S

93202A

932021

SUPERVISOR'S USE ONLY

OUTSTANDING SCHOLARSHIP EXEMPLAR

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tick this box if you have NOT written in this booklet

Scholarship 2022
Calculus

Time allowed: Three hours
Total score: 40

ANSWER BOOKLET

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Write your answers in this booklet.

Make sure that you have Formulae Booklet S–CALCF.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Answers developed using a CAS calculator require **ALL commands to be shown**. Correct answers only will not be sufficient.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area. This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Question	Score
ONE	
TWO	
THREE	
FOUR	
FIVE	
TOTAL	

ASSESSOR'S USE ONLY

1a. Let $z = x + yi$

$\Rightarrow |(x+a) + yi| = \sqrt{a}\,|(x+1) + yi|$

$\Rightarrow |(x+a) + yi|^2 = a\,|(x+1) + yi|^2$

$\therefore (x+a)^2 + y^2 = a[(x+1)^2 + y^2]$

$x^2 + 2ax + a^2 + y^2 = ax^2 + 2ax + a + ay^2$

$\Rightarrow (a-1)x^2 + (a-1)y^2 = a^2 - a$

$(a-1)(x^2+y^2) = a(a-1)$

As $a \neq 1 \Rightarrow x^2 + y^2 = |z|^2 = a$

hence $|z| = \sqrt{a}$

1b. From (A), $y = \frac{\pi}{4} - x$

Substitute into (B) gives:

$\tan x + \tan\left(\frac{\pi}{4} - x\right) = 1$

$\tan x + \dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x} = 1$

$\tan x + \dfrac{1 - \tan x}{1 + \tan x} = 1$

$\tan x + \tan^2 x + 1 - \tan x = 1 + \tan x$

$\tan^2 x - \tan x = 0$

$\tan x(\tan x - 1) = 0$

$\Rightarrow \tan x = 0, \tan x = 1$

Hence $(x, y) = (k\pi, \frac{\pi}{4} - k\pi), k \in \mathbb{Z}$ or $(x, y) = (\frac{\pi}{4} - k\pi, k\pi), k \in \mathbb{Z}$

1c. Given $x^4 + x^3 - 4x^2 + x + 1 = 0$, notice $x = 1$ is a root.

$\Rightarrow \cancel{(x-1)(x^3}$

Apply synthetic division

	1	1	-4	1	1
1		1	2	-2	-1
	1	2	-2	-1	0

$\Rightarrow (x-1)(x^3+2x^2-2x-1) = 0$

Given $x<0$, $x \neq 1$

$\Rightarrow x^3+2x^2-2x-1 = 0$, notice $x=1$ is a root

$\Rightarrow (x-1)(x^2+3x+1) = 0$

Given $x<0$, $x-1 \neq 0$

$\Rightarrow x^2+3x+1 = 0$

Roots are $x = \frac{-3 \pm \sqrt{5}}{2}$

For $x = \frac{-3+\sqrt{5}}{2}$, $x^3 = \frac{1}{8}(\sqrt{5}-3)^3$

$= \frac{1}{8}[5\sqrt{5} - 45 + 27\sqrt{5} - 27]$

$= \frac{1}{8}[32\sqrt{5} - 72]$

$= 4\sqrt{5} - 9$

$\Rightarrow x^3 + \frac{1}{x^3} = 4\sqrt{5} - 9 + \frac{1}{4\sqrt{5}-9}$

$= 4\sqrt{5} - 9 + \frac{4\sqrt{5}+9}{-1}$

$= -18$

For $x = \frac{-3-\sqrt{5}}{2}$, $x^3 = \frac{-1}{8}(3+\sqrt{5})^3$

$= -\frac{1}{8}[27 + 27\sqrt{5} + 45 + 5\sqrt{5}]$

$= -\frac{1}{8}[72 + 32\sqrt{5}]$

$= -9 - 4\sqrt{5}$

$\Rightarrow x^3 + \frac{1}{x^3} = -9 - 4\sqrt{5} - \frac{1}{9+4\sqrt{5}}$

$= -9 - 4\sqrt{5} - \frac{9-4\sqrt{5}}{1}$

$= -18$

Hence $x^3 + \frac{1}{x^3} = -18$

2a. $x^2 - 4x + 10 = kx^2 + 2kx + k$

$\Rightarrow (k-1)x^2 + (2k+4)x + (k-10) = 0$

Two distinct real roots $\Rightarrow \nabla = b^2 - 4ac > 0$

$\Rightarrow (2k+4)^2 - 4(k-1)(k-10) > 0$

$4k^2 + 16k + 16 - 4(k^2 - 11k + 10) > 0$

$60k - 24 > 0$

$k > \frac{2}{5}$

Must also have roots with same sign:

~~Hence~~ Roots are $\frac{-(2k+4) \pm \sqrt{60k-24}}{2k-2}$

$x_{12} = \frac{-(k+2) \pm \sqrt{15k-6}}{2k-2}$

~~Case 1: $k > 1 \Rightarrow$ Denominator is positive~~

~~Must have $-(k+2) - \sqrt{15k-6} > 0$ for both positive~~

Must have numerator having same signs.

Know $-(k+2) + \sqrt{15k-6} > -(k+2) - \sqrt{15k-6}$.

Hence either $-(k+2) + \sqrt{15k-6} < 0$ or $-(k+2) - \sqrt{15k-6} > 0$

$-(k+2) + \sqrt{15k-6} < 0$

$\Rightarrow \sqrt{15k-6} < k+2$

$15k - 6 < k^2 + 4k + 4$

$k^2 - 11k + 10 > 0$

$(k-1)(k-10) > 0$

$k < 1$ or $k > 10$

~~Given $k > \frac{2}{5}$, domain is $(\frac{2}{5}, 1) \cup (1,$~~

$-(k+2) - \sqrt{15k-6} > 0$

$\Rightarrow -\sqrt{15k-6} > k+2$

$\sqrt{15k-6} < -(k+2)$

Hence must have $|-(k+2)| > |\sqrt{15k-6}|$

$|k+2| > \sqrt{15k-6}$

$k^2 + 4k + 4 > 15k - 6$

$$k^2 - 11k + 10 > 0$$
$$(k-1)(k-10) > 0$$
$$\Rightarrow k < 1, \quad k > 10.$$

Given $k > \frac{2}{5}$, desired interval for k is $\left(\frac{2}{5}, 1\right) \cup (10, \infty)$

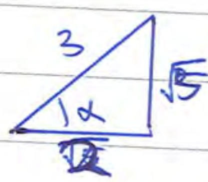
2b. Using sine rule:

$$\frac{\sin(2\alpha)}{16} = \frac{\sin\alpha}{12}$$

$$\Rightarrow \frac{2\sin\alpha\cos\alpha}{16} = \frac{\sin\alpha}{12}$$

$$\frac{\cos\alpha}{8} = \frac{1}{12}$$

$$\cos\alpha = \frac{2}{3}$$

Sketch up triangle 

, this gives $\sin\alpha = \frac{\sqrt{5}}{3}$

$\angle BAC = \pi - 3\alpha$, as $\angle$s in Δ Σ to π radians

$$A_\Delta = \cancel{\tfrac{1}{2}\ldots\sin} \ \tfrac{1}{2}|AB||AC|\sin\angle BAC$$
$$= \tfrac{1}{2}(16)(12)\sin(\pi - 3\alpha)$$
$$= 96\sin 3\alpha$$
$$= 96\sin(a + 2\alpha)$$
$$= 96\left[\sin\alpha\cos 2\alpha + \cos\alpha\sin 2\alpha\right]$$
$$= 96\left[\sin\alpha(2\cos^2\alpha - 1) + 2\sin\alpha\cos^2\alpha\right]$$
$$= 96\left[\frac{\sqrt{5}}{3}\left(\frac{8}{9} - 1\right) + 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{4}{9}\right)\right]$$
$$= 96\left[-\frac{\sqrt{5}}{27} + \frac{8\sqrt{5}}{27}\right]$$
$$= 96 \times \frac{7\sqrt{5}}{27}$$
$$= \frac{224\sqrt{5}}{9}$$

2c. Without loss of generality, let common ratio $r \geq 1$ such that sides of Δ in ascending order is a, ar, ar^2

Let angle opposite a be β and angle opposite ar^2 be γ

$$\Rightarrow \frac{\sin\beta}{a} = \frac{\sin\alpha}{ar} = \frac{\sin\gamma}{ar^2}$$

$$\Rightarrow \sin\alpha = r\sin\beta = \frac{1}{r}\sin\gamma$$

$$\Rightarrow \sin\gamma = r\sin\alpha$$

By Triangle inequality, $ar + a > ar^2$

$$\Rightarrow r + 1 > r^2$$

$$r^2 - r - 1 < 0$$

Roots at $r = \frac{1 \pm \sqrt{5}}{2}$

Based on restrictions, $1 \leq r < \frac{1+\sqrt{5}}{2}$

As $\sin\alpha = \frac{\sin\gamma}{r}$, and $\alpha < 90^\circ$ as α does not correspond to longest side, $\sin\alpha$ has maximum value equal to $\frac{1}{r}$ i.e. when $\sin\gamma = 1$

$$a^2r^2 = a^2 + a^2r^4 - 2a^2r^2\cos\alpha$$

$$r^2 = 1 + r^4 - 2r^2\cos\alpha$$

$$\cos\alpha = \frac{r^4 - r^2 + 1}{2r^2} \leq 1$$

$$\Rightarrow r^4 - r^2 + 1 \leq 2r^2$$

$$r^4 - 3r^2 + 1 \leq 0$$

Roots at $r^2 = \frac{3 \pm \sqrt{5}}{2}$

3a. $f(x) = \frac{e^{5x}\sqrt{x+1}}{e^{\sqrt{x+1}}}$, $f(0) = \frac{e^{0}\sqrt{1}}{e} = \frac{1}{e}$

$\Rightarrow \ln[f(x)] = \ln e^{5x} + \frac{1}{2}\ln(x+1) - \sqrt{x+1}$

$\Rightarrow \frac{f'(x)}{f(x)} = 5 + \frac{1}{2(x+1)} - \frac{1}{2\sqrt{x+1}}$

$\Rightarrow f'(x) = f(x)\left[5 + \frac{1}{2(x+1)} - \frac{1}{2\sqrt{x+1}}\right)$

$\therefore f'(0) = f(0)\left[5 + \frac{1}{2} - \frac{1}{2}\right]$

$= \frac{5}{e}$

3b. Let depth in dripper at any moment in time in cm be h

$\Rightarrow$ Volume of dripper at any given height h, $V_{dripper} = \frac{1}{3}81\pi h = 27\pi h$

Hence $\frac{dV_{dripper}}{dh} = 27\pi$.

Know $\frac{dV_{dripper}}{dt} = -50 \text{ cm}^3 \text{ min}^{-1}$ (negative as decreasing)

Hence $\frac{dh}{dt} = \frac{dh}{dV_{dripper}} \cdot \frac{dV_{dripper}}{dt} = \frac{-50}{27\pi} \text{ cm min}^{-1}$

For beaker, let depth of coffee be H, then $V = 81\pi H$, $\frac{dV}{dH} = 81\pi$

$\frac{dV_{beaker}}{dt} = 50 \text{ cm}^3 \text{ min}^{-1}$

$\frac{dH_{beaker}}{dt} = \frac{50}{81\pi} \text{ cm min}^{-1}$.

$\therefore$ Ratio of $\frac{dH}{dt}$ to $\frac{dh}{dt}$ when the dripper depth is 9cm is $\frac{50}{81\pi} \times \frac{-27\pi}{50} = -\frac{1}{3}$ (negative as one is increasing while the other is decreasing, and independent of depth).

3c. Let the distance AE be x, then $EC = r - x$

$\angle AEF = 90^\circ$ as $EG \parallel AB$ so $\angle AEF = \angle EAB$, co int $\angle$s $\parallel$ lines are equal.

$\angle ACB = 45^\circ$ as isos Δ have equal base angles.

Hence length $EF = (\cancel{r}-x)\tan(45^\circ) = r-x$

Length $EG = \sqrt{AG^2 - AF^2}$

$= \sqrt{r^2-x^2}$

Length $FG = EG - EF$

$l_{FG} \overset{set}{=} L = \sqrt{r^2-x^2} - r + x$

$\frac{dL}{dx} = \frac{-2x}{2\sqrt{r^2-x^2}} + 1 \overset{set}{=} 0$

$\Rightarrow \frac{x}{\sqrt{r^2-x^2}} = 1$

$x^2 = r^2 - x^2$

$2x^2 = r^2$

$x = \frac{r}{\sqrt{2}}$ as $x > 0$ (length).

Maximum length of $FG = \sqrt{r^2 - \frac{r^2}{2}} - r + \frac{r}{\sqrt{2}}$

$= \frac{r}{\sqrt{2}} - r + \frac{r}{\sqrt{2}}$

$= (\sqrt{2}-1)r$

4a. Notice that $\cos\theta + i\sin\theta$

$= \frac{e^{i\theta}+e^{-i\theta}}{2} + \frac{e^{i\theta}-e^{-i\theta}}{2}$

$= e^{i\theta}$

$\Rightarrow \cos^6\theta = \frac{1}{64}(e^{i\theta}+e^{-i\theta})^6$

$= \frac{1}{64}\left[e^{i6\theta} + 6e^{i4\theta} + 15e^{i2\theta} + 20 + 15e^{i(-2\theta)} + 6e^{i(4\theta)} + e^{i(-6\theta)}\right]$

$= \frac{1}{64}\left[\cos(6\theta) + i\sin 6\theta + 6(\cos 4\theta + i\sin 4\theta) + 15(\cos 2\theta + i\sin 2\theta)\right.$
$+ 21 + 15(\cos(-2\theta) + i\sin(-2\theta)) + 6(\cos(-4\theta) + i\sin(-4\theta))$
$\left. + \cos(-6\theta) + i\sin(-6\theta)\right]$

Know $\cos(-x) = \cos(x)$, even function, $\sin(-x) = -\sin(x)$, odd function.

$\Rightarrow \cos^6\theta = \frac{1}{64}\left[2\cos(6\theta) + 12\cos 4\theta + 30\cos(2\theta) + 20\right]$

$= \frac{1}{32}\cos(6\theta) + \frac{3}{16}\cos(4\theta) + \frac{15}{32}\cos(2\theta) + \frac{5}{16}$ as required.

4b. By symmetry of astroid, note that $x^{\frac{2}{3}} + y^{\frac{2}{3}} = \cos^2 t + \sin^2 t = 1$

$\Rightarrow y^{\frac{2}{3}} = 1 - x^{\frac{2}{3}}$

$y = \sqrt{(1-x^{\frac{2}{3}})^3}$ is the curve in the first quadrant

$= (1-x^{\frac{2}{3}})^{\frac{3}{2}}$

Hence area of astroid $= 4\int_0^1 y\,dx$

$= 4\int_0^1 (1-x^{\frac{2}{3}})^{\frac{3}{2}}\,dx$, let $x = \sin^3\theta$

$\Rightarrow dx = 3\sin^2\theta\cos\theta\,d\theta$

When $x = 0$, $\theta = 0$

When $x = 1$, $\theta = \frac{\pi}{2}$

$\Rightarrow 4\int_0^{\frac{\pi}{2}} (1-\sin^2\theta)^{\frac{3}{2}} \cdot 3\sin^2\theta\cos\theta\,d\theta$

$= 12\int_0^{\frac{\pi}{2}} \cos^3\theta \sin^2\theta \cos\theta\,d\theta$

$\cancel{= 12\int_0^{\frac{\pi}{2}} \cos 4\theta - \cos 6\theta\,d\theta}$

$= 12\int_0^{\frac{\pi}{2}} \cos^4\theta - \cos^6\theta\,d\theta$

Using method in part (a), $\cos^4\theta = \frac{1}{16}(e^{i\theta} + e^{-i\theta})^4$

$= \frac{1}{16}\left(e^{i4\theta} + 4e^{i2\theta} + 6 + 4e^{i(-2\theta)} + e^{i(-4\theta)}\right)$

$= \frac{1}{16}\left(2\cos(4\theta) + 8\cos(2\theta) + 6\right)$

$= \frac{1}{8}\cos(4\theta) + \frac{1}{2}\cos(2\theta) + \frac{3}{8}$

Hence $\cos^4\theta - \cos^6\theta = -\frac{1}{32}\cos(6\theta) - \frac{1}{16}\cos(4\theta) + \frac{1}{32}\cos(2\theta) + \frac{1}{16}$

$\Rightarrow 12\int_0^{\frac{\pi}{2}} \cos^4\theta - \cos^6\theta\,d\theta = 12\left[\frac{-1}{192}\sin(6\theta) - \frac{1}{64}\sin(4\theta) + \frac{1}{64}\sin(2\theta) + \frac{1}{16}\theta\right]_0^{\frac{\pi}{2}}$

$= 12\left[0 - 0 + 0 + \frac{\pi}{32} - (0)\right]$

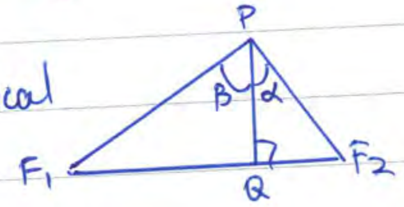
$= \frac{3\pi}{8}$

4c. Length $PF_1 = \sqrt{(x_0+c)^2+(y_0)^2}$, length $PF_2 = \sqrt{(x_0-c)^2+(y_0)^2}$

Length $F_1F_2 = 2c \Rightarrow 4c^2 = (x_0+c)^2 + y_0^2 + (x_0-c)^2 + (y_0)^2 - 2\sqrt{((x_0+c)^2+y_0^2)((x_0-c)^2+y_0^2)}\cos\theta$

$\Rightarrow 4c^2 = 2x_0^2 + 2c^2 + 2y_0^2 - 2\sqrt{(x_0^2-c^2)^2 + y_0^2(x_0^2+c^2)}\cos\theta$

$c^2 = x_0^2 + y_0^2 - \sqrt{(x_0^2-c^2)^2 + y_0^2(x_0^2+c^2)}\cos\theta$



Let Q be point on x-axis such that PQ is vertical

Let α and β be angle as shown

Then $\tan\alpha = \dfrac{c-x_0}{y_0}$, $\tan\beta = \dfrac{x_0+c}{y_0}$

$\tan(\theta) = \tan(\alpha+\beta) = \dfrac{\frac{2c}{y_0}}{1-\frac{c^2-x_0^2}{y_0^2}} = \dfrac{2cy_0}{y_0^2-c^2+x_0^2}$, Note $F_1F_2 = 2c$ hence area of $\Delta PF_1F_2 = cy_0 \overset{\text{set}}{=} A$

$\Rightarrow \dfrac{2\tan(\frac{\theta}{2})}{1-\tan^2(\frac{\theta}{2})} = \dfrac{2A}{x_0^2+y_0^2-c^2}$

$(x_0^2+y_0^2-c^2)\tan\left(\frac{\theta}{2}\right) = A - A\tan^2\left(\frac{\theta}{2}\right)$

~~A =~~

$A\tan^2\left(\frac{\theta}{2}\right) + (x_0^2+y_0^2-c^2)\tan\left(\frac{\theta}{2}\right) - A = 0$

$\tan\left(\frac{\theta}{2}\right) = \dfrac{-(x_0^2+y_0^2-c^2) + \sqrt{(x_0^2+y_0^2-c^2)^2+4A^2}}{2A}$

5a. Given $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$\Rightarrow \tan(A)\tan(B) = \frac{\cos(A-B) - \cos(A+B)}{\cos(A+B) + \cos(A-B)}$$

Hence $\tan(x)\tan(90^\circ - x) = \dfrac{\cos(2x-90^\circ) - \cos(90^\circ)}{\cos(90^\circ) + \cos(2x-90^\circ)}$

$$= \frac{\cos(2x-90^\circ)}{\cos(2x-90^\circ)}$$

$$= 1$$

$$\therefore \cancel{\int_0^a \prod_{k=1}} \int_0^a \left(\tan(1^\circ)\tan(2^\circ)\ldots\tan(89^\circ)\right) dx$$

$$= \int_0^a \prod_{k=1}^{44} \tan(k^\circ)\tan(90^\circ - k^\circ) \times \tan(45^\circ)\, dx$$

$$= \int_0^a dx$$

$$= a$$

b. $\displaystyle\int \frac{1}{\sqrt{1+x^2}}\, dx$, let $x = \tan\theta$

$$\Rightarrow dx = \sec^2\theta\, d\theta$$

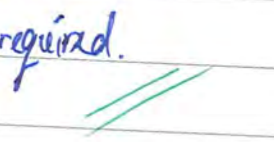
$$\Rightarrow \int \frac{1}{\sqrt{1+\tan^2\theta}} \sec^2\theta\, d\theta$$

$$= \int \frac{1}{\sqrt{\sec^2\theta}} \sec^2\theta\, d\theta$$

$$= \int \sec\theta\, d\theta$$

$= \displaystyle\int \frac{\sec\theta(\sec\theta + \tan\theta)}{\sec\theta + \tan\theta}\, d\theta$, notice $\frac{d}{d\theta}(\sec\theta + \tan\theta) = \sec\theta\tan\theta + \sec^2\theta$ which is the numerator.

$= \ln|\sec\theta + \tan\theta| + C$, $\tan\theta = x$



From triangle, deduce $\sec\theta = \sqrt{x^2+1}$, $\tan\theta = x$

$\therefore \displaystyle\int \frac{1}{\sqrt{1+x^2}}\, dx = \ln\left|\sqrt{x^2+1} + x\right| + C$ as required.

5c. Let $u(x) = \frac{dy}{dx}$

Then $x\frac{dy}{dx} = \frac{v_1}{\sqrt{2}}\sqrt{1+u^2}$

$\Rightarrow \frac{1}{\sqrt{1+u^2}}\,du = \frac{v_1}{\sqrt{2}\cancel{x}}\,dx$

$\Rightarrow \int \frac{1}{\sqrt{1+u^2}}\,du = \int \frac{v_1}{\sqrt{2}}x\,dx$

$\ln\left|\sqrt{1+u^2}+u\right| = \frac{v_1}{2\sqrt{2}}x^2 + C_1$

Given $f'(1) = u(1) = 0$

$\Rightarrow \ln\left|\sqrt{1}+0\right| = \frac{v_1}{2\sqrt{2}}(1) + C_1$

$C_1 = \frac{-v_1}{2\sqrt{2}}$

$\Rightarrow \ln\left|\sqrt{1+\left(\frac{dy}{dx}\right)^2}+\frac{dy}{dx}\right| = \frac{v_1}{2\sqrt{2}}x^2 - \frac{v_1}{2\sqrt{2}}$

$\sqrt{1+\left(\frac{dy}{dx}\right)^2}+\frac{dy}{dx} = e^{\frac{-v_1}{2\sqrt{2}}}e^{\frac{v_1}{2\sqrt{2}}x^2}$

$\Rightarrow \int \frac{1}{\sqrt{1+u^2}}\,du = \int \frac{v_1}{\sqrt{2}x}\,dx$

$\ln\left|\sqrt{1+u^2}+u\right| = \frac{v_1}{\sqrt{2}}\ln|x| + C_1$

Know when $x=1$, $u=0$

$\Rightarrow \ln\left|\sqrt{1}+0\right| = \frac{v_1}{\sqrt{2}}\ln|1| + C_1$

Hence $C_1 = 0$

$\ln\left|\sqrt{1+\left(\frac{dy}{dx}\right)^2}+\frac{dy}{dx}\right| = \frac{v_1}{\sqrt{2}}\ln|x|$

$\sqrt{1+\left(\frac{dy}{dx}\right)^2}+\frac{dy}{dx} = x^{\frac{v_1}{\sqrt{2}}}$

93202A

Subject	Scholarship Calculus	*Standard*	93202	*Total score*	36
Q	***Grade score***	***Annotation***			
1	8	The candidate showed strong algebraic skills in factorising polynomials in 1b. They demonstrated ability of setting out work logically and with clarity.			
2	7	The candidate demonstrated logical thinking skills in 2a and successfully transformed the question into solving a quadratic inequality. They showed ability of manipulating sine rule, cosine rule and area of triangle involving compound angles in 2b. They showed understanding of geometric sequence and successfully found an expression for the angle that need to be optimised in 2c. They would have solved it if they could identify it's a quadratic expression in disguise.			
3	7	The candidate showed elegance in their dealing with transforming a geometry question into an algebraic function and then optimised it using calculus in 3c.			
4	7	The candidate showed competence in drawing new ideas from previous question 4a in integrating trig expression involving high powers in 4b. The candidate recognised the relation between the required angle and the coordinates of the point but abandoned the work pre-maturely in 5c.			
5	7	The candidate demonstrated in-depth understanding of trig identities and trig integration in 5b. They successfully transformed the 2nd order differential equation into a separable first order equation through substitution in 5a, however they stopped for further progression.			