum area discussed. OR As evidence for E8 but graph is discrete or of poor quality. **E8 / T2** Evidence of table of values. AND Graph drawn. AND Formula for area provided. AND At least three valid non-trivial comments.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE question attempted towards solution.	1u	2u	3u	2r	3r	1t	2t

Question One

x	$y = 240 - 2x$	$A = x(240 - 2x)$
0	240	0
20	200	4000
40	160	6400
60	120	7200
80	80	6400
100	40	4000
120	0	0

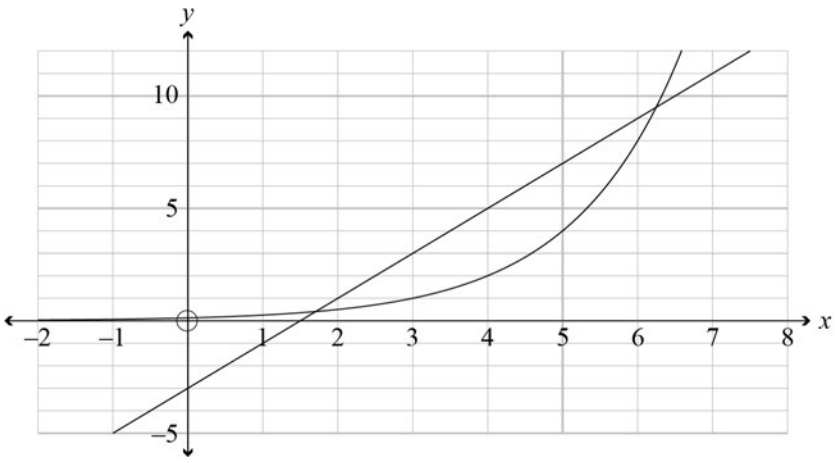

Area
7500
7000
6500
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
10
20
30
40
50
60
70
80
90
100
110
120
130
140

Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
TWO (a)	$y = -(x-3)^2 + 6$ OR $y = -x^2 + 6x - 3$ OR $y = -(x-1)(x-5) + 2$ Allow other equivalent solutions.	• Correct equation.		
(b)	Draw the graph $y = 2^{x-3}$ Draw the graph $y = 2x - 3$ Intersection at $x = 1.7$ and $x = 6.25$ Allow margin of error in the accuracy, consistent with the graph drawing. Allow an error of ±0.2.	• Drawing the graph of $y = 2^{x-3}$ OR Consistent solutions from minor error. OR Only one solution provided. OR CAO	• Two values of x found from the intersection of the two graphs. • Graph must show both intersection points	
(c)(i)	Graph drawn, as discrete points, for $0 < x \le 8$ Allow for minor error in graph	• Graph drawn, as a continuous graph. Do not penalise negative values included.	• Graph drawn as discrete points for $0 \le x \le 8$ OR $0 < x \le 8$	E7 / T1 Correct discrete graph drawn for $0 \le x \le 8$ OR $0 < x \le 8$ AND Correct justified equation, but not with an appropriate domain. OR Correct justified equation, with appropriate domain BUT with a continuous graph E8 / T2 Correct discrete graph drawn for $0 \le x \le 8$ OR $0 < x \le 8$ AND Correct justified equation, with appropriate domain
(ii)	Equation found, with some justification, e.g. second difference of +4 indicated in the table. $F = 2n^2 + 4n$ OR $F = 2n(n+2)$ OR $F = 2(n+1)^2 - 2$ AND Domain for $1 \le x \le 8$ (with integer points) Also allow for x > 0 or equivalent Allow equation given in terms of y and x.	• Recognition that the equation is a quadratic with the coefficient of x^2 as 2. OR Table of first and second differences shown and indication that the equation is a quadratic. OR C.A.O.	• Correct equation, with some valid justification.	
(iii)	Exponential equation of $F = 4^{n-1}$. OR $F = 0.25 \times 4^n$ Allow other equivalent solutions. Allow C.A.O.	• Equation identified as an exponential, with base of 4.	• Correct equation of $F = 4^{n-1}$.	

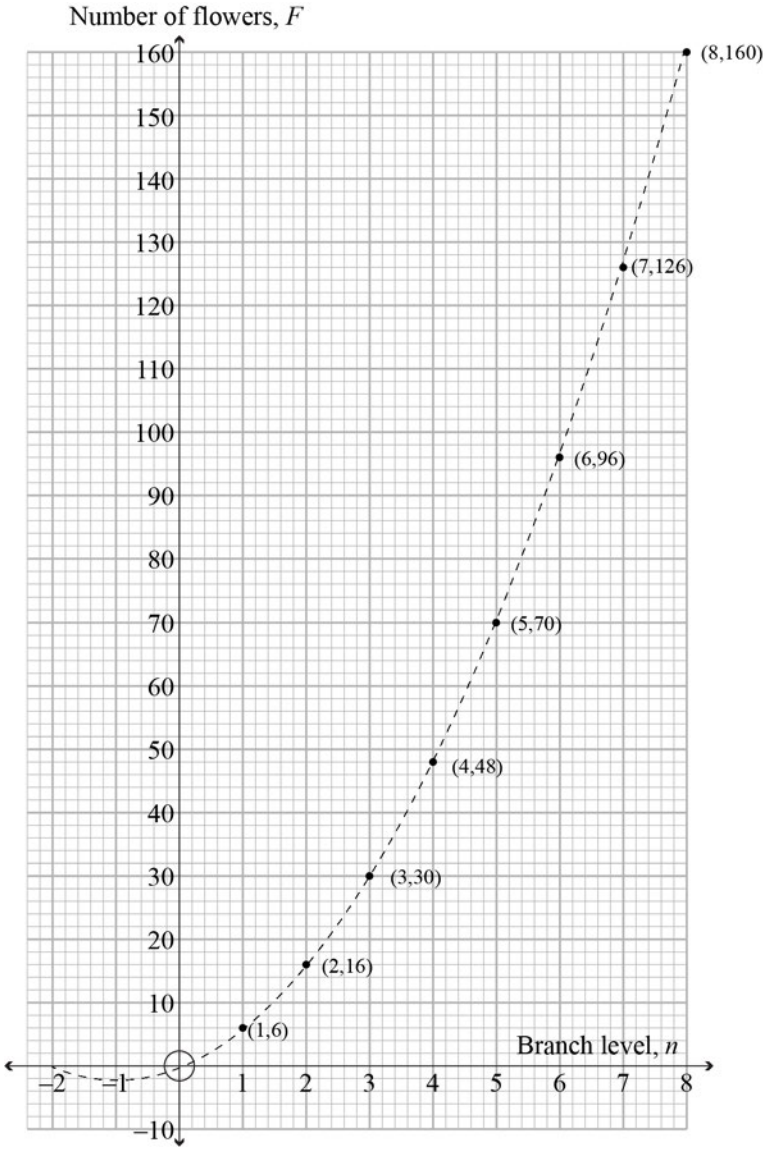
(iv)	Sample comments regarding the false claim are: • No, it is only one set of results from one tree, so cannot be generalised to claim that this will always occur. • No, these results are from only one tree in one particular place, so cannot be generalised to all trees in NZ. • No, different growing conditions in different locations will lead to different results and hence a different formula. • No, the results are only for up to branch 8 flowers. Other-sized trees may not follow the same pattern. • No, trees may be diseased, which would affect the number of flowers on the branches. Allow other valid reasons.	• Recognising that the claim is false. AND With ONE valid comment.		

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE question attempted towards solution.	1u	2u	3u	2r	3r	1t	2t

Question Two (b)



Question Two (c)(i)

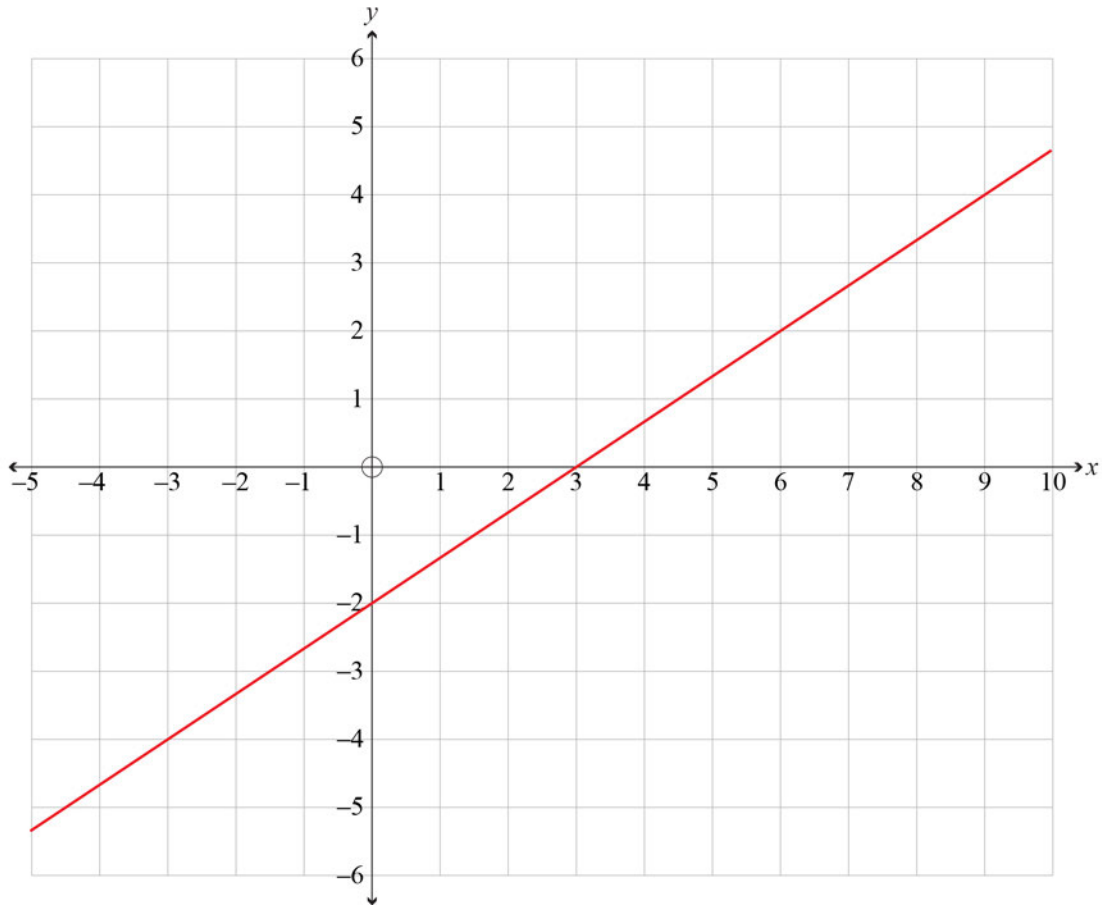


Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
THREE (a)(i)	Correct equation of $y = 2^{x+2} - 4$ OR equivalent.	• Equation identified as an exponential, with base of 2.	• Correct equation.	
(ii)	Straight line graph drawn, showing x-axis intercept at (3,0) AND y-axis intercept at (0,–2).	• Straight line with only one axis-intercept correct.	• Accurate graph drawn.	
(b)(i)	Correct equation of $y = 1000 \times 1.2^x$ OR $S = 1000 \times 1.2^t$	• Included 1000 in the exponential equation. OR Recognised that the base is 1.2.	• Correct equation.	

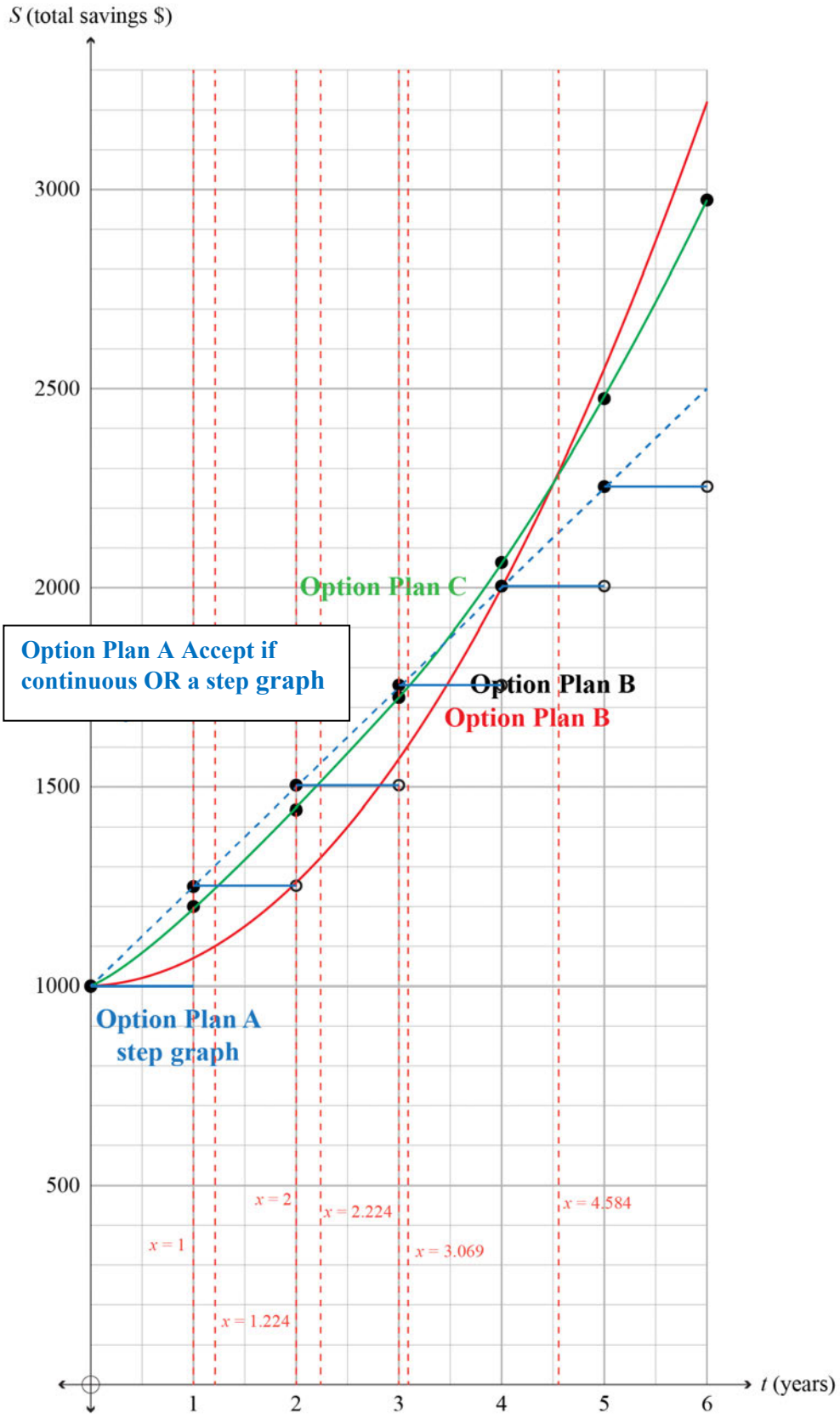
(ii)	Graph drawn for Savings Plan A.	• Continuous graph drawn of $y = 250x + 1000$		**E7/T1** Savings Plan A drawn as a continuous graph OR a correct step-graph AND Equation for savings plan C AND Savings Plan B drawn accurately. AND At least two valid comparisons made before and after the intercept point of $t = 4.6$ years. **E8 / T2** As for E7 AND At least three valid comparisons of the options made both before and after the intercept point of $t = 4.6$ years.
	Graph drawn for Savings Plan B Parabola continuous graph.	• Parabola graph drawn, showing (0,1000) and at least THREE other values drawn, but lacking accuracy.	• Parabola graph drawn correctly	
(iii)	Table of values for Savings Plan A and Savings Plan B produced. Valid comparisons made between the various Option Plans, including the evidence of dates, at least. Examples of possible comparison comments are: • Generally, Savings Plan C will be the best if $t < 4.6$ years (approximately). • Generally, Savings Plan B will be the best if $t > 4.6$ years (approximately). • As the years increase, so Savings Plan B will become better and better compared to the other savings plans. • Occasionally, but for only short time periods, Savings Plan A is the best, just after 1 year and just after 2 years. • Savings Plan A is generally the weakest choice. Other non-trivial valid comparisons acceptable.	• Table for Savings Plan A OR Savings Plan B correct AND ONE valid, non-trivial comparison made.	• Savings Plan A AND Savings Plan B correct in Table AND TWO valid non-trivial comparisons made before and after $t = 4.6$ years. (Can use whole number of years)	

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE question attempted towards solution.	1u	2u	3u	2r	3r	1t	2t

Question Three (a)(ii)



Question Three (b)(ii)



Question Three (b)(iii)

End of Year (t)	Total Savings Option Plan A	Total Savings Option Plan B	Total Savings Option Plan C
0	1000	1000	1000
1	1250	1070	1200
2	1500	1260	1440
3	1750	1570	1728
4	2000	2000	2073.60
5	2250	2550	2488.32
6	2500	3220	2985.98

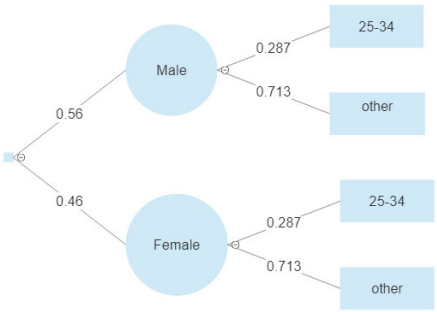
Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 13	14 – 18	19 – 24

Assessment Schedule – 2023

Mathematics and Statistics: Demonstrate understanding of chance and data (91037)

Evidence

Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
ONE (a)(i)	18.8 % + 10.9 % = 29.7 % or 0.297	• Correct answer.		
(ii)	Prob (female and 25–34) = 0.46 × 0.287 = 0.132 OR  	• C.A.O. OR Partial tree diagram.	• Correct probability, with working.	
(iii)	• Expected value = 63 × 0.188 = 11.84 The expected number of aged 55+ users is 12 people. (Allow 11 people; Allow 11 or 12 people; Allow about 12 people.) .Niko's percent is 12.6% • Because the sample size is quite small out of the huge number of Spotify users, Niko should expect to see quite a large variation. • The result of 8 Spotify users is only 3 Spotify users less than the expected value of 11 Spotify users. • Niko's claim is correct, as this small difference between his result and the actual expected value is an acceptable variation within his relatively small sample size. • Candidate queries whether the sample chosen by Niko is actually a representative random sample as the survey members have been selected from only Nico's family and work colleagues. This selection may cause bias in the results.	• Stating sample size is quite small. OR The sample may have possible bias.	• Calculated expected value. OR Calculating probabilities	**T1 / E7** As for r AND Discussed the large sampling variability based on small sample size. OR Discussed issues regarding the sample selection and possible bias.

(b)(i)	Feb 2017, when there were approximately 185-195 million users.	• Correct answer, with some evidence.		
(ii)	**Trend** The long-term trend has increased from approximately 210 million users in May 2016 to just over 280 million users in Aug 2020. **Unusual** • There is a large spike in late 2017, jumping from 190 million users in Feb to all time high of 320 million users in Dec 2017. • A second noticeable spike was in Dec 2019, reaching around 287 million users. • Do not accept any reference to non-existent repeating patterns. • Irregular scale. • No obvious seasonality.	• Any one sensible feature identified. Accept omission of justification.	• Any two sensible features identified, with attempt to justify.	**T1 / E7** Two valid features with clear numerical evidence, justification, units. **T2 / E8** Gaining T1 AND identifying a grade r – quality misleading feature on the graph (part (iii)). OR Three valid features with clear numerical evidence, justification, units.
(iii)	The timeline is not on a linear scale (regular interval). The vertical scale starts at 175, but scales should be starting at 0. Because the vertical scale starts at 175, this would have the effect of exaggerating the actual rises in the data. Accept other non-trivial valid comments.	• Identifying a non-trivial valid comment relating to the graph being misleading.	• Identifying two non-trivial valid comments relating to the graph being misleading.	

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response or no relevant evidence.	One question part attempted.	1u	2u	3u	1r	2r	1t	2t

Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
TWO (a)(i)	$\frac{17}{68}\times\frac{17}{68}=\frac{1}{16}=0.0625$ OR $\frac{17}{68}\times\frac{16}{67}=\frac{4}{67}=0.0597$ OR $0.25\times0.25=0.0625$	• Recognised the need to use $\frac{17}{68}$ or 0.25.	• Correct probability. (Allow sampling with or without replacement.)	
(ii)	**Centre** The median of pop music tempo is 123 bpm, which is higher than the classical music tempo of 97 bpm **Shift / Overlap** The middle 50% box for the pop music tempo is further up on the scale than the classical music tempo (must state values). There is some overlap between the two middle 50% boxes (must state values). **Shape** The distribution of pop music tempo is almost symmetrical, whereas the classical music tempo is slightly right skewed. **Spread** The IQR of classical music is 57 bpm compared to 45 bpm for pop music. OR Range classical 142, pop 136 **Clusters** Identifying classical at 70-90 **Unusual point (outlier)** 199 (or 198) for classical	• ONE significant feature compared.	• TWO different significant features compared, with some numerical evidence included.	• THREE different significant features compared, including appropriate relevant numerical evidence.
(iii)	Thom's claim is false because pop music tempo tends to be faster / higher than classical music tempo on Spotify, because the median tempo of classical music (97 bpm) is lower than the LQ tempo (102 bpm) of pop music, AND including numerical justification. I am confident in my conclusion because the sample size is big enough for me to use ½ and ¾ rule. OR Thom's claim is false, however, I am not very confident because the median is only just outside the pop music middle 50% box. Sampling variability suggests that a different sample may show both medians within each other's box, in which case Thom would be correct. (Do not accept that the sample size is not sufficiently large.)	• Rejected claim with a valid attempt to justify.	• Decision that the claim is false, including a correct conclusion, with reasoning based on classical median outside classical box OR DBM/OVS	• Response as in Merit, including supporting numerical justification. AND Confidence level is clearly discussed.

(b)(i)	$\frac{1-570}{1000}=\frac{430}{1000}=\frac{43}{100}=0.43$	• Correct answer.		
(ii)	Because the **sample size** of the graph is 1000 is large enough to allow an estimate for the centre of graph, I would expect to see a **similar** distribution.	• Similar distribution, with ONE distribution feature. OR New sample is likely to have 4 slow, 110 medium, 60 fast, 26 very fast pop songs. OR Sample size discussed.	• Similar distribution, with examples AND Sample size discussed	

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response or no relevant evidence.	One question part attempted.	1u	2u	3u	1r	2r	1t	2t

Q	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
THREE (a)(i)	Danceability score value that is approximately 0.6. Allow the response given as a range, e.g. score would be between 0.3 to 0.7. Allow indicated on the graph. Allow evidence of averaging the 3 or 4 data points for the score of 0.1. I am not confident in this result because there are only 3 results for the score of 0.1 and they have a big variation.	• Valid answer.	• Valid answer for the expected "Danceability" score. AND Comment expressing a lack of confidence in the prediction with valid reasoning.	
(ii)	There appears to be a positive linear relationship. as "Happy music" score increases then so does the "Danceability" score. The relationship is weak, because the points are scattered away from the line of the best fit. Accept there is no relationship.	• One statement made for the relationship, with some valid justification.	• TWO statements made for the relationship, in context, with valid justification.	
(iii)	**Overall**, as the relationship is weak, I would not be very confident in my predictions from these results. **But**, I would be more confident to predict a danceability score for music with "Happy music" score higher than 0.75. Because the points are much **closer to the line of best fit** between 0.75 and 1 (showing a stronger relationship) than below 0.4. **But**, I would be reluctant and very unconfident to predict a danceability score for music with "Happy music" score less than 0.4. Because most of the points are **scattered away from the line**. This section is showing the weakest relationship.		• Correct conclusion, with some correct justification. E.g. The relationship is weaker between 0 and 0.4.	• Correct conclusion, made with clear comparison of at least two sections AND with supporting justification.

(b)(i)	$\frac{92}{500} = \frac{23}{125} = 0.184 = 18.4\%$	• Correct probability.		
(ii)	Prob(less than 2 hours, given a free subscription) $= \frac{40}{200} = 0.2 = 20\%$ Prob(less than 2 hours, given a premium subscription) $= \frac{21}{300} = 0.07 = 7\%$ Must have valid numerical reasoning, e.g. because 0.2 > 0.07 or 20 % > 7 % It is more likely that free subscribers will use less than 2 hours per week compared to the premium subscribers, because the proportion / probability / percentage of free subscribers is much higher than that of the premium subscribers.	• ONE conditional probability calculated.	• TWO conditional probabilites calculated.	• Both probabilities correct AND compared AND fully justified with conclusion made.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	One partial solution	1 of u	2 of u	3 of u	1 of r	2 of r	t1	t2

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 14	15 – 19	20 – 24

<table>
<tr>
<td>(c)</td>
<td>
$20x^2 + 20x = (2x+5)^2$
$20x^2 + 20x = 4x^2 + 20x + 25$
$16x^2 = 25$
$x^2 = \frac{25}{16}$
$x = \pm\frac{5}{4}$
Accept $x = \pm\sqrt{\frac{25}{16}}$
OR
Alternative method:
$20x^2 + 20x = (2x+5)^2$
$20x^2 + 20x = 4x^2 + 20x + 25$
$16x^2 - 25 = 0$
$(4x-5)(4x+5) = 0$
$x = \pm\frac{5}{4}$
Allow answer in any form.
</td>
<td>
For award of u:
ONE of:
• expansion and simplification of RHS
• consistently solves, giving both solutions

For award of r:
• correctly solves for both solutions.
</td>
</tr>
<tr>
<td>(d)</td>
<td>
Let F be the first part of the journey and S be the second part of the journey.
$F + S = 1600$
$\Rightarrow S = 1600 - F$
$\frac{1}{3}F = \frac{1}{5}S$
$\Rightarrow 5F = 3S$
$5F = 3(1600 - F)$
$5F = 4800 - 3F$
$8F = 4800$
$F = 600$
Then $S = 1600 - 600 = 1000$ km
OR
Alternative method:
Let x be the distance covered in the first part of the journey; then $1600 - x$ is the distance remaining.
$\frac{x}{3} = \frac{1600 - x}{5}$
$5x = 3(1600 - x)$
$5x = 4800 - 3x$
$8x = 4800$
$x = 600$
Remaining distance is 1600 – 600 = 1000 km
Units not needed.
Allow alternative algebraic methods.
</td>
<td>
For award of u:
ONE of:
• forms the equation $\frac{1}{3}F = \frac{1}{5}S$ or $5F = 3S$
• consistent combining of their equations in one variable.

For award of r:
ONE of:
• combining of the equations in one variable
• consistent distances found for both parts of the journey.

For award of t:
• correct distances found for both parts of the journey.
</td>
</tr>
</table>

(e)

$\left[(2n+3)^2-2(2n+3)+5\right]-\left[(2n+1)^2-2(2n+1)+5\right]$

$=\left[4n^2+12n+9-4n-6+5\right]-\left[4n^2+4n+1-4n-2+5\right]$

$=\left[4n^2+8n+8\right]-\left[4n^2+4\right]$

$=8n+4$ #1

$=4(2n+1)$

This expression has a factor of 4, it is divisible by 4.

OR

Alternative method:

Assume n is odd – not required to be stated.

$\left[(n+2)^2-2(n+2)+5\right]-\left[n^2-2n+5\right]$

$=\left[n^2+4n+4-2n-4+5\right]-\left[n^2-2n+5\right]$

$=\left[n^2+2n+5\right]-\left[n^2-2n+5\right]$

$=4n$

This expression has a factor of 4, it is divisible by 4.

OR

Alternative method:

Assume n is even – not required to be stated.

$\left[(n+3)^2-2(n+3)+5\right]-\left[(n+1)^2-2(n+1)+5\right]$

$=\left[n^2+6n+9-2n-6+5\right]-\left[n^2+2n+1-2n-2+5\right]$

$=\left[n^2+4n+8\right]-\left[n^2+4\right]$

$=4n+4$

$=4(n+1)$

This expression has a factor of 4, it is divisible by 4.

Accept any order of the differences considered.

Allow alternative algebraic methods.

OR

Allow other valid algebraic methods used by considering other consecutive odd terms, e.g. $2n-1$ and $2n+1$.

For award of u:

ONE of:

- correct expression for the difference between two successive odd-numbered patterns
- consistent expansion and simplification of one of the square shaped brackets
- consistently simplifies to an expression without any brackets (equivalent to #1)
- any valid numerical working (using one example) with a clear justification.

For award of r:

ONE of:

- correctly simplifies to an expression without any brackets #1
- consistently uses algebraic reasoning and explanation to justify clearly that the result is divisible by 4
- Any valid numerical working (using two or more examples) with a clear justification.

For award of t:

- uses algebraic reasoning and explanation to justify clearly that the result is divisible by 4.

Q	Expected Coverage	Grade (generated by correctly demonstrating the procedures listed in EN4) Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent.
TWO (a)	$40x^2+11x-2=(5x+2)(8x-1)$ i.e. $y=8x-1$ Allow C.A.O. Accept $y=\frac{40x^2+11x-2}{5x+2}$	For award of u: • stating that $y = 8x - 1$ or $y=\frac{40x^2+11x-2}{5x+2}$
(b)	$(6x+5)(4x-1)=(2x+1)(12x-1)$ $24x^2-6x+20x-5=24x^2-2x+12x-1$ $24x^2 + 14x - 5 = 24x^2 + 10x - 1$ #1 $14x-5=10x-1$ $14x-10x=-1+5$ $4x=4$ $x=1$	For award of u: • expansion and simplification of a pair of brackets (LHS or RHS), forming a quadratic expression with three terms #1 For award of r: • equation solved to find $x = 1$.
(c)	$\frac{2(4x+1)-5(3x-4)}{10}\geq 5$ $\frac{8x+2-15x+20}{10}\geq 5$ $\frac{-7x+22}{10} \geq 5$ #1 $-7x+22\geq 50$ $-7x\geq 28$ $x\leq\frac{28}{-7}$ $x\leq -4$ Accept $x \leq \frac{-28}{7}$ or $x \leq -4$ or $-4 \geq x$ as the final answer.	For award of u: ONE of: • correct arrangement for both numerator and denominator (does not need to be expanded or simplified). Accept $8x + 2 - 15x - 20$ for the numerator • consistent solution found (with either $\leq$, $\geq$ or = sign). For award of r: ONE of: • correct linear inequation at #1 • equation solved to find $x = -4$ • inequation solved to find $x \geq -4$ • consistently reverses inequality sign due to mult/div of a negative number. For award of t: • inequation solved to find $x \leq -4$.

<table>
<tr>
<td>(d)</td>
<td>
$\frac{4w}{5} = \frac{v(w+3)}{4}$
$16w = 5v(w+3)$ #1
$16w = 5vw + 15v$
$16w - 5vw = 15v$ #2
$w(16-5v) = 15v$ #3
$w = \frac{15v}{16-5v}$ #4
Accept other equivalent solutions.
</td>
<td>
For award of u:
ONE of:
• cross-multiply #1
• consistently collecting terms involving w and terms not involving w on different sides of the equation #2
• consistently factorising the pair of terms involving w #3
• consistently rearranging by dividing by the bracket #4.

For award of r:
TWO of:
• cross-multiply #1
• consistently collecting terms involving w and terms not involving w on different sides of the equation #2
• consistently factorising the pair of terms involving w #3
• consistently rearranging by dividing by the bracket #4
OR
• correctly states v in terms of w.

For award of t:
• correct rearrangement.
</td>
</tr>
<tr>
<td>(e)</td>
<td>
$(3x-1)(3x+2) - x(x+1) = 8$
$9x^2 + 6x - 3x - 2 - x^2 - x - 8 = 0$
$8x^2 + 2x - 10 = 0$
$4x^2 + x - 5 = 0$
$(4x+5)(x-1) = 0$
Either $4x+5=0$
$x = -\frac{5}{4}$ Ignore as not appropriate.
OR
$x - 1 = 0$
$x = 1$
Units not required.
</td>
<td>
For award of u:
ONE of:
• forming correct expression $(3x - 1)(3x + 2)$ simplified to $9x^2 + 3x - 2$
• forming correct equation for the shaded area
• consistent simplification to a quadratic equation in three terms.

For award of r:
ONE of:
• simplification to a quadratic equation in three terms
• consistent solving of their quadratic equation, with evidence of negative value disregarded

For award of t:
• correct positive solution found for the question, with evidence of negative value disregarded.
</td>
</tr>
</table>

Q	Expected Coverage	Grade (generated by correctly demonstrating the procedures listed in EN4) Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent.
THREE (a)	$\text{Perimeter} = 6(3x+4) = 60$ $18x + 24 = 60$ $18x = 36$ $x = \frac{36}{18} = 2$ OR Alternative method: $\text{Perimeter} = 6(3x+4) = 60$ $3x + 4 = 10$ $3x = 6$ $x = \frac{6}{3} = 2$ Allow solution as an unsimplified fraction.	For award of u: • correct solution for the value of x Accept C.A.O.
(b)	$\frac{2x}{2x-3} = \frac{x+4}{x+2}$ $2x(x+2) = (x+4)(2x-3)$ $2x^2 + 4x = 2x^2 - 3x + 8x - 12$ $4x = 5x - 12$ $x = 12$ OR Alternative method: $\frac{2x(x+2) - (x+4)(2x-3)}{(2x-3)(x+2)} = 0$ $\frac{2x^2 + 4x - (2x^2 + 5x - 12)}{(2x-3)(x+2)} = 0$ $2x^2 + 4x - 2x^2 - 5x + 12 = 0$ $-x + 12 = 0$ $x = 12$	For award of u: ONE of: • correct arrangement for both numerator and denominator (does not need to be expanded or simplified). Accept $2x^2 + 4x - 2x^2 + 5x + 12$ for the numerator • consistent solution found. For award of r: • correct value for x found.

(c)	$\frac{1}{2} \times (2x + 8)(x + 2) = 24$ $(x + 4)(x + 2) = 24$ $x^2 + 6x + 8 = 24$ $x^2 + 6x - 16 = 0$ $(x + 8)(x - 2) = 0$ Either $x = -8$ Ignore as not appropriate Or $x = 2$ OR Alternative method: $\frac{1}{2} \times (2x+8)(x+2) = 24$ $(2x+8)(x+2) = 48$ $2x^2 + 4x + 8x + 16 - 48 = 0$ $2x^2 + 12x - 32 = 0$ $x^2 + 6x - 16 = 0$ $(x+8)(x-2) = 0$ Either $x = -8$ Ignore as not appropriate. Or $x = 2$	For award of u: ONE of: • form and simplify a quadratic expression for the area of the triangle, with or without the 24 being used • consistent solution found, with evidence of the invalid value disregarded. For award of r: • correct solution found, with evidence of the invalid value disregarded.

<table>
<tr>
<td>(d)</td>
<td>$(2^2)^{x-2} \times 2^{x+1} = (2^5)^x$
$2^{2x-4} \times 2^{x+1} = 2^{5x}$
$2^{2x-4+x+1} = 2^{5x}$ #1
$2^{3x-3} = 2^{5x}$
$3x - 3 = 5x$ #2
$-3 = 2x$
$x = -\frac{3}{2}$</td>
<td>For award of u:
ONE of:
• recognition of powers of 2 on both sides
• LHS or RHS correct at stage #1.

For award of r:
ONE of :
• forming the linear equation #2
• consistently forming an equation and solving for their x-value.

For award of t:
• correct solution found.</td>
</tr>
<tr>
<td>(e)</td>
<td>Let T be the number of $20 notes.
Let F be the number of $50 notes.
$T + F = 40$
$20T + 50F = 1700$

$T + F = 40$
$2T + 5F = 170$

$2T + 2F = 80$
$2T + 5F = 170$
Subtracting gives:
$3F = 90$
$F = 30$ and $T = 10$

Value of $20 notes will be 10 × $20 = $200

Value of $50 notes will be 30 × $50 = $1500

or alternative method:

$20(40 - F) + 50F = 1700$
$F = 30$
$T = 10$

Candidate could use x and y as the variables.
Allow alternative algebraic methods.</td>
<td>For award of u:
ONE of:
• forms both equations
• consistent combining of their equations into one variable.

For award of r:
ONE of:
• consistent number of either $20 notes of $50 notes
• correct value for either $20 notes or $50 notes
• correct combining of the equations into one variable
• consistent value for both $20 notes and $50 notes.

For award of t:
ONE of:
• correct number of both $20 and $50 notes
• correct value for both $20 notes and $50 notes.</td>
</tr>
</table>