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SCHOLARSHIP EXEMPLAR

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

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Scholarship 2021
Calculus

Time allowed: Three hours
Total score: 40

ANSWER BOOKLET

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Write your answers in this booklet.

Make sure that you have Formulae Booklet S–CALCF.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Answers developed using a CAS calculator require **ALL commands to be shown**. Correct answers only will not be sufficient.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Question	Score
ONE	
TWO	
THREE	
FOUR	
FIVE	
TOTAL	

ASSESSOR'S USE ONLY

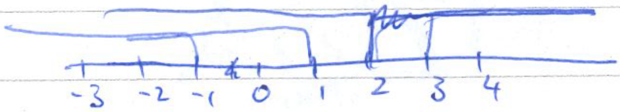
$\frac{x^2-x-2-x^2+2x+3}{x+1}$

1a) $f(x)=\frac{x^2-x-2}{x^2-2x-3} = \frac{(x-2)(x+1)}{(x-3)(x+1)} = \cancel{\frac{x-2}{x-3}>0}$ $= 1-\frac{x+1}{(x-3)(x+1)}$

$(x-2)(x-1)>0$ $\quad (x-3)(x+1)>0$

$x<2,\ x>1$ $\quad x<2,\ x>0$ $\quad x>3,\ x<-1$

$1-\frac{1}{x-3}>0$



$x-3-1>\frac{a}{x-3}$ $\quad x<2$

$x<4$ $\quad x-3>1$

$\frac{1}{x-3}<1$

$1<x-3$

$x>4$

$\cancel{x<1,\ x>3}$

$x<2\ ,x>3$

1b) $x^{x\sqrt{x}}=x^{2x}$ let $\sqrt{x}=a$

$x^{a^3}=x^{2a^2}$

$a^3-2a^2=0$

$a^2(a-2)=0$

$a=0,\ a=2$

$\sqrt{x}=0,$

$x=0$

$\sqrt{x}=2$

$x=4$

$x\sqrt{x}=2x$

$x^2x=4x^2$

$x^3=4x^2$

$x^3-4x^2=0$

$x^2(x-4)=0$

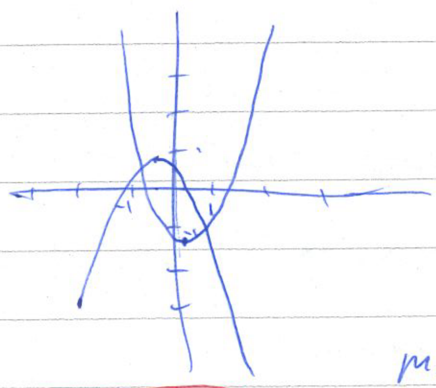
$x=0,4$

1c) $y_1=2(x^2-\frac{1}{2}x)-1$

$=2(x-\frac{1}{4})^2-\frac{9}{8}$

$y_2=-2(x^2+\frac{1}{2}x)+1$

$=-2(x+\frac{1}{4})^2+\frac{7}{8}$



$2x^2-x-1=-2x^2-x+1$

$4x^2=2$

$x^2=\frac{1}{2}$

$x=\pm\frac{1}{\sqrt{2}}$

midpoint of x-intercepts is ... area:

So $x=0$

ASSESSOR'S USE ONLY

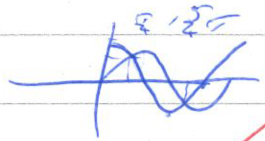
$u-1 = x+1-1$
$u-1 = x$

1d) $\int_0^2 \frac{x}{\sqrt{x+1}}\,dx$ let $u = x+1$ $du = dx$ $x = u-1$

$\sqrt{u}$

$\int_1^3 \left(\frac{u-1}{\sqrt{u}}\right)du$ $\frac{u^1}{u^{\frac12}} = u^{\frac12}$ $\frac{1}{u^{\frac12}} = -u^{-\frac12}$

$= \int_1^3 \left(u^{\frac12} - u^{-\frac12}\right)du$



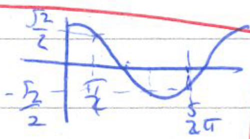
$= \left[\frac23 u^{\frac32} - 2u^{\frac12}\right]_1^3$

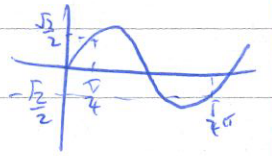
$= \left(\frac23\sqrt{27} - 2\sqrt3\right) - \left(\frac23 - 2\right)$

$= \left(\frac23 \cdot 3\sqrt3 - 2\sqrt3\right) - \cancel{\left(-\frac73\right)} \left(-\frac43\right)$

~~$= 0 + \frac73 = \frac73$~~ $= 0 + \frac43$

$= \frac43$



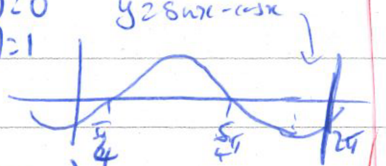


1e) ~~$\int |\sin x - \cos x|\,dx = \begin{cases} \sin x - \cos x & (1) \\ -\sin x + \cos x & (2) \end{cases}$~~

~~When (1) $\int_0^{2\pi}$~~

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt2}{2} = \sin\left(\frac{\pi}{4}\right)$ $\sin(2\pi) = 0$
$\cos\left(\frac54\pi\right) = -\frac{\sqrt2}{2} = \sin\left(\frac54\pi\right)$ $\cos(2\pi) = 1$

$y = \sin x - \cos x$



So $-\int_0^{\frac{\pi}{4}}(\sin x - \cos x) + \int_{\frac{\pi}{4}}^{\frac54\pi}(\sin x + \cos x) - \int_{\frac54\pi}^{2\pi}(\sin x + \cos x)$

$= -\left[-\cos x - \sin x\right]_0^{\frac{\pi}{4}} + \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac54\pi} - \left[-\cos x\ \sin x\right]_{\frac54\pi}^{2\pi}$

$= -\left[\left(-\frac{\sqrt2}{2} - \frac{\sqrt2}{2}\right) - (-1-0)\right] + \left[\left(+\frac{\sqrt2}{2} + \frac{\sqrt2}{2}\right) - \left(-\frac{\sqrt2}{2} - \frac{\sqrt2}{2}\right)\right] - \left[(-1-0) - \left(\frac{\sqrt2}{2} + \frac{\sqrt2}{2}\right)\right]$

$= -\left[\left(-\sqrt2 + 1\right)\right] + \left(\sqrt2 + \sqrt2\right) - \left(-1 - \sqrt2\right)$

$\sqrt2 - \cancel{1} + \sqrt2 + \sqrt2 \cancel{+1} + \sqrt2$

$= \cancel{4\sqrt2}$

$= 4\sqrt2$ units^2

2a) $\log_{\frac{a}{b}} b = 5$ so $\left(\frac{a}{b}\right)^5 = b$

$\frac{a^5}{b^5} = b$

$a^5 = b^6 \Rightarrow a = b^{\frac{6}{5}}$

$\frac{\log b}{\log \frac{a}{b}} = 5$

$\frac{\log b}{\log a - \log b} = 5$

$\log b = 5\log_a - 5\log b$

$6\log_b = 5\log_a$

$\log_{\frac{a}{b}}(b^{\frac{1}{3}} \cdot a^{\frac{1}{4}}) = k$

$\left(\frac{a}{b}\right)^k = a^{\frac{1}{4}} \cdot b^{\frac{1}{3}}$

$k\log\left(\frac{a}{b}\right) = \log(a^{\frac{1}{4}} \cdot b^{\frac{1}{3}})$

$k(\log a - \log b) = \log(a^{\frac{1}{4}} \cdot b^{\frac{1}{3}})$

$k\left(\log \frac{b^{\frac{6}{5}}}{b}\right) = \log(b^{\frac{6}{5} \cdot \frac{1}{4}} \cdot b^{\frac{1}{3}})$

$= k(\log b^{\frac{1}{5}}) = \log(b^{\frac{3}{10}} \cdot b^{\frac{1}{3}})$

$= \log(b^{\frac{19}{30}})$

$\frac{1}{5}k\log b = \frac{19}{30}\log_b$

so $\frac{1}{5}k = \frac{19}{30}$

$k = \frac{19}{6}$

$\therefore \log_{\frac{a}{b}}(\sqrt[5]{b} \times \sqrt[4]{a}) = \frac{19}{6}$

2b) let $a+b=11$ ~~b = 11-a~~ $a = 11-b$

$a^2b^3 = y$ where a is first number, b is second number

maximise y

sub b into y:

~~$y = a^2(11-a)^3$~~

~~$= 11a^2 - a^3$~~

~~$\frac{dy}{da} = 22a - 3a^2 = 0$~~

~~$a(22-3a) = 0$~~

~~$a = 0, \frac{22}{3}$~~

~~$\frac{d^2y}{da^2} = 22 - 6a$~~

~~$\left.\frac{d^2y}{da^2}\right|_{a=0} > 0$~~

$\left.\frac{d^2y}{da^2}\right|_{a=\frac{22}{3}} < 0$, so $a = \frac{22}{3}$ is max

~~$y = a^2(1331 - 363a +$~~

~~$\frac{dy}{dx} = 2a(11-a)^3 - 3a^2(11-a)^2$~~

~~$= a(11-a)^2[22 - 2a - 3a]$~~

~~$= a(11-a)^2(22-5a)$~~

~~$a = 11, \frac{22}{5}, 0$~~

~~$\frac{d^2y}{dx^2} =$~~

ASSESSOR'S USE ONLY

~~$a = \frac{22}{3}$, $b = 11 - \frac{22}{3} = \frac{11}{3}$~~

~~$\frac{11}{3}, \frac{22}{3}$~~

~~∴ Pair of numbers = $\frac{22}{3}, \frac{11}{3}$ ($\frac{11}{3}$ being the first number and $\frac{22}{3}$ being the 2nd number)~~

~~1a 3.074 2651.065~~ 5 302.09

Sub a into y:

$y = (11-b)^2 b^3$
$= (121 - 22b + b^2)b^3$
$= b^5 - 22b^4 + 121b^3$

$\frac{dy}{dx} = 5b^4 - 88b^3 + 363b^2$
$= b^2(5b^2 - 88b + 363)$
$= b^2(5b-33)(b-11)$

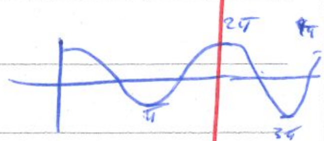
5   -33
1   -11

$b = 0, 11, \boxed{\frac{33}{5}}$ ($b = 11, 0$ would minimise as $a = 0, 11$ and $10a^2b^3$ would equal 0)

when $b = \frac{33}{5}$,
$a = 11 - \frac{33}{5}$
$= \frac{22}{5}$

∴ $a = \frac{22}{5}$, $b = \frac{33}{5}$

2c) $f(x) = a\sin(\pi x + \alpha) + b\cos(\pi x + \alpha) + 1$



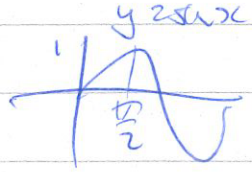
$= a(\sin\pi x \cos\alpha + \cos\pi x \sin\alpha) + b(\cos\pi x\cos\alpha + \sin\pi x \sin\alpha) + 1$
$= a(\cos\pi x \sin\alpha) + b(\cos\pi x \cos\alpha) + 1$

$f(2020) = a(\cos 2020\pi)(\sin\alpha) + b(\cos 2020\pi)(\cos\alpha) + 1 = 10$
$= a\sin\alpha + b\cos\alpha + 1 = 10$
$a\sin\alpha + b\cos\alpha = 9$ ①

$f(2021) = a(\cos 2021\pi)(\sin\alpha) + b(\cos 2021\pi)(\cos\alpha) + 1$
$= -a\sin\alpha - b\cos\alpha + 1$
$= -(a\sin\alpha + b\cos\alpha) + 1$

Sub ①: $= -9 + 1 = -8$

2d) $f(x) = (x^2+1)^{\sin x}$ let $y = f(x)$



$y = (x^2+1)^{\sin x}$

$\cancel{y\,dx}\ \ln y = \sin x \ln(x^2+1)$

$\frac{1}{y}\frac{dy}{dx} = \cos x \ln(x^2+1) + \sin x \cdot \frac{2x}{x^2+1}$

$\frac{dy}{dx} = y\left(\cos x \ln(x^2+1) + \sin x \cdot \frac{2x}{x^2+1}\right)$

When $\cancel{x}$ $x = \frac{\pi}{2}$, $f(x) = y = \left[\left(\frac{\pi}{2}\right)^2+1\right]^1 = \frac{\pi^2}{4}+1 = \frac{4+\pi^2}{4}$

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2},\, y=\frac{4+\pi^2}{4}} = \frac{\pi^2+4}{4}\left(0 + 1\cdot\frac{\pi}{\frac{\pi^2}{4}+1}\right)$

$= \frac{\pi^2+4}{4}\left(\frac{\pi}{\frac{\pi^2+4}{4}}\right)$

$= \frac{\pi^2+4}{4} \times \frac{4\pi}{\pi^2+4}$

$= \pi$

2e) let $\log_2 x = A$, $y = A^2 + 6mA + n$

$\cancel{dx}\ \cancel{y} = \frac{dy}{dx} = 2A + 6m = 0$

$A = -\frac{6m}{2} = -3m$ $\log_2 x = -3m$

$2^{-3m} = x$

$x = \frac{1}{8^m}$

sub $(\frac{1}{8}, -2)$ into ~~eq~~ original eq:

$-2 = 9 - 18m + n$

$\cancel{n=18}$ $\cancel{n=18+}$

$18m - n = 11$

ASSESSOR'S USE ONLY

3a) $-\frac{b}{a}$ = sum of roots as roots are $\sin\theta, \cos\theta$

$\sin\theta + \cos\theta = -\frac{b}{a}$

$$LHS = \frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta} = \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\sin\theta-\cos\theta} + \frac{\cos^2\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta-\cos\theta} - \frac{\cos^2\theta}{\sin\theta-\cos\theta}$$

$$= \frac{(\sin^2\theta-\cos^2\theta)}{(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta+\cos\theta)\cancel{(\sin\theta-\cos\theta)}}{\cancel{(\sin\theta-\cos\theta)}}$$

$$= \sin\theta + \cos\theta$$

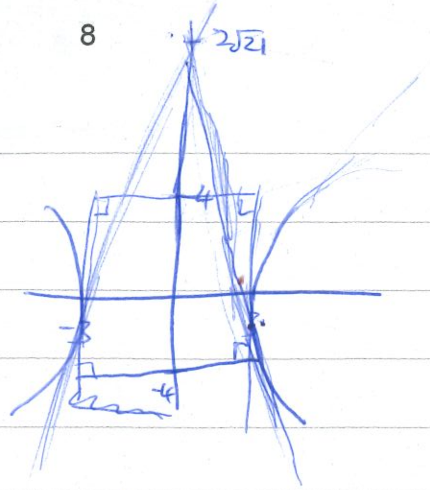
and ~~so $\sin\theta+\cos\theta = -\frac{b}{a}$ QED~~

so we must prove $\sin\theta + \cos\theta = -\frac{b}{a}$

which is true as roots are $\sin\theta, \cos\theta$

$\therefore$ It is proved that

$$\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta} = -\frac{b}{a}$$



3b) $y = mx + 2\sqrt{21}$

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

$9 - \frac{3}{2} = \frac{15}{2}$

$\frac{3}{4}x^2 \quad \frac{3}{2} \left(\sqrt{\frac{15}{2}} + \frac{3}{4}\right)^2$

$16x^2 - 9y^2 = 144$

$32x - 18y\frac{dy}{dx} = 0$

$18y\frac{dy}{dx} = 32x$

$\frac{dy}{dx} = \frac{32x}{18y} = \frac{16x}{9y}$

$m = \frac{16x}{9y}$

$\cancel{y = \frac{16x}{19y} + 2\sqrt{21}}$ $y =$

$19y^2 = 16x + 38\sqrt{21}y$

$19y^2 = 16\sqrt{9 + \frac{9}{16}y^2} + 38\sqrt{21}y$

$16x^2 - 9y^2 = 144$

$16x^2 = 144 + 9y^2$

$x^2 = 9 + \frac{9}{16}y^2$

$x = \sqrt{9 + \frac{9}{16}y^2}$

ASSESSOR'S USE ONLY

3c) $f(x) = ax^3 - bx = x(ax^2 - b)$

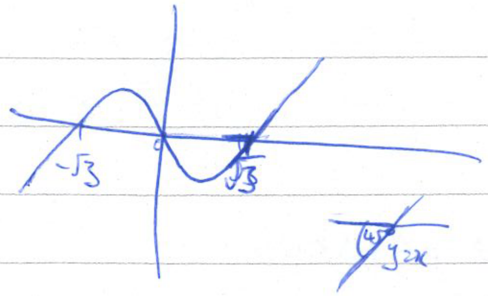
$= \cancel{ax(x^2-b)}$ $\quad x^2 = \frac{b}{a}$

$x = \pm\sqrt{\frac{b}{a}}$

~~$x=0, \pm\sqrt{b}$~~, so $b = \sqrt{3}$

~~$f(x) = ax^3 - 3x$~~ $\quad$ so $\frac{b}{a} = 3$

$b = 3a$



$\frac{dy}{dx}$ at $x = \sqrt{3}$ is 1 as 45° angle

$\frac{dy}{dx} = \cancel{3ax^2 - 3}$

$\frac{dy}{dx} = 3ax^2 - 3$

at $x = \sqrt{3}$,

$\frac{dy}{dx} = 9a - 3 = 1$

$9a = 4$

$a = \frac{4}{9}$

so $f(x) = \frac{4}{9}x^3 - 3x$

$f'(x) = \frac{4}{3}x^2 - 3$

$f'(0) = -3$

$y = ax^3 - 3ax$

$\frac{dy}{dx} = 3ax^2 - 3a$

$\left.\frac{dy}{dx}\right|_{x=\sqrt{3}} = 9a - 3a = 1$

$6a = 1$

$a = \frac{1}{6}$

so $y = \frac{1}{6}x^3 - \frac{1}{2}x$

$\left.\frac{dy}{dx}\right|_{x=0} = -3a$

$= -3 \times \frac{1}{6}$

$= -\frac{1}{2}$

d i) $5P5 = 120$

ii) G . B (B) B B . B . B

$GP1 \times 2 \times 5P5 = 1440$

iii) G B G B B . B B

$5P5 \times 6P2$

$= 3600$

1 2 3 $\quad$ 1 3 2 $\quad$ 2 1 3

2 3 1

3 1 2

3 2 1

1 2 3 4

$\times 4 \times 6 = 24$

4a) $\frac{dA}{dt} = 0.16A + D$ $\quad t=0,\ A = 76{,}000$

at $t=10$, is $A \geq 500{,}000$?

$$\frac{dA}{dt} = 0.16A + 5000$$
$$= \frac{4}{25}A + 5000$$
$$\frac{dA}{dt} = \frac{4A + 125{,}000}{25}$$

$$\int \frac{25}{4A+125{,}000} \frac{dA}{dt} dt = \int 1 \, dt$$

$$= \frac{25}{4} \ln|4A + 125{,}000| = t + c$$

sub $t=0$, $A = 76{,}000$

$$\frac{25}{4} \ln|179{,}000| = c$$

$$\frac{25}{4} \ln|4A+125{,}000| = t + \frac{25}{4} \ln|179{,}000|$$

when $t=10$,

~~886592.804~~

$$\frac{25}{4} \ln|4A + 125{,}000| = 10 + \frac{25}{4} \ln|179{,}000|$$

$$\ln(4A + 125{,}000) = \frac{8}{5} + \ln|179{,}000|$$ CON

$$4A + 125000 = 179{,}000e^{\frac{8}{5}}$$
$$4A = 179{,}000e^{\frac{8}{5}} - 125{,}000$$
$$A = \frac{179{,}000e^{\frac{8}{5}} - 125{,}000}{4}$$
$$= 190{,}398.201$$

$\therefore$ After 10 years, they will have \$190, 398.201, which is not enough to buy the house.

ASSESSOR'S USE ONLY

4bi) $\frac{dy}{dx} = (x-1)y^3$ ~~$\frac{1}{2y}$ -2 y^3~~

$\int \frac{1}{y^3}\frac{dy}{dx}dx \int (x-1)\,dx$

$-\frac{1}{2y^2} = \frac{1}{2}x^2 - x + c$

~~$f(0)=$~~ $x=0,\ y=a$

$-\frac{1}{2a^2} = c$

$-\frac{1}{2y^2} = \frac{1}{2}x^2 - x - \frac{1}{2a^2}$

$\frac{1}{2y^2} = \frac{1}{2a^2} + x - \frac{1}{2}x^2$ mei

$\frac{1}{2y^2} = \frac{1 + 4a^2x - a^2x^2}{2a^2}$

$2y^2 = \frac{2a^2}{1+4a^2x - a^2x^2}$

$y^2 = \frac{a^2}{1+4a^2x - a^2x^2}$

$y = \sqrt{\frac{a^2}{1+4a^2x-a^2x^2}}$

$1 + a^2x - a^2x^2 = 0$

$1 + a^2x(\cancel{1-x}) = 0$

4b ii)

$\overbrace{3,\ 7,}^{4}\ \overbrace{13,}^{6}\ \overbrace{21}^{8}$

1 4 9 16

2, 3, 4, 5

n^2 1, 4, 9

$n^2 + n + 1$

1 4 9 16

2 6 12 20

4, 6, 8

12 2

$n^2 + n$

n=1 n=2 n=3 n=4

4c) $T_n = \sqrt{1 + 1 + \frac{1}{4}} + \sqrt{1 + \frac{1}{4} + \frac{1}{9}} + \sqrt{1 + \frac{1}{9} + \frac{1}{16}} + \sqrt{1 + \frac{1}{16} + \frac{1}{25}}$

$= \sqrt{\frac{9}{4}} + \sqrt{\frac{49}{36}} + \sqrt{\frac{169}{144}} + \frac{441}{400}$

$= \frac{3}{2} + \frac{7}{6} + \frac{13}{12} + \frac{21}{20} + \frac{31}{30}$

$= \frac{n^2 + n + 1}{n^2 + n} = 1 + \frac{1}{n^2 + n}$

$n^2(n^2 + 2n + 1)$

$T_n = \sqrt{\frac{n^2(n+1)^2 + (n+1)^2 + n^2}{n^2(n+1)^2}} = \sqrt{\frac{n^4 + 2n^3 + n^2 + n^2 + 2n + 1 + n^2}{n^2(n+1)^2}} = \sqrt{\frac{n^4 + 2n^3 + 3n^2 + 2n + 1}{n^2(n+1)^2}}$

$S_{n=2}$ $T_1 = \frac{3}{2}$

$T_2 = \frac{8}{3}$

$T_3 = \frac{15}{4}$

$T_4 = \frac{25}{4}$

$S_n = \frac{3}{2}, \frac{8}{3}, \frac{15}{4}, \frac{25}{4}$

$= \frac{n^2 + n + 1}{n^2 + n}$ $T_n = \frac{n^2 + n + 1}{n^2 + n} = 1 + \frac{1}{n^2 + n}$

$S_{un} = \frac{3}{2}, \frac{8}{3}, \frac{15}{4}, \frac{25}{4} \dots \frac{3}{2}, \frac{16}{6}, \frac{45}{12}, \frac{125}{20}, \frac{437}{60}$

$\frac{16}{9}$ $\frac{48}{32}$ $\frac{5}{3}$

3, 16, 45, 125

13 29 80

16 51

~~2, 15, 44, 124~~

ASSESSOR'S USE ONLY

$A=(1,2)$ $B=(3,4)$

5a) let $C=(x,y)$

~~BC=~~ $|BC|=\sqrt{(3-x)^2+(4-y)^2}$

$|AB|=\sqrt{(1-x)^2+(2-y)^2}$

$|BC|=4|AB|$

$\sqrt{(3-x)^2+(4-y)^2}=4\sqrt{(1-x)^2+(2-y)^2}$

~~BC~~

$9-6x+x^2+16-8y+y^2=16(1-2x+x^2+4-4y+y^2)$

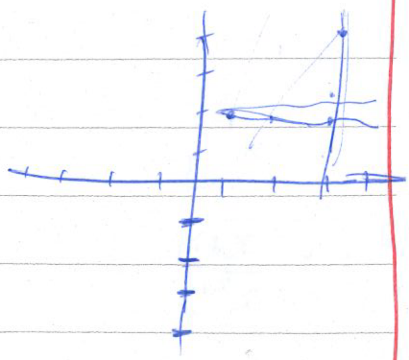
~~x^2+~~

$x^2-6x+y^2-8y+25=16(x^2-2x+y^2-4y+5)$

$\cancel{x^2-6x+y^2-8y}+25=16x^2-32x+16y^2-64y+80$

$15x^2-26x+15y^2-56y+55=0$

~~$15x^2-26x+15($~~



$m_{BC}=\frac{4-y}{3-x}$ $m_{AB}=\frac{2-y}{1-x}$

$\frac{4-y}{3-x}=-\frac{1-x}{2-y}$

$(4-y)(2-y)=-(1-x)(3-x)$

$8-6y+y^2=-(3-4x+x^2)$

$8-6y+y^2=-3+4x-x^2$

$(y-3)^2-1=-3+4x-x^2$

$(y-3)^2=-x^2+4x-2$

$y-3=\sqrt{-x^2+4x-2}$

$y=\sqrt{-x^2+4x-2}+3$

$15x^2-26x+15y^2-56y=-55$

$15(x^2-\frac{26}{15}x)+15(y^2-\frac{56}{15}y)=-55$

$15(x-\frac{26}{30})^2-\frac{169}{15}+15(y-\frac{56}{30})^2-\frac{784}{15}=-55$

~~$15(x-\frac{26}{30}$~~

$15(x-\frac{13}{15})^2+15(y-\frac{28}{15})^2=\frac{128}{15}$

$(x-\frac{13}{15})^2+(y-\frac{28}{15})^2=128$

~~So~~ $\therefore$ C is an looking for the locus: $(x-\frac{13}{15})^2+(y-\frac{28}{15})^2=128$

(a circle with centre $(\frac{13}{15},\frac{28}{15})$ and radius of $8\sqrt{2}$

5b) let $z = x+iy \quad \bar{z} = x-iy$

$$x+iy + \frac{1}{x-iy} = x-iy + \frac{1}{x+iy}$$

$$x+iy + \frac{x+iy}{x^2+y^2} = x-iy + \frac{x-iy}{x^2+y^2}$$

$$\frac{(x+iy)(x^2+y^2)+x+iy}{x^2+y^2} = \frac{(x-iy)(x^2+y^2)+(x-iy)}{x^2+y^2}$$

$$\cancel{x^3}+\cancel{xy^2}+x^2yi+y^3i+\cancel{x}+iy = \cancel{x^3}+\cancel{xy^2}-x^2yi-y^3i+\cancel{x}-iy$$

$$\cancel{2x^2yi+2y^3i+2iy}$$

$$x^2yi+y^3i+iy = -x^2yi-y^3i-iy$$

$$x^2yi+y^3i+iy = -(x^2yi+y^3i+iy)$$

$$x^2y+y^3+y=0$$

$$y(x^2+y^2+1)=0$$

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Annotated Scholarship Exemplar Template

Subject	Scholarship Calculus	*Standard*	93202	*Total score*	26

Q	*Score*	*Annotation*
1	**7**	The candidate demonstrated a thorough understanding of modulus function in 1e by integrating the function over separate limits. They failed to check on the domains of function in solving equation/inequality in 1a and 1b.
2	**8**	The candidate successfully applied logarithm rules in solving 2a. They also displayed ability in applying trig composite angle formula in 2c, and showed understanding of properties of trig functions. In 2e, although the final answer is not given, but they have identified the two equations needed in solving the problem with logarithm rules used correctly.
3	**7**	The candidate managed to prove the trig identity successfully in 3a. They also showed good insight in using permutation principles in 3d. They could have solved 3b if they persevered in solving the simultaneous equations following their method.
4	**4**	The candidate showed ability in applying reverse chain rule in solving differential equation in 4a. Although the candidate made a minor error in their calculation, but has managed to work consistently afterwards; they have also answered the question in context. The candidate could have continued in solving 4b if they were able to identify the function involved is a quadratic under the root.
5	**0**	The candidate could establish the equality relating the two distances, however, could not reduce it to a single variable. They could have continued finishing it by writing the equation of the straight line of BC. In 5b, the candidate has managed to rationalise the complex denominator, and achieved as far as simplifying it to a concise form, however, they could not make meaningful conclusion from it.