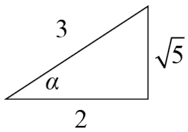
Assessment Schedule – 2022

Scholarship Calculus (93202)

Evidence Statement

Q	Solution
ONE (a)	Let $z = x + \mathrm{i}y$, then $\lvert x+a+\mathrm{i}y\rvert^2 = a\lvert x+1+\mathrm{i}y\rvert^2$ $(x+a)^2 + y^2 = a\left\lvert (x+1)^2 + y^2 \right\rvert$ $x^2 + 2ax + a^2 + y^2 = ax^2 + 2ax + a + ay^2$ $x^2 + y^2 - a\left(x^2+y^2\right) = a - a^2$ $\lvert z\rvert^2 (1-a) = a(1-a)$ $\lvert z\rvert = \sqrt{a}$ **Alternate solution** $\lvert z+a\rvert^2 = (z+a)(\bar{z}+a) = z\bar{z} + za + \bar{z}a + a^2$ Since $\lvert z+a\rvert^2 = a\lvert z+1\rvert^2$ $z\bar{z} + za + \bar{z}a + a^2 = a(z+1)(\bar{z}+1) = az\bar{z} + az + a\bar{z} + a$ $z\bar{z}(1-a) = a(1-a)$ $\lvert z\rvert^2 = a$ $\lvert z\rvert = \sqrt{a}$ (in this context the negative root is not valid.)
(b)	$\tan x + \tan\left(\dfrac{\pi}{4} - x\right) = 1$ $\tan x + \dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x} = 1$ $\dfrac{1-\tan x}{1+\tan x} = 1 - \tan x$ $(1-\tan x)(1+\tan x) = (1-\tan x)$ $1 - \tan^2 x = 1 - \tan x$ $\tan x(\tan x - 1) = 0$ Solution set (i) $\tan x = 1 \rightarrow x_1 = n\pi + \dfrac{\pi}{4}(n \in Z)$ $y_1 = -n\pi$ Solution set (ii) $\tan x = 0 \rightarrow x_2 = k\pi(k \in \mathbb{Z})$ $y_2 = -k\pi + \dfrac{\pi}{4}(k \in \mathbb{Z})$ Since $\tan\left(\dfrac{\pi}{4} + n\pi\right) = \tan\left(\dfrac{\pi}{4} - n\pi\right)$, the complete solution set is $\left(\pm n\pi + \dfrac{\pi}{4}, \mp n\pi\right) n \in Z$ and $\left(\mp k\pi, \pm k\pi + \dfrac{\pi}{4}\right) k \in Z.$

(c)	$x^4 + x^3 - 4x^2 + x + 1 = 0$ $x^2 + x - 4 + \frac{1}{x} + \frac{1}{x^2} = 0$ $\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 4 = 0 \rightarrow \left(\left(x + \frac{1}{x}\right)^2 - 2\right) + \left(x + \frac{1}{x}\right) - 4 = 0$ $\left(x + \frac{1}{x}\right)^2 + \left(x + \frac{1}{x}\right) - 6 = 0$ $x + \frac{1}{x} = 2 \text{ or } -3$ However, $x + \frac{1}{x} = 2$ has solution $x = 1$ $x + \frac{1}{x} = -3$ has solutions $\frac{-3 \pm \sqrt{5}}{2}$ which are both negative. We use -3 only $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right)$ $= \left(x + \frac{1}{x}\right)\left(\left(x + \frac{1}{x}\right)^2 - 3\right) = -18$ **Alternate solution :** $P(1) = 0;\ P(x) = (x-1)^2\left(x^2 + 3x + 1\right) = 0$ $x = \frac{-3 \pm \sqrt{5}}{2}$ $x^3 = \left(\frac{-3 \pm \sqrt{5}}{2}\right)^3 = \left(\frac{-27 \pm 27\sqrt{5} - 45 \pm 7\sqrt{5}}{8}\right) = \pm 4\sqrt{5} - 9$ $\frac{1}{x^3} = \mp 4\sqrt{5} - 9$ $x^3 + \frac{1}{x^3} = -18$

Q	Solution
TWO (a)	$x^2-4x+10=k\left(x+1\right)^2$ $\left(1-k\right)x^2-\left(4+2k\right)x+\left(10-k\right)=0$ For distinct real roots: $\left(4+2k\right)^2-4\times\left(1-k\right)\left(10-k\right)>0$ $60k-24>0$ $k>\frac{2}{5}$ For roots of the same sign: $\frac{10-k}{1-k}>0$ i.e. $k>10$ *or* $k<1$ However, for the roots to be real, the condition becomes: $k>10$ or $\frac{2}{5}<k<1$ **Alternate solution :** $b^2-4ac>0$ and $\lvert b\rvert>\sqrt{b^2-4ac}$ $b^2-4ac>0\rightarrow k>\frac{2}{5}$ $\lvert b\rvert>\sqrt{b^2-4ac}\rightarrow ac>0:\left(1-k\right)\left(10-k\right)>0$ i.e. $k>10$ or $k<1$ However, for the roots to be real, the condition becomes: $k>10$ or $\frac{2}{5}<k<1$
(b)	$\frac{12}{\sin\alpha}=\frac{16}{\sin 2\alpha}=\frac{16}{2\sin\alpha\cos\alpha}\rightarrow$  Which gives $\frac{\sin\alpha}{12}=\frac{2\sin\alpha\cos\alpha}{16}$ and $\frac{8}{12}=\cos\alpha$ from which $\sin\alpha=\frac{\sqrt{5}}{3}$ $\sin 2\alpha=2\sin\alpha\cos\alpha=\frac{4\sqrt{5}}{9}$ $\cos 2\alpha=\cos^2\alpha-\sin^2\alpha=-\frac{1}{9}$ Area $=\frac{1}{2}\times16\times12\times\sin\left(180°-3\alpha\right)$ $=96\sin 3\alpha$ $=96\times\left(\sin\alpha\cos 2\alpha+\sin 2\alpha\cos\alpha\right)$ $=96\times\left(-\frac{\sqrt{5}}{27}+\frac{8\sqrt{5}}{27}\right)$ $=96\times\frac{7\sqrt{5}}{27}$ units $=\frac{224\sqrt{5}}{9}$ units

(c)	Let the three sides be lr^{-1}, l, and lr. Then l is the side opposite α. Using the cosine rule: $l^2 = \left(\frac{l}{r}\right)^2 + (lr)^2 - 2\left(\frac{l}{r}\right)(lr)\cos\alpha$ $l^2 = \frac{l^2}{r^2} + l^2r^2 - 2l^2\cos\alpha$ $1 - \frac{1}{r^2} - r^2 = -2\cos\alpha$ $\frac{1}{2}\left(r^2 - 1 + \frac{1}{r^2}\right) = \cos\alpha$ $\frac{1}{2}\left(r - \frac{1}{r}\right)^2 + \frac{1}{2} = \cos\alpha$ When $r = 1$, the LHS function in terms of r is minimum. Since cosine curve is a decreasing function between 0° and 90°, α will be a max when $\cos\alpha$ is minimum. $\cos\alpha = \frac{1}{2} \rightarrow \alpha$ = 60°: equilateral triangle

Q	Solution
THREE (a)	Let $y=\dfrac{\mathrm{e}^{5x}\times\sqrt{x+1}}{\mathrm{e}^{\sqrt{x+1}}}=\left(\mathrm{e}^{5x-(x+1)^{\frac{1}{2}}}\right)\times(x+1)^{\frac{1}{2}}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\left(5-\dfrac{1}{2}(x+1)^{-\frac{1}{2}}\right)\mathrm{e}^{5x-(x+1)^{\frac{1}{2}}}\times(x+1)^{\frac{1}{2}}+\left(\mathrm{e}^{5x-(x+1)^{\frac{1}{2}}}\right)\times\dfrac{1}{2}(x+1)^{-\frac{1}{2}}$ $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=0}=\left(5-\dfrac{1}{2}\right)\mathrm{e}^{-1}\times1+\mathrm{e}^{-1}\left(\dfrac{1}{2}\right)=\dfrac{5}{\mathrm{e}}$ **Alternate solution** Let $y=f(x)$ then $\ln y=\ln\mathrm{e}^{5x}+\ln(x+1)^{\frac{1}{2}}-\ln e^{(x+1)^{\frac{1}{2}}}$ $\ln y=5x+\dfrac{1}{2}\ln(x+1)-(x+1)^{\frac{1}{2}}$ Differentiate both sides $\dfrac{1}{y}\times\dfrac{\mathrm{d}y}{\mathrm{d}x}=5+\dfrac{1}{2(x+1)}-\dfrac{1}{2}(x+1)^{-\frac{1}{2}}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=y\left(5+\dfrac{1}{2(x+1)}-\dfrac{1}{2}(x+1)^{-\frac{1}{2}}\right)$ Since $y(0)=\dfrac{1}{\mathrm{e}}$ $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=0}=\dfrac{1}{\mathrm{e}}\left(5+\dfrac{1}{2}-\dfrac{1}{2}\right)=\dfrac{5}{\mathrm{e}}$

(b)

Beaker:

The flow rate from the dripper is constant, as is the radius of the beaker. So the rate at which the depth of coffee increases is constant. Let the depth of coffee in the beaker be x.

$V = \pi r^2 x$ and

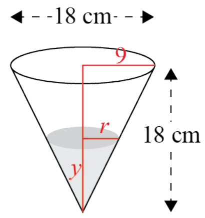
$$\frac{dV}{dt} = \pi r^2 \frac{dx}{dt}$$

$$50 = \pi \times 9^2 \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{50}{\pi \times 81} \text{ cm min}^{-1}$$

Dripper:

Let the depth of the coffee in the dripper be y and the radius of the surface of the liquid, r. Then, by similar triangles:



$$\frac{r}{y} = \frac{9}{18} = \frac{1}{2} \text{ and } r = \frac{y}{2}$$

$$V = \frac{1}{3}\pi r^2 y = \frac{1}{3}\pi\left(\frac{y}{2}\right)^2 y = \frac{1}{3}\pi\frac{y^3}{4}$$

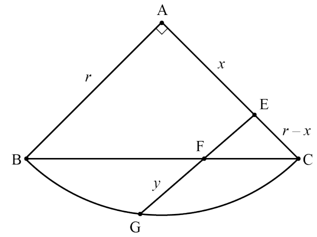
$$-\frac{dV}{dt} = \pi\frac{y^2}{4}\frac{dy}{dt}$$

$$-50 = \frac{\pi}{4} \times 9^2 \times \frac{dy}{dt}$$

$$\frac{dy}{dt} = -\frac{50 \times 4}{\pi \times 81} \text{ cm min}^{-1}$$

$$\text{Ratio} \left|\frac{dx}{dt}\right| : \left|\frac{dy}{dt}\right| = \frac{50}{\pi \times 81} : \frac{50 \times 4}{\pi \times 81} = 1:4$$

(c)



Let AE $= x$ and FG $= y$.
Then CE $= r - x$.
Since ΔABC ~ ΔEFC (∠BAC= ∠FEC and ∠ABC = ∠EFC as BA || GE)
ΔEFC is isosceles and EF = $r - x$
ΔAEG is right angled

GE $= \sqrt{r^2 - x^2}$ so

$$y = \sqrt{r^2 - x^2} - (r - x) = \sqrt{r^2 - x^2} - r + x$$

$$\frac{dy}{dx} = \frac{1}{2}\left(r^2 - x^2\right)^{-\frac{1}{2}} \times (-2x) + 1$$

Let $\frac{dy}{dx} = 0$ for max.

$$\frac{1}{2}\left(r^2 - x^2\right)^{-\frac{1}{2}} \times (-2x) + 1 = 0$$

$$\left(r^2 - x^2\right)^{-\frac{1}{2}} = \frac{1}{x}$$

$$r^2 - x^2 = x^2$$

$$r^2 = 2x^2$$

$$\Rightarrow x = \frac{1}{\sqrt{2}} r$$

So max is

$$y = \sqrt{r^2 - \frac{1}{2}r^2} - r + \frac{1}{\sqrt{2}} r$$

$$= \frac{1}{\sqrt{2}} r - r + \frac{1}{\sqrt{2}} r$$

$$= \frac{2}{\sqrt{2}} r - r$$

Alternate Solution

Let GT ⊥ BC, T is the foot on BC
Since BA || GE, ΔGTF is right angled isosceles triangle

GF = $\sqrt{2}$ GT

Max GF is when GT is max: i.e. T is the intersection of AG and BC

max GT = $r - \frac{r}{\sqrt{2}}$

max GF = $\left(\sqrt{2} - 1\right) r$.

Q	Solution
FOUR (a)	$\cos^6\theta = \left(\frac{e^{i\theta}+e^{-i\theta}}{2}\right)^6 = \frac{\left(e^{i\theta}+e^{-i\theta}\right)^6}{64}$ $= \frac{1}{64}\left\{\begin{array}{l}\left(e^{i\theta}\right)^6+6\left(e^{i\theta}\right)^5\left(e^{-i\theta}\right)+15\left(e^{i\theta}\right)^4\left(e^{-i\theta}\right)^2+20\left(e^{i\theta}\right)^3\left(e^{-i\theta}\right)^3+\\ 15\left(e^{i\theta}\right)^2\left(e^{-i\theta}\right)^4+6\left(e^{i\theta}\right)\left(e^{-i\theta}\right)^5+\left(e^{-i\theta}\right)^6\end{array}\right\}$ $= \frac{1}{64}\left\{e^{6i\theta}+6e^{4i\theta}+15e^{2i\theta}+20+15e^{-2i\theta}+6e^{-4i\theta}+e^{-6i\theta}\right\}$ $= \frac{1}{32}\left(\frac{e^{6i\theta}+e^{-6i\theta}}{2}\right)+\frac{6}{32}\left(\frac{e^{4i\theta}+e^{-4i\theta}}{2}\right)+\frac{15}{32}\left(\frac{e^{2i\theta}+e^{-2i\theta}}{2}\right)+\frac{20}{64}$ $= \frac{1}{32}\cos 6\theta+\frac{3}{16}\cos 4\theta+\frac{15}{32}\cos 2\theta+\frac{5}{16}$
(b)	Using symmetry, $A = 4\int_0^1 y\,dx$ $y = \sin^3 t$ $x = \cos^3 t$ and $\frac{dx}{dt} = -3\cos^2 t\sin t$ When $x = 0 \rightarrow t = \frac{\pi}{2}$ and $x = 1 \rightarrow t = 0$ The integral becomes $A = 4\times\int_{\frac{\pi}{2}}^{0} -3\sin^4 t\cos^2 t\,dt = 3\int_0^{\frac{\pi}{2}}\sin^2 2t\sin^2 t\,dt = 3\int_0^{\frac{\pi}{2}}\sin^2 2t\times\frac{1-\cos 2t}{2}\,dt$ $= \frac{3}{2}\int_0^{\frac{\pi}{2}}\frac{1-\cos 4t}{2}\,dt - \frac{3}{4}\int_0^{\frac{\pi}{2}}\sin^2 2t\,d(\sin 2t) = \frac{3}{2}\left(\frac{\pi}{4}-\frac{1}{2}+\frac{1}{2}\right)-\left(\frac{3}{4}\times\frac{1}{3}\times\sin^3 2t\right)_0^{\frac{\pi}{2}} = \frac{3\pi}{8}$

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Alternate solution:

$A = 4\int_0^{\frac{\pi}{2}} \left(\sin^3 t\right)\left(3\cos^2 t \sin t\right) \mathrm{d}t = 12\int_0^{\frac{\pi}{2}} \left(\sin^4 t\right)\left(\cos^2 t\right) \mathrm{d}t$

$12\int_0^{\frac{\pi}{2}} \left(\sin^4 t\right)\left(\cos^2 t\right) \mathrm{d}t$

$12\int_0^{\frac{\pi}{2}} \left(1-\cos^2 t\right)^2 \left(\cos^2 t\right) \mathrm{d}t$

$12\int_0^{\frac{\pi}{2}} \left(\cos^2 t - 2\cos^4 t + \cos^6 t\right) \mathrm{d}t$

$\cos^2 t = \frac{1}{2}\left(\cos 2t + 1\right) = \frac{1}{2}\cos 2t + \frac{1}{2}$

$\cos^6 t = \frac{1}{32}\cos 6t + \frac{3}{16}\cos 4t + \frac{15}{32}\cos 2t + \frac{5}{16}$

$\cos^4 t = \left(\frac{\mathrm{e}^{\mathrm{i}t} + \mathrm{e}^{-\mathrm{i}t}}{2}\right)^4$

$= \frac{1}{16}\left(\mathrm{e}^{4\mathrm{i}t} + 4\mathrm{e}^{2\mathrm{i}t} + 6 + 4\mathrm{e}^{-2\mathrm{i}t} + \mathrm{e}^{-4\mathrm{i}t}\right)$

$= \frac{1}{8}\cos 4t + \frac{1}{2}\cos 2t + \frac{3}{8}$

$2\cos^4 t = \frac{1}{4}\cos 4t + \cos 2t + \frac{3}{4}$

So

$\cos^2 t - 2\cos^4 t + \cos^6 t$

$= \frac{-1}{32}\cos 2t - \frac{1}{16}\cos 4t + \frac{1}{32}\cos 6t + \frac{1}{16}$

$= 12\int_0^{\frac{\pi}{2}} \left(-\frac{1}{32}\cos 2t - \frac{1}{16}\cos 4t + \frac{1}{32}\cos 6t + \frac{1}{16}\right) \mathrm{d}t$

$= 12\left[-\frac{1}{64}\sin 2t - \frac{1}{64}\sin 4t + \frac{1}{192}\sin 6t + \frac{t}{16}\right]_0^{\frac{\pi}{2}}$

$= 12\left[0 - 0 + 0 + \frac{\pi}{32} + 0 + 0 + 0 + 0\right]$

$= \frac{12\pi}{32}$

$= \frac{3\pi}{8}$

(c)	Using the cosine rule: $\left(\mathrm{F_1F_2}\right)^2 = \mathrm{PF_1}^2 + \mathrm{PF_2}^2 - 2\mathrm{PF_1PF_2}\cos\theta$ But $\mathrm{F_1F_2} = 2c$ and $\mathrm{PF_1} + \mathrm{PF_2} = 2a$ and $b^2 = a^2 - c^2$ for this ellipse $\therefore \left(2c\right)^2 = \mathrm{PF_1}^2 + \mathrm{PF_2}^2 - 2\mathrm{PF_1}\cdot\mathrm{PF_2}\cdot\cos\theta$ $= \left(\mathrm{PF_1} + \mathrm{PF_2}\right)^2 - 2\mathrm{PF_1PF_2}\left(1+\cos\theta\right) \therefore \mathrm{PF_1}\cdot\mathrm{PF_2} = \dfrac{2\left(a^2-c^2\right)}{1+\cos\theta}$ Since area $\Delta\mathrm{PF_1F_2} = \dfrac{1}{2}\cdot\mathrm{PF_1}\cdot\mathrm{PF_2}\cdot\sin\theta$ $= \dfrac{1}{2}\times\dfrac{2\left(a^2-c^2\right)}{1+\cos\theta}\sin\theta$ $= \dfrac{b^2\cdot 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}}{2\cos^2\dfrac{\theta}{2}}$ $= b^2\tan\dfrac{\theta}{2}$

Q	Solution
FIVE (a)	$\tan 45° = 1$ And since $\tan(90° - x) = \cot x$ $\tan 1° \times \tan 2° \times \tan 3° \times \ldots \times \tan 88° \times \tan 89°$ $= \tan 1° \times \tan 2° \times \tan 3° \times \ldots \times \tan(90° - 3°) \times \tan(90° - 2°) \times \tan(90° - 1°)$ $= \tan 1° \times \tan 2° \times \tan 3° \ldots \tan 45° \ldots\ldots \cot 3° \times \cot 2° \times \cot 1° = 1$ I.e. we have $\int_0^a 1\,\mathrm{d}t = a$
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\sqrt{x^2+1}+x\right)\right]$ $= \dfrac{\frac{x}{\sqrt{x^2+1}}+1}{\sqrt{x^2+1}+x}$ $= \dfrac{\frac{x}{\sqrt{x^2+1}}+\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}}{\sqrt{x^2+1}+x}$ $= \dfrac{\frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}}{\sqrt{x^2+1}+x}$ $= \dfrac{1}{\sqrt{x^2+1}}$

(c)

Let $\dfrac{dy}{dx} = A$ then

$$x\frac{dA}{dx} = \frac{v_1}{v_2}\sqrt{1+A^2}$$

$$\int \frac{dA}{\sqrt{1+A^2}} = \frac{v_1}{v_2}\int \frac{1}{x}dx$$

Using the given hint:

$$\int \frac{1}{\sqrt{1+x^2}}dx = \ln\left|\sqrt{1+x^2}+x\right| + c$$

$$\ln\left|\sqrt{1+A^2}+A\right| = \frac{v_1}{v_2}\ln x + K$$

Back substituting for A

$$\ln\left|\sqrt{1+\left(\frac{dy}{dx}\right)^2}+\frac{dy}{dx}\right| = \frac{v_1}{v_2}\ln x + K$$

Now $y'(1) = 0$

$$\ln\left|\sqrt{1+0}+0\right| = \frac{V_1}{V_2}\ln 1 + K$$

$$0 = K$$

$$\ln\left|\sqrt{1+\left(\frac{dy}{dx}\right)^2}+\frac{dy}{dx}\right| = \frac{v_1}{v_2}\ln x$$

$$\ln\left|\sqrt{1+\left(\frac{dy}{dx}\right)^2}+\frac{dy}{dx}\right| = \ln x^{\frac{v_1}{v_2}}$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2}+\frac{dy}{dx} = x^{\frac{v_1}{v_2}}$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = x^{\frac{v_1}{v_2}} - \frac{dy}{dx}$$

$$1+\left(\frac{dy}{dx}\right)^2 = x^{2\frac{v_1}{v_2}} - 2x^{\frac{v_1}{v_2}}\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^2$$

$$x^{\frac{2v_1}{v_2}} - 2x^{\frac{v_1}{v_2}}\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2}\left[x^{\frac{v_1}{v_2}} - x^{-\frac{v_1}{v_2}}\right]$$

	If $v_1 = v_2$ $\frac{dy}{dx} = \frac{1}{2}\left[x - x^{-1}\right]$ $\int dy = \frac{1}{2}\int\left[x - x^{-1}\right]dx$ $y = \frac{1}{2}\times\frac{1}{2}x^2 - \frac{1}{2}\ln x + k$ since $y(1) = 0$ $0 = \frac{1}{4} + k$ $k = -\frac{1}{4}$ $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x - \frac{1}{4}$ For $v_1 \neq v_2$ $\frac{dy}{dx} = \frac{1}{2}\left[x^{\frac{v_1}{v_2}} - x^{-\frac{v_1}{v_2}}\right]$ $\int dy = \frac{1}{2}\int\left[x^{\frac{v_1}{v_2}} - x^{-\frac{v_1}{v_2}}\right]dx$ $y = \frac{1}{2}\left[\frac{1}{\frac{v_1}{v_2}+1}x^{\left(\frac{v_1}{v_2}+1\right)} - \frac{1}{1-\frac{v_1}{v_2}}x^{\left(1-\frac{v_1}{v_2}\right)}\right] + k$ $y = \frac{1}{2}\left[\frac{v_2}{v_1+v_2}x^{\left(\frac{v_1}{v_2}+1\right)} + \frac{v_2}{v_1-v_2}x^{\left(1-\frac{v_1}{v_2}\right)}\right] + k$ Since $y(1) = 0$ $0 = \frac{1}{2}\left[\frac{v_2}{v_1+v_2} + \frac{v_2}{v_1-v_2}\right] + k$ $-k = \frac{1}{2}\left[\frac{v_1v_2 - v_2^2 + v_1v_2 + v_2^2}{v_1^2 - v_2^2}\right]$ $-k = \frac{1}{2}\left[\frac{2v_1v_2}{v_1^2 - v_2^2}\right]$ $k = \frac{v_1v_2}{v_2^2 - v_1^2}$ Finally $y = \frac{1}{2}\left[\frac{v_2}{v_1+v_2}x^{\left(\frac{v_1}{v_2}+1\right)} + \frac{v_2}{v_1-v_2}x^{\left(1-\frac{v_1}{v_2}\right)}\right] + \frac{v_1v_2}{v_2^2 - v_1^2}$	1 for $v_1 \neq v_2$

Sufficiency Statement

Score 1–4, no award	Score 5–6, Scholarship	Score 7–8, Oustanding Scholarship
Shows understanding of relevant mathematical concepts, and some progress towards solutions to problems.	Application of high-level mathematical knowledge and skills, leading to partial solutions to complex problems.	Application of high-level mathematical knowledge and skills, perception, and insight / convincing communication shown in finding correct solutions to complex problems.

Cut Scores

Scholarship	Outstanding Scholarship
21 – 32	33 – 40