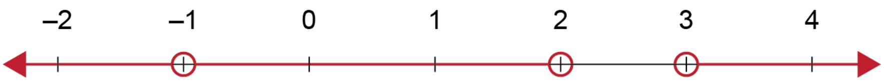
Assessment Schedule – 2021

Scholarship Calculus (93202)

Evidence Statement

Q	Solution
ONE (a)	$f(x)=\frac{x^2-x-2}{x^2-2x-3}=\frac{(x+1)(x-2)}{(x+1)(x-3)}=\frac{x-2}{x-3}$ $f(x)>0$ for $x<2$ or $x>3$ and $x\neq -1$ 
(b)	$x\neq 0$, as 0^0 is undefined. Now, since $x>0$ $\ln x^{x\sqrt{x}}=\ln x^{2x}$ so, $x\sqrt{x}\ln x=2x\ln x$ $x\left(\sqrt{x}-2\right)\ln x=0$ $x=4,$ or $x=1$ The solutions are $x = 1$ and $x = 4$.
(c)	$-2x^2-x+1=2x^2-x-1$ $x=\pm\frac{1}{\sqrt{2}}$ Since $-2x^2-x+1-(2x^2-x-1)=-4x^2+2$ is an even function, therefore the y-axis, i.e. the line $x=0$, divides the required area into equal parts.
(d)	Let $\sqrt{x+1}=u$. Then, when $x=0, u=1$, and when $x=2, u=\sqrt{3}$. Also, $x+1=u^2$, $\mathrm{d}x=2u\,\mathrm{d}u$. Substituting: $\int_1^{\sqrt{3}}\frac{\left(u^2-1\right)2u\,\mathrm{d}u}{u}=2\int_1^{\sqrt{3}}\left(u^2-1\right)\mathrm{d}u$ $=\left[2\left(\frac{u^3}{3}-u\right)\right]_1^{\sqrt{3}}=2\left(\frac{\left(\sqrt{3}\right)^3}{3}-\sqrt{3}\right)-2\left(\frac{1^3}{3}-1\right)=2\sqrt{3}-2\sqrt{3}+\frac{4}{3}=\frac{4}{3}$

(e)

$$\text{Area} = 2\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \left(\sin x - \cos x\right) \mathrm{d}x$$

$$= 2\left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$= 2\left(\frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}\right) = \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

Alternate solution :

$$\int_0^{2\pi} \left|\sin x - \cos x\right| \mathrm{d}x =$$

$$\int_0^{\frac{\pi}{4}} \left(\cos x - \sin x\right)\mathrm{d}x + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \left(\sin x - \cos x\right)\mathrm{d}x + \int_{\frac{5\pi}{4}}^{2\pi} \left(\cos x - \sin x\right)\mathrm{d}x$$

$$= \left[\sin x + \cos x\right]_0^{\frac{\pi}{4}} + \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{5}{4\pi}} + \left[\sin x + \cos x\right]_{\frac{5\pi}{4}}^{2\pi}$$

$$= \left(\frac{2}{\sqrt{2}} - 1\right) + \left(\frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}\right) + \left(1 + \frac{2}{\sqrt{2}}\right) = \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

Q	Solution
TWO (a)	$\log_{\frac{a}{b}}\left(\frac{a}{b}\right)=1,$ i.e. $\log_{\frac{a}{b}} a-\log_{\frac{a}{b}} b=1$ Since $\log_{\frac{a}{b}} b=5$, we have $\log_{\frac{a}{b}} a=6$ $\log_{\frac{a}{b}}\left(\sqrt[3]{b}\times\sqrt[4]{a}\right)=\frac{1}{3}\log_{\frac{a}{b}} b+\frac{1}{4}\log_{\frac{a}{b}} a=\frac{5}{3}+\frac{6}{4}=\frac{19}{6}$ **Alternate solution :** $\left(\frac{a}{b}\right)^5=b\Rightarrow a=b^{\frac{6}{5}}$ $\Rightarrow\log_{\frac{a}{b}}(a)=\log_{\frac{a}{b}}\left(b^{\frac{6}{5}}\right)=\frac{6}{5}\log_{\frac{a}{b}}(b)=6$ $\therefore\log_{\frac{a}{b}}\left(\sqrt[3]{b}\times\sqrt[4]{a}\right)=\frac{1}{3}\log_{\frac{a}{b}} b+\frac{1}{4}\log_{\frac{a}{b}} a=\frac{5}{3}+\frac{6}{4}=\frac{19}{6}$
(b)	Let x and y be the numbers. Then $x+y=11$. We must maximise $P=x^2y^3=(11-y)^2y^3$ Clearly $0\le y\le 11$. $\frac{\mathrm{d}P}{\mathrm{d}y}=(11-y)^2(3y^2)+y^3\left[2(11-y)(-1)\right]$ $=(11-y)y^2\left[3(11-y)-2y\right]$ $=(11-y)y^2(33-5y)$ Critical numbers are 0, 11 and $\frac{33}{5}$ $P(0)=P(11)=0$ Absolute max is when $y=\frac{33}{5}$. Solution set $x=\frac{22}{5}$ and $y=\frac{33}{5}$.
(c)	Since 2020π is a multiple of 2π, $f(2020)=a\sin\alpha+b\cos\alpha+1=10$ Let $y=2020\pi+\alpha$ $f(2021)=a\sin(y+\pi)+b\cos(y+\pi)+1$ $=-a\sin y-b\cos y+1$ $=-(a\sin\alpha+b\cos\alpha)+1=-8$ **Alternate solution.** Use the composite angle formula.

(d)	$\ln f(x) = \sin x \times \ln(x^2+1)$ $\frac{f'(x)}{f(x)} = \cos x \times \ln(x^2+1) + \frac{2x \times \sin x}{x^2+1}$ $f'(x) = (x^2+1)^{\sin x} \times \left[\cos x \times \ln(x^2+1) + \frac{2x \times \sin x}{x^2+1}\right]$ $f'\left(\frac{\pi}{2}\right) = \left(\left(\frac{\pi}{2}\right)^2 + 1\right)^1 \times \left[0 + \frac{\pi}{\left(\frac{\pi}{2}\right)^2 + 1}\right] = \pi$
(e)	Since $\log_2 x$ increases uniformly on $(0,\infty)$, let $\log_2 x = A$. Then $f(A) = A^2 + 6mA + n$ and $f'(A) = 2A + 6m$, which has a min when $A = -3m$. So, $f(x) = (\log_2 x)^2 + 6m(\log_2 x) + n$ has a minimum when $\log_2 x = -3m$. $\log_2 \frac{1}{8} = -3m$ and $-3 = -3m$ or $m = 1$ Since $f\left(\frac{1}{8}\right) = -2$, $-2 = 9 - 18 + n$ $n = 7$ **Alternate solution.** $\frac{\mathrm{d}}{\mathrm{d}x}(\log_2 x) = \frac{1}{x \cdot \ln 2}$ $\frac{\mathrm{d}f(x)}{\mathrm{d}x} = 2\log_2 x \cdot \frac{1}{x \cdot \ln 2} + 6m \cdot \frac{1}{x \cdot \ln 2}$ $x = \frac{1}{8} : \frac{-6}{\frac{1}{8}\ln 2} + \frac{6m}{\frac{1}{8}\ln 2} = 0 \rightarrow m = 1$ $-2 = (-3)^2 + 6(1)(-3) + n \rightarrow n = 7$

Q	Solution
THREE (a)	$\dfrac{\sin\theta}{1-\cot\theta}+\dfrac{\cos\theta}{1-\tan\theta}=\dfrac{\sin^2\theta}{\sin\theta-\cos\theta}+\dfrac{\cos^2\theta}{\cos\theta-\sin\theta}$ $=\dfrac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$ $=\dfrac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{(\sin\theta-\cos\theta)}$ $=\sin\theta+\cos\theta=\text{sum of roots}=-\dfrac{b}{a}$
(b)	$y^2=m^2x^2+4\sqrt{21}mx+84$ $\therefore 16x^2-9m^2x^2-36\sqrt{21}mx-756=144$ $x^2(16-9m^2)-36\sqrt{21}mx-900=0$ Require $b^2-4ac=0$ $\Rightarrow 5184m^2=57\,600\Rightarrow m^2=\dfrac{100}{9}$ or $m=\pm\dfrac{10}{3}$
(c)	$y'=3ax^2-b$ Now at $x=\sqrt{3}$: $y'\|_{x=\sqrt{3}}=1$ since $\tan 45°=1$ $\Rightarrow 9a-b=1$ Now at P and Q, $y(\pm\sqrt{3})=0:\sqrt{3}3a-\sqrt{3}b=0$ $\Rightarrow 3a-b=0$ $\therefore 6a=1,\ a=\dfrac{1}{6},\ b=\dfrac{1}{2}$, and $y'(0)=-b=-\dfrac{1}{2}$
(d)(i)	$5!=120$
(ii)	$6!\times 2=1440$
(iii)	$7!-6!\times 2=3600$

Q	Solution
FOUR (a)	$\frac{dA}{dt} = 0.16A + D$ $\int \frac{dA}{0.16A + D} = \int 1 dt$ $\frac{1}{0.16} \ln\lvert 0.16A + D \rvert = t + c$ The initial deposit $A(t=0) = 76000$, then $\frac{1}{0.16} \ln\lvert 0.16 \times 76000 + 5000 \rvert = c$ $(c = 60.94)$ and when $t = 10$: $\frac{1}{0.16} \ln\lvert 0.16A + 5000 \rvert = 10 + c$ $\frac{1}{0.16} \ln\lvert 0.16A + 5000 \rvert - \frac{1}{0.16} \ln\lvert 0.16 \times 76000 + 5000 \rvert = 10$ $\ln \frac{0.16A + 5000}{0.16 \times 76000 + 5000} = 1.6$ $A = \left(0.16 \times 76000 + 5000\right) e^{1.6} = 499962.73$ Only \$37.27 short, so will be fine. **Alternate solutions** $\frac{dA}{dt} = 0.16A + D$ $\int \frac{dA}{0.16A + D} = \int 1 dt$ $\frac{1}{0.16} \ln\lvert 0.16A + D \rvert = t + c$ Let $A(0) = x$ be the initial deposit required to meet their goal. Also, $A(10) = 500\,000$. Then $\frac{1}{0.16} \ln\lvert 0.16x + 5000 \rvert = c$ and $\frac{1}{0.16} \ln\lvert 0.16 \times 500\,000 + 5000 \rvert = 10 + c$ so $\frac{1}{0.16} \ln\lvert 0.16x + 5000 \rvert = \frac{1}{0.16} \ln\lvert 0.16 \times 500\,000 + 5000 \rvert - 10$ $\ln\lvert 0.16x + 5000 \rvert = \ln 85\,000 - 1.6$ $= 9.75041$ $0.16x + 5000 = e^{9.75041}$ $x = \$76\,006.82$ Although short by \$6.82, realistically this initial investment will be sufficient .

Alternate solution :

If the initial investment is correct then,

$$\int_0^{10} \mathrm{d}t = \int_{76000}^{500000} \frac{1}{0.16A + D}\mathrm{d}A$$

LHS is clearly 10

Consider the RHS

$$t = \frac{1}{0.16}\Big[\ln\big(16A + D\big)\Big]_{76000}^{500000}$$

$$= \frac{1}{0.16}\Big[\ln\big(16 \times 500000 + 5000\big)\Big] - \Big[\ln\big(16 \times 76000 + 5000\big)\Big]$$

$$= 70.9400 - 60.9396$$

$$= 10.0004$$

Which is about 3.5 hours more than ten years.

The initial deposit of \$76000 will be sufficient.

(b)(i)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (x-1)y^3$$

$$\int y^{-3}\,\mathrm{d}y = \int (x-1)\mathrm{d}x$$

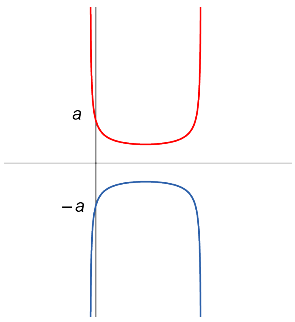
$$-\frac{y^{-2}}{2} = \frac{x^2}{2} - x + c$$

At $x = 0$ $y = a$: $c = -\frac{1}{2a^2}$

$$y^{-2} = -x^2 + 2x + \frac{1}{a^2}$$

$$y = \pm\left(\frac{1}{a^2} - x^2 + 2x\right)^{-\frac{1}{2}}$$

Which, when graphed for $a \neq 0$ would give:



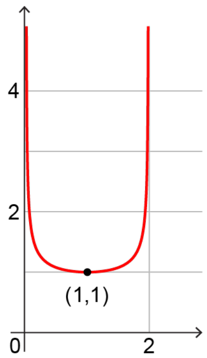
However, since $a > 0$, we consider only the positive root; hence the function required is:

$$y(x) = +\left(\frac{1}{a^2} - x^2 + 2x\right)^{-\frac{1}{2}}$$

(ii)

For a finite and positive, the condition $\frac{1}{a^2} - x^2 + 2x > 0$ or $x^2 - 2x - \frac{1}{a^2} < 0$ must be satisfied for a *real* domain to exist. The quadratic has roots $x = 1 \pm \sqrt{1 + \frac{1}{a^2}}$.

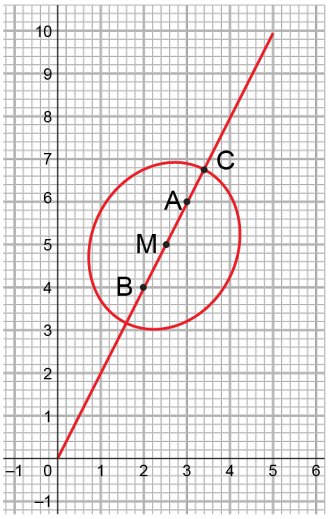
The natural domain of $y(x)$ *is* $\left(1 - \sqrt{1 + \frac{1}{a^2}}, 1 + \sqrt{1 + \frac{1}{a^2}}\right)$.

	Range: As $x \to \left(1 \mp \sqrt{1+\frac{1}{a^2}}\right)$, $y(x) \to +\infty$. The minimum value of $y(x)$ occurs at the turning point of $x^2 - 2x - \frac{1}{a^2}$, i.e. when $x = 1$ and $y(1) = \left(1+\frac{1}{a^2}\right)^{-\frac{1}{2}}$. The range is $\left(1+\frac{1}{a^2}\right)^{-\frac{1}{2}} \le y$. i.e. $y \ge \frac{a}{\sqrt{1+a^2}}$.
(iii)	$\lim\limits_{a\to+\infty}\left[\left(\frac{1}{a^2} - x^2 + 2x\right)^{-\frac{1}{2}}\right] = \left(-x^2+2x\right)^{-\frac{1}{2}}$ Which is defined if $2x - x^2 > 0$, i.e., as $a \to +\infty$, the domain approaches $0 < x < 2$. The range: As $x \to 0^+$, $y(x) \to +\infty$ and as $x \to 2^-$, $y(x) \to +\infty$. The minimum value will occur when $-x^2 + 2x$ takes on its max value, which is when $x = 1$ and $y(1) = +\left(-1^2 + 2\times 1\right)^{-\frac{1}{2}} = 1$. 

(c)	$T_1 = \frac{3}{2} = 1 + \frac{1}{1\times 2} = 1 + 1 - \frac{1}{2}$ $T_2 = \frac{7}{6} = 1 + \frac{1}{2\times 3} = 1 + \frac{1}{2} - \frac{1}{3}$ $\vdots$ $T_{2021} = \frac{2021\times 2022 + 1}{2021\times 2022} = 1 + \frac{1}{2021\times 2022} = 1 + \frac{1}{2021} - \frac{1}{2022}$ Therefore $\sum_{n=1}^{2021} T_n = 2021 + 1 - \frac{1}{2022} = \frac{2022^2 - 1}{2022}$ or $\frac{2021\times 2023}{2022} = 2021\frac{2021}{2022}$ Or in general: $\sum_{r=1}^{n}\sqrt{1 + \frac{1}{r^2} + \frac{1}{\left(r+1\right)^2}} = \sum_{r=1}^{n}\sqrt{\frac{r^2\left(r+1\right)^2 + \left(r+1\right)^2 + r^2}{r^2\left(r+1\right)^2}}$ $= \sum_{r=1}^{n}\sqrt{\frac{r^2(r+1)^2 + 2r(r+1) + 1}{r^2\left(r+1\right)^2}}$ $= \sum_{r=1}^{n}\sqrt{\frac{\left(r^2 + r + 1\right)^2}{r^2\left(r+1\right)^2}}$ $= \sum_{r=1}^{n}\frac{r^2 + r + 1}{r\left(r+1\right)}$ $= \sum_{r=1}^{n}\left(1 + \frac{1}{r\left(r+1\right)}\right) = \sum_{r=1}^{n}\left(1 + \frac{1}{r} - \frac{1}{r+1}\right)$ Since $\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right)$ $= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \ldots + \frac{1}{n-1} - \frac{1}{n} + \frac{1}{n} - \frac{1}{n+1} = 1 - \frac{1}{n+1}$ $\sum_{r=1}^{n}\sqrt{1 + \frac{1}{r^2} + \frac{1}{\left(r+1\right)^2}} = \sum_{r=1}^{n}\left(1 + \frac{1}{r} - \frac{1}{r+1}\right) = n + 1 - \frac{1}{n+1}$ Therefore, $\sum_{r=1}^{2021} T_r = 2021 + 1 - \frac{1}{2022} = 2021\frac{2021}{2022}$

Q	Solution
FIVE (a)	$m_{AB} = 1,\ m_{BC} = -1$ To make BC = 4AB, B can be translated by $\begin{pmatrix} 8 \\ -8 \end{pmatrix}$ or $\begin{pmatrix} -8 \\ 8 \end{pmatrix}$ Therefore C is (11, – 4) or (– 5, 12) **Alternate solution:** $\lvert \mathrm{AB} \rvert = \sqrt{2^2 + 2^2} = \sqrt{8}$ The line $\overline{\mathrm{BC}}$ is given by: $y - 4 = -1(x - 3)$ $y = -x + 7$ For the required magnitude we want: $\sqrt{(x-3)^2 + (4 - (-x+7))^2} = 4\sqrt{8}$ $2x^2 - 12x + 18 = 128$ $x^2 - 6x - 55 = 0$ $(x - 11)(x + 5) = 0$ $x = 11$ or $x = -5$ The points defining C are (11,–4) or (–5,12).
(b)	Multiply $z\bar{z}$ to the equation: $z^2\bar{z} + z = z\bar{z}^2 + \bar{z}$ Note that $z\bar{z} = x^2 + y^2$, $(x^2 + y^2)z + z = (x^2 + y^2)\bar{z} + \bar{z}$ $(x^2 + y^2 + 1)(z - \bar{z}) = 0$ Therefore, $z = \bar{z} \to y = 0, x \in \mathbb{R},\ x \neq 0$ **Alternate solution:** $\dfrac{z\bar{z} + 1}{z} = \dfrac{z\bar{z} + 1}{\bar{z}}$ $z = \bar{z} \to y = 0, x \in \mathbb{R}, x \neq 0$ **Alternate solution:** $x + \mathrm{i}y + \dfrac{x + \mathrm{i}y}{x^2 + y^2} = x - \mathrm{i}y + \dfrac{x - \mathrm{i}y}{x^2 + y^2}$ $(x^2 + y^2)(x + \mathrm{i}y) + x + \mathrm{i}y = (x^2 + y^2)(x - \mathrm{i}y) + x - \mathrm{i}y$ $(x^2 + y^2)[2\mathrm{i}y] + 2\mathrm{i}y = 0$ $[2\mathrm{i}y][x^2 + y^2 + 1] = 0$ $y = 0$ or $x^2 + y^2 = -1$ so $y = 0$ Back substituting into the original equation gives $x + \dfrac{1}{x} = x + \dfrac{1}{x}$, which is true for all real $x \neq 0$. The solution set is the Real axis with the exclusion of 0.

(c)(i) The locus is traced by a point moving in the Argand plane so that the sum of its distances from the points (2,4i) and (3,6i) is constant. The locus is an ellipse with foci (2,4i) and (3,6i).



(ii) The foci are collinear with the origin. The principle axis of the ellipse is therefore the line $y = 2x$. So, max $|z|$ is found where the line intersects with the ellipse. Let C be the vertex furthest from the origin, through which the line $y = 2x$ will pass. max $|z|$ = the distance of point C from the origin.

Distance OM from origin to midpoint of the major axis = $\sqrt{\left(\frac{5}{2}\right)^2 + 5^2} = \frac{5}{2}\sqrt{5}$

So, distance from origin to opposite vertex $= \frac{5}{2}\sqrt{5} + 2.$

Alternate solution.

Let the origin be O. Then $\max|z| = \left|\overline{\text{OA}}\right| + \left|\overline{\text{AC}}\right|$.

$\left|\overline{\text{OA}}\right| = \sqrt{3^2 + 6^2} = \sqrt{45} = 3\sqrt{5}$

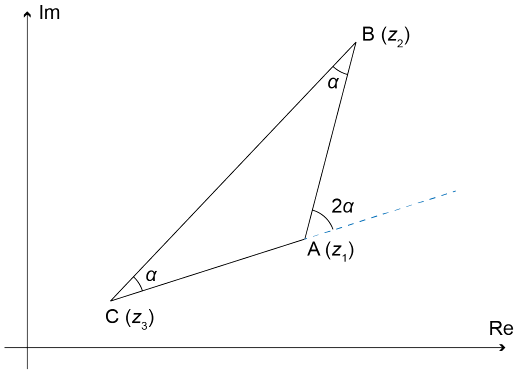
$\left|\overline{\text{AC}}\right| + \left|\overline{\text{BC}}\right| = 4$ from the definition of the elipse.

$\left|\overline{\text{BA}}\right| + 2\left|\overline{\text{AC}}\right| = 4$

$\left|\overline{\text{AC}}\right| = 2 - \frac{1}{2}\left|\overline{\text{BA}}\right| = 2 - \frac{1}{2} \times \sqrt{5}$

$\max|z| = \overline{\text{OA}} + \overline{\text{AC}} = 3\sqrt{5} + 2 - \frac{1}{2} \times \sqrt{5} = \frac{5}{2}\sqrt{5} + 2$

(d)



Let $\arg(z_1 - z_3) = \beta$, then $\arg(z_2 - z_3) = \alpha + \beta$ and $\arg(z_2 - z_1) = 2\alpha + \beta$

Note that $\arg(z_2 - z_3)^2 = 2\alpha + 2\beta$,

$$\therefore \arg\left(\frac{1}{2}(z_2 - z_3)\sec\alpha\right)^2 = \arg(z_2 - z_3)^2 = \arg\left(z_2 - z_1\right)\left(z_1 - z_3\right)$$

As for the modulus, $\left|z_2 - z_1\right| = \left|z_1 - z_3\right| = \frac{1}{2}\left|z_2 - z_3\right|\sec\alpha$

$$\therefore \left|\left(z_2 - z_1\right)\left(z_1 - z_3\right)\right| = \left(\frac{1}{2}\left|z_2 - z_3\right|\sec\alpha\right)^2$$

Therefore,

$$\left(z_2 - z_1\right)\left(z_1 - z_3\right) = \left(\frac{1}{2}(z_2 - z_3)\sec\alpha\right)^2$$

	Alternate solution. AB = AC so $\lvert z_2 - z_1\rvert = \lvert z_1 - z_3\rvert$ and $\arg(z_2 - z_1) - \arg(z_1 - z_3) = 2\alpha$ Therefore $z_2 - z_1 = (z_1 - z_3)(\cos 2\alpha + \mathrm{i}\sin 2\alpha)$ **(A)** In the given triangle $\overline{\mathrm{BC}}^2 = \overline{\mathrm{AC}}^2 + \overline{\mathrm{AB}}^2 - 2\overline{\mathrm{AC}}\cdot\overline{\mathrm{AB}}\cdot\cos(180° - 2\alpha)$ $\overline{\mathrm{BC}}^2 = 2\overline{\mathrm{AC}}^2 - 2\overline{\mathrm{AC}}^2(-\cos 2\alpha)$ $\overline{\mathrm{BC}}^2 = 2\overline{\mathrm{AC}}^2(1 + \cos 2\alpha) = 4\overline{\mathrm{AC}}^2\cos^2\alpha$ and $\overline{\mathrm{BC}} = 2\overline{\mathrm{AC}}\cos\alpha$ So: $\lvert z_2 - z_3\rvert = 2\lvert z_1 - z_3\rvert\cos\alpha$ and $\arg(z_2 - z_3) - \arg(z_1 - z_3) = \alpha$, which gives $z_2 - z_3 = 2(z_1 - z_3)(\cos\alpha + \mathrm{i}\sin\alpha)\cos\alpha$ **(B)** Since $(\cos 2\alpha + \mathrm{i}\sin 2\alpha) = (\cos\alpha + \mathrm{i}\sin\alpha)^2$ From **(A)**: $\dfrac{z_2 - z_1}{z_1 - z_3} = (\cos\alpha + \mathrm{i}\sin\alpha)^2$ From **(B)**: $\dfrac{z_2 - z_3}{2(z_1 - z_3)\cos\alpha} = \cos\alpha + \mathrm{i}\sin\alpha$ Which, after equating gives $\dfrac{z_2 - z_1}{z_1 - z_3} = \left[\dfrac{z_2 - z_3}{2(z_1 - z_3)\cos\alpha}\right]^2$ i.e. $(z_2 - z_1)(z_1 - z_3) = \left(\dfrac{1}{2}(z_2 - z_3)\sec\alpha\right)^2$

Sufficiency Statement

Score 1–4, no award	Score 5–6, Scholarship	Score 7–8, Oustanding Scholarship
Shows understanding of relevant mathematical concepts, and some progress towards solution to problems.	Application of high-level mathematical knowledge and skills, leading to partial solutions to complex problems.	Application of high-level mathematical knowledge and skills, perception, and insight / convincing communication shown in finding correct solutions to complex problems.

Cut Scores

Scholarship	Outstanding Scholarship
21 – 33	34 – 40