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QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tick this box if you have NOT written in this booklet

Scholarship 2022
Physics

Time allowed: Three hours
Total score: 32

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should answer ALL the questions in this booklet.

For all 'describe' or 'explain' questions, the answers should be written or drawn clearly with all logic fully explained.

For all numerical answers, full working must be shown and the answer must be rounded to the correct number of significant figures and given with the correct SI unit.

Formulae you may find useful are given on page 3.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–20 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area. This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Question	Score
ONE	
TWO	
THREE	
FOUR	
TOTAL	

ASSESSOR'S USE ONLY

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The examination starts on page 4.

The formulae below may be of use to you.

$v_f = v_i + at$	$T = 2\pi\sqrt{\frac{l}{g}}$	$F = BIL$
$d = v_i t + \frac{1}{2}at^2$	$T = 2\pi\sqrt{\frac{m}{k}}$	$V = BvL$
$d = \frac{v_i + v_f}{2}t$	$E_p = \frac{1}{2}ky^2$	$\phi = BA$
$v_f^2 = v_i^2 + 2ad$	$F = -ky$	$\varepsilon = -\frac{\Delta\phi}{\Delta t}$
$F_g = \frac{GMm}{r^2}$	$a = -\omega^2 y$	$\varepsilon = -L\frac{\Delta I}{\Delta t}$
$F_c = \frac{mv^2}{r}$	$y = A\sin\omega t$ $y = A\cos\omega t$	$\frac{N_p}{N_s} = \frac{V_p}{V_s}$
$\Delta p = F\Delta t$	$v = A\omega\cos\omega t$ $v = -A\omega\sin\omega t$	$E = \frac{1}{2}LI^2$
$\omega = 2\pi f$	$a = -A\omega^2\sin\omega t$ $a = -A\omega^2\cos\omega t$	$\tau = \frac{L}{R}$
$d = r\theta$	$\Delta E = Vq$	$I = I_{MAX}\sin\omega t$
$v = r\omega$	$P = VI$	$V = V_{MAX}\sin\omega t$
$a = r\alpha$	$V = Ed$	$I_{MAX} = \sqrt{2}\,I_{rms}$
$W = Fd$	$Q = CV$	$V_{MAX} = \sqrt{2}\,V_{rms}$
$F_{net} = ma$	$C_T = C_1 + C_2$	$X_C = \frac{1}{\omega C}$
$p = mv$	$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$	$X_L = \omega L$
$x_{COM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$	$E = \frac{1}{2}QV$	$V = IZ$
$\omega = \frac{\Delta\theta}{\Delta t}$	$C = \frac{\varepsilon_o\varepsilon_r A}{d}$	$f_0 = \frac{1}{2\pi\sqrt{LC}}$
$\alpha = \frac{\Delta\omega}{\Delta t}$	$\tau = RC$	$v = f\lambda$
$L = I\omega$	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$	$f = \frac{1}{T}$
$L = mvr$	$R_T = R_1 + R_2$	$n\lambda = \frac{dx}{L}$
$\tau = I\alpha$	$V = IR$	$n\lambda = d\sin\theta$
$\tau = Fr$		$f' = f\frac{V_W}{V_W \pm V_S}$
$E_{K(ROT)} = \frac{1}{2}I\omega^2$		$E = hf$
$E_{K(LIN)} = \frac{1}{2}mv^2$		$hf = \phi + E_K$
$\Delta E_p = mg\Delta h$		$E = \Delta mc^2$
$\omega_f = \omega_i + \alpha t$		$\frac{1}{\lambda} = R\left(\frac{1}{S^2} - \frac{1}{L^2}\right)$
$\omega_f^2 = \omega_i^2 + 2\alpha\theta$		$E_n = -\frac{hcR}{n^2}$
$\theta = \frac{(\omega_i + \omega_f)}{2}t$		
$\theta = \omega_i t + \frac{1}{2}\alpha t^2$		

QUESTION ONE: PHOTONS

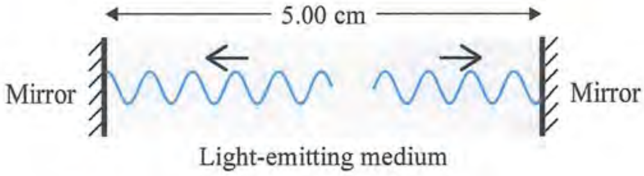
Radius of Earth	$= 6.37 \times 10^{6}$ m	Mean Earth-Sun distance	$= 1.50 \times 10^{11}$ m
Mass of Earth	$= 5.98 \times 10^{24}$ kg	Mass of Sun	$= 1.99 \times 10^{30}$ kg
Speed of light	$= 3.00 \times 10^{8}$ m s^{-1}	Planck's constant	$= 6.63 \times 10^{-34}$ J s
Surface area of a sphere	$= 4\pi r^2$		

(a) The description of the photoelectric effect and the Bohr model of the atom both involve the concept of the quantisation of energy. There are similarities and differences in how this concept is applied in these contexts.

Describe ONE difference in the use of the concept of the quantisation of energy between the photoelectric effect and the Bohr model of the atom.

The quantisation of energy in the Bohr model involves the quantised energy states of the atom's electrons – they occupy discrete, fixed positions. with discrete angular momentum. In the photoelectric effect, the quantisation of ~~eg~~ energy is seen by modelling light as photons – packets (quanta) of a discrete amount of electromagnetic energy.

(b) A laser typically consists of a medium that emits light placed between two mirrors that form a cavity, as illustrated on the right. The cavity is similar to a closed box for the emitted light waves.

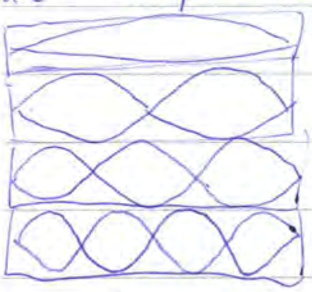


The light-emitting medium can emit a continuous spectrum of light within a narrow range of wavelengths. The minimum wavelength of emitted light is 480 nm (4.80×10^{-7} m), and the maximum wavelength is 490 nm (4.90×10^{-7} m). The cavity is 5.00 cm long.

Some wavelengths within this range are able to form standing waves within the cavity. These are called standing wave modes.

Calculate the total number of standing wave modes possible in the cavity within the emitted 480–490 nm range.

When reflected off mirror, the mirror effectively acts as a fixed end to form nodes due to 180° phase change. Standing wave modes occur as follows:



$L = \frac{1}{2}\lambda \Rightarrow \lambda = \frac{2L}{1}$

$L = \lambda \Rightarrow \lambda = \frac{2}{2}L$

$L = \frac{3}{2}\lambda \Rightarrow \lambda = \frac{2}{3}L$

$L = \frac{4}{2}\lambda \Rightarrow \lambda = \frac{2}{4}\lambda$

$\therefore \lambda = \frac{2L}{n}$ where $n \in \mathbb{Z}$.

consider then: $n = \frac{2L}{\lambda}$.

• For $\lambda = 480\text{nm}$, $n = \frac{2(5.00 \times 10^{-2})}{480 \times 10^{-9}} = 208\,333\ldots$ (0dp).

• For $\lambda = 490\text{nm}$, $n = \frac{2(5.00 \times 10^{-2})}{490 \times 10^{-7}} = 204\,081$ (0dp)

number of possible modes: $n \in \{204082, 204083 \ldots 208332, 208333\}$

$\therefore$ # standing wave modes $= 208333 - 204081 = \boxed{4252}$

(c) The process of nuclear fusion in the Sun releases energy which spreads through space in the form of electromagnetic radiation. The photons that make up this radiation carry momentum as well as energy, with the momentum per photon given by:

$$p = \frac{h}{\lambda} \rightarrow E = \frac{hc}{\lambda} = pc.$$

, a given mass converted to photons has same momentum independent of λ.

where h is Planck's constant, and λ is the photon wavelength.

The Sun loses 4.30×10^9 kg of mass each second due to nuclear reactions.

Estimate the force exerted on the Earth by the photons it receives from the Sun.

Assume each photon has a wavelength of 550 nm (5.50×10^{-7} m), and that every photon that reaches Earth is absorbed.

~~$\frac{\Delta m}{\Delta t} = 4.30 \times 10^9 \text{ kg s}^{-1}$. But if $\Delta p = \Delta(mv) = \frac{\Delta m}{\Delta t}$~~

~~But $F = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = \frac{\Delta m}{\Delta t} c$.~~

~~$\therefore F = \frac{\Delta m}{\Delta t} c$~~

~~$F = (4.30 \times 10^9$~~

~~$p = \frac{h}{\lambda}$ 6.63×10~~

[continued at back of booklet].

(d) (i) Comment on the significance of the size of the force exerted on the Earth by the photons.

5.82×10^8 N is ~~significant it is a continued force that is~~ continuously applied on Earth. Although large, the Earth's mass of 5.98×10^{24} kg indicates only an acceleration of $\frac{F}{m} = \frac{5.82 \times 10^8}{5.98 \times 10^{24}} = 9.7 \times 10^{-17} \text{ ms}^{-2}$ occurs which is insignificant. if Earth is "pushed back" it will simply receive less photons, so F decreases.

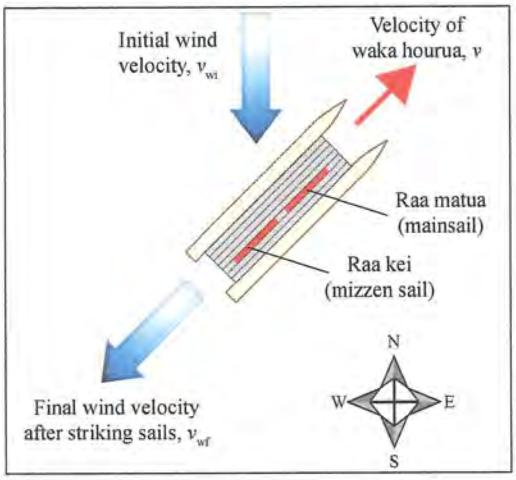
(ii) If the Earth were covered by ice it would be more reflective.

Explain how this would affect your answer to part (c).

Less of the photons would be absorbed as a large portion is reflected. $\therefore$ ~~[illegible]~~ However, for the photon to be reflected, it must then travel in the opposite direction (towards the sun). The Earth must then have twice the momentum change (by the conservation of momentum) per these photons ~~per second~~. $\therefore$ Greater "F" would occur. (emitted photons towards sun, when reflected, carry momentum. $\therefore$ Earth must have greater force exerted on it by these reflected photons)

QUESTION TWO: WAKA HOURUA

The history of sailing in New Zealand goes right back to the original settlement by ancestors of Māori, more than 700 years ago. The ancestors, from Polynesia, designed double-hulled boats with triangular sails, called waka hourua, that were strong, stable, and most importantly, able to sail into the wind. This allowed them to carry out exploratory voyages. From such exploration, they were able to plan and carry out their migration to Aotearoa. Waka hourua are able to sail against the wind by heading at an angle to the wind, as shown in the simplified diagram below right.



Adapted from: www.sciencelearn.org.nz/images/701-te-aurere

As measured by a stationary observer, the initial wind velocity is 10.0 m s^{-1} from the north, the velocity of the waka hourua is 6.00 m s^{-1} to the north-east, and the final wind velocity is 10.0 m s^{-1} towards the south-west, in the opposite direction to the velocity of the waka hourua.

(a) (i) By considering the wind direction before and after striking the sails, use impulse and momentum to explain how the wind produces a force on the sail.

Assume no external force, i.e. conservation of momentum applies.

Then initially, p_{system}: | Finally: | Wind collides with sails to produce impulse. This is done at an angle, so the resultant force on the waka is oblique.

p_{waka} p_{wind} p_{system} | p_{waka} p_{wind}

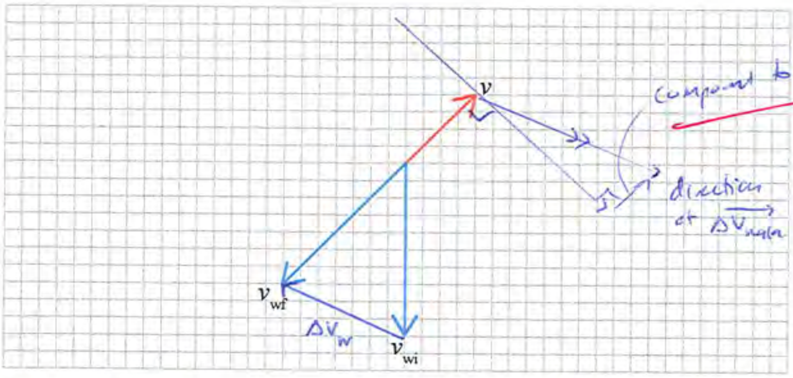
~~Note that,~~

The wind changes direction from Southward to South-west after striking the sail. ∴ It must have ~~expected~~ experienced a force ($F_{[illegible]}$) from the sails to change the direction but not the size of velocity. By Newtons 3rd law, this ~~[illegible]~~ means the sails also experience an equal and opposite ~~force~~ force over ~~[illegible]~~ same period of ~~[illegible]~~ time. ∴ experiences a ~~change~~ change in momentum, i.e. impulse. ($\Delta p = F\Delta t$).

(ii) Explain how the wind produces a force on the waka hourua that has a component in the direction that the waka hourua is travelling.

Use the grid below to draw a vector diagram to help your explanation. The vectors for the initial wind velocity v_{wi}, the final wind velocity v_{wf}, and the velocity v of the waka hourua are drawn on the grid to help you.
Start your answer by finding the vector for the change Δv_w in the velocity of the wind.

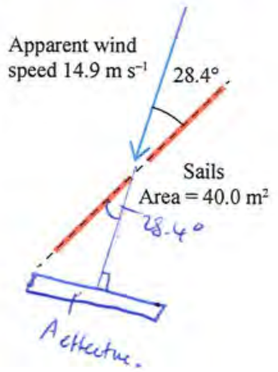
[numerically shown at back of booklet]



If you need to redraw your response, use the diagram on page 14.

Note that $\overrightarrow{\Delta V_w}$ is travelling with a Northern and Western component. But then $\overrightarrow{\Delta p_w} = (\overrightarrow{\Delta V_w})m$. By conservation of momentum then, $\overrightarrow{\Delta p_{waka}} = -(\overrightarrow{\Delta p_w}) = (\overrightarrow{\Delta V_{waka}})(m_{waka})$. Thus in opposite directions, the impulse and change in velocity of the waka has a Southern and Easterly component. Since West component ($\overrightarrow{\Delta V_w}$) > Northern component ($\overrightarrow{\Delta V_w}$), Eastern component ($\overrightarrow{\Delta V_{waka}}$) > Southern component ($\overrightarrow{\Delta V_{waka}}$).
∴ $\overrightarrow{\Delta V_{waka}}$ in direction of NE is positive, so will produce force with NE component.

(b) The motion of the waka hourua combined with the motion of the wind changes both the apparent speed and direction of the wind hitting the sails. On board the moving waka hourua, the wind appears to have a higher speed, and to come from a direction further towards the front of the waka hourua. The captain measures the wind velocity as 14.9 m s^{-1} that hits the sails at an angle of 28.4°, as shown in the diagram on the right. The sails have a combined area of 40.0 m^2. The density of air is 1.23 kg m^{-3}.



Calculate the mass of air that hits the sails each second.

~~From perspective of~~ Volume of air arriving each second, consider effective area, $A_{effective} = (40.0)(\sin 28.4°) = 19.0\ m^2$.

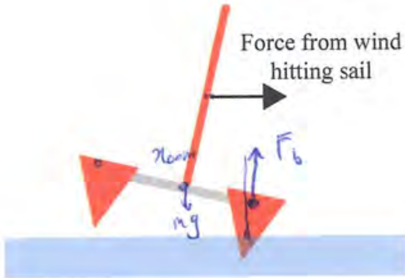
$\therefore \frac{\Delta V}{\Delta t} = A_{effective} \times v$

$= (19.0)(14.9) = 283\ m^3 s^{-1}$.

But $m = \rho V$, ∴ mass per second $= (1.23)(283)$

$= 349\ kg\ s^{-1}$ (3sf)

(c) The sideways force of the wind on the sails causes the waka hourua to tilt over sideways. In strong winds, the upwind side of the hiwi (hull) may lift out of the water altogether.



(i) Explain how the double-hulled design of the waka hourua helps it stay upright in strong winds.

You may wish to add information to the diagram to illustrate your answer.

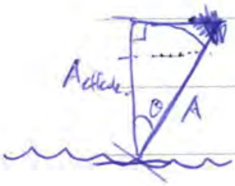
consider the shape in which, the pivot for tilting is one ~~one~~ of the hulls. (rather than the centre.

Then the weight force of the waka (mg) produces an anticlockwise torque as does the upwards buoyant force on the other hull. Otherwise, ~~one~~ one hull would ~~...~~ mean that the "mg" ~~produces~~ easily produce an anticlockwise torque. Thus with great τ_a, opposing τ_c by wind, waka hourua stays upright.

(ii) Explain how the tilting of the waka hourua will affect the force produced by the wind hitting the sails.

As the waka tilts, the effective ~~area~~ cross-sectional area to the plane at which the wind collides with decreases.

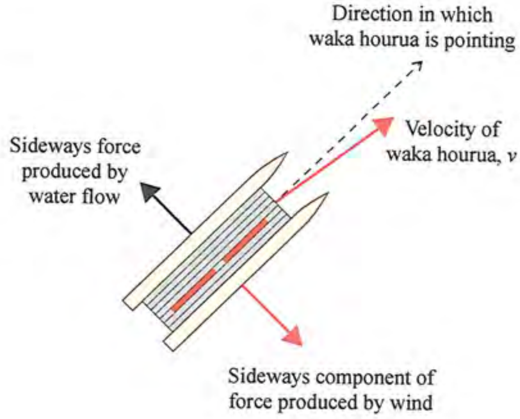
[i.e. $A_{effective} = A\cos\theta$, so as θ increases $A_{effective}$ decreases].



But that means less wind ~~...~~ mass elements collide with the sail, ∴ will produce less force as the waka ~~hourua~~ hourua tilts more.

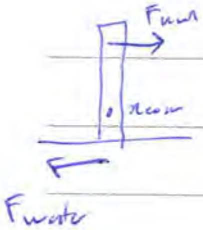
only tilts when $\tau_c > \tau_a$, which is difficult as x_{com} is further from the pivot point such that for $\tau = Fd_\perp$, larger $d_\perp$ increases τ_a.

(d) Due to the sideways component of the force from the wind, the velocity of the waka hourua is in a slightly different direction to the direction that the hull is pointing. This slight difference in directions creates an asymmetrical flow of water around the sides of the hull, which produces a sideways force on the hull of the waka hourua, as shown in the diagram on the right.



Explain the effect the sideways force produced by water flow around the hull will have on the direction of motion, and on the ability of the waka hourua to stay upright in strong winds.

Sideways force provided by waterflow is opposite in direction to sideways component ~~of the~~ produced by the wind, ~~so will cancel~~.
Whereas the sideways component of force increases with stronger winds, the sideways force produced by water flow increases with greater velocity of the waka hourua.



However, the forces together produce a clockwise torque when the force by water is below x_{com}, and sideways force by wind is above x_{com}. ∴ ~~as~~ ~~[illegible]~~ in stronger winds, both $F_{wind\ sideways}$ ~~$F_{wind\ sideways}$~~ increases, but also since this accelerates the waka, v increases $\Rightarrow$ $F_{water\ sideways}$ increases. Hence τ_c increases.

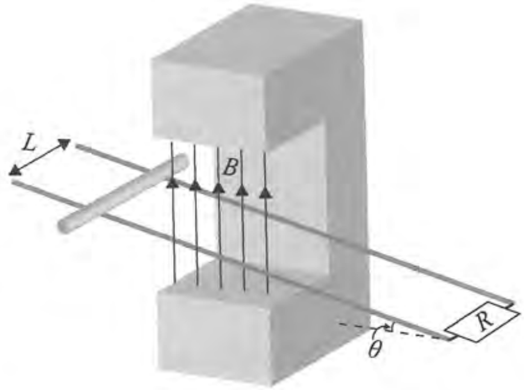
- This reduces the ability of the waka to stay upright in strong winds.

- The direction the waka travels may have minimal impact <u>if</u> the ~~horizontal~~ $F_{water\ sideways}$ ~~is~~ is equal in size to $F_{wind\ sideways}$ $\Rightarrow$ as indicated by vector diagram given.

QUESTION THREE: MAGNET SLIDER

Acceleration due to gravity = 9.81 m s^{-2}

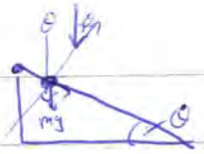
A metal roller of mass m slides without friction down parallel conducting rails of negligible electrical resistance. The rails are separated by a distance L, and are connected to each other at the bottom by a resistance R, forming a closed rectangular conducting loop with the rails and the roller. The plane of the rails makes an angle θ with the horizontal, and a uniform vertical magnetic field B exists throughout the region.



As the metal roller slides down the rails through the magnetic field, it reaches a constant velocity v.

(a) (i) Show that the constant velocity achieved by the roller through the magnetic field is given by the relationship:

$$v = \frac{mgR\tan\theta}{B^2L^2\cos\theta} \qquad \textbf{Equation \#1}$$



acceleration by gravity down slope:

$F_{slope\,g} = mg\sin\theta$.

In magnetic field, $F = BIL\cos\theta$, but $I = \frac{V}{R} = \frac{\varepsilon}{R} = \frac{\frac{\Delta\phi}{\Delta t}}{R}$. $[\varepsilon = \frac{\Delta\phi}{\Delta t}]$.

since $\frac{\Delta\phi}{\Delta t} = \frac{\Delta BA\cos\theta}{\Delta t} = BL\cdot v\cdot\cos\theta \Rightarrow I = \frac{BLv}{R}\cos\theta$

$\Rightarrow F = B\left(\frac{BLv}{R}\cdot\cos\theta\right)L\cdot\cos\theta$.

$F_{impede} = \frac{B^2L^2v\cos^2\theta}{R}$.

But at constant velocity, $F_{slope\,g} = F_{impede}$.

$\therefore mg\sin\theta = \frac{B^2v\cos^2\theta\cdot L^2}{R}$

[continued at back]

(ii) Explain what difference, if any, it makes to the constant velocity v, if the magnetic field is in the opposite direction.

If magnetic field is in opposite direction, the induced emf will also be in the opposite direction but it will have the ~~same~~ same size, as the initial movement through B is the same speed from g.
$\therefore$ induced current in circuit and subsequent force is the same size.
There is no change to the constant velocity v.
(induced current will be in opposite direction to original set up, i.e. clockwise).
~~We see this in the equation #1 as B~~

For small values of θ, Equation #1 may be approximated as:

$$v = \frac{mgR}{B^2L^2} \times \left(\theta + \frac{5\theta^3}{6}\right)$$ ah yes taylor series. **Equation #2**

(b) An experiment is set up with $B = 2.00$ T, $L = 0.500$ m, $m = 5.00 \times 10^{-3}$ kg, and $R = 10.0\ \Omega$.

[redone at back of booklet]

Determine the accuracy of **Equation #2**, compared with **Equation #1**, at $\theta = 25.0°$ (0.436 radians).

① $v = \frac{(5.00\times10^{-3})(9.81)(10.0)}{(2.00)^2(0.500)^2} \cdot \frac{\tan(25.0°)}{\cos(25.0°)} = 0.252\ ms^{-1}$ (3sf)

② $v = \frac{(5.00\times10^{-3})(9.81)(10.0)}{(2.00)^2(0.500)^2} \cdot \left(0.436 + \frac{5(0.436)^3}{6}\right) = 0.248\ ms^{-1}$ (3sf)

~~uncertainty~~ error ~~Accuracy~~ % $= \frac{0.252 - 0.229}{0.252} \times 100\% = \pm 1.84\%$ (2dp) - this is very accurate.

alternatively, compare "$\frac{\tan\theta}{\cos\theta}$" to "$\theta + \frac{5\theta^3}{6}$" $\Rightarrow \left(\frac{\frac{\tan(25.0°)}{\cos 25.0°} - 0.436 - \frac{5}{6}(0.436)^3}{\left(\frac{\tan(25.0°)}{\cos(25.0°)}\right)} \times 100\%\right) = \pm$ 1.84% (error %)

(c) By calculating the velocity at the high angle of $\theta = 85.0°$, explain if this equipment would be suitable for testing whether **Equation #1** is accurate at high angles of θ.

$v = \frac{(5.00\times10^{-3})(9.81)(10.0)}{(2.00)^2(0.500)^2} \cdot \frac{\tan(85.0°)}{\cos(85.0°)} = 64.3\ ms^{-1}$ (3sf)

If testing via reaction time, no. ~~5.61 ms⁻¹ is too fast~~ 64.3 ms⁻¹ is too fast, as every second 64.3 m is traversed (reaction time for humans ≈ 0.2–0.3s). Even if ~~Unless~~ a ~~light gate is used~~, $\theta = 85.0°$ is not suitable to test equation ①. (even if a light gate is used).

(d) Explain whether **Equation #1** remains valid if the roller rolls rather than slides down the slope.

~~If the roller rolls, then this means that the argument of $F_{g\ slope} - F_{impedance}$~~

~~When sliding, all~~ Rolling requires rotational kinetic energy, gained via loss of GPE. However, ~~once this is supplied sufficiently,~~ $F_{g\ slope}$ and $F_{impedance}$ calculations & derivations are still valid so equation ① should remain valid.

Steady state with constant v is reached when all GPE being lost in the motion is dissipated as heat in the resistor, this occurs for $F_{g\ slope} = F_{impedance}$ which by the same derivation method yields the same v. ∴ yes, valid.

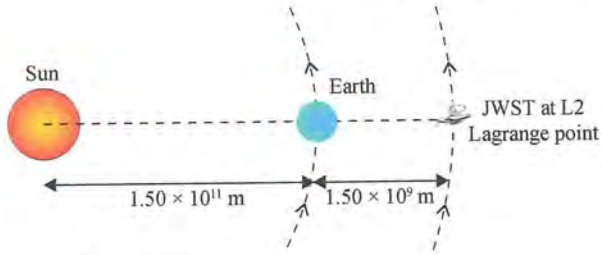
[continued at back]

QUESTION FOUR: ORBITAL DYNAMICS

Universal Gravitational Constant $= 6.67 \times 10^{-11}$ N m^2 kg^{-2}
Period of Earth's orbit $= 365.25$ days

Mass of Sun $= 1.99 \times 10^{30}$ kg
Mass of Earth $= 5.98 \times 10^{24}$ kg

The James Webb Space Telescope (JWST) has a mass of 6.16×10^3 kg. It was launched on Christmas Day 2021 and is now orbiting at a point called the L2 Lagrange point, where it will remain in a direct line with the Sun and Earth as shown right.



(a) (i) State the period of the orbit of the JWST around the Sun.

365.25 days (same as Earth's orbit).

(ii) Calculate the net force acting on the JWST at the L2 point.

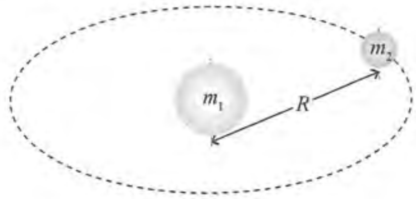
$F = \frac{mv^2}{r} = m\omega^2 r = (6.16 \times 10^3)\left(\frac{2\pi}{365.25 \times 24 \times 3600}\right)^2 (1.50 \times 10^{11} + 1.50 \times 10^9)$

~~$F = 1.86 \times 10^8$ N (3sf)~~

$F = 37.0$ N (3sf)

An approximation for two bodies in orbit around each other is that the period T of the orbit can be determined using the relationship:

$$T^2 = \frac{4\pi^2 R^3}{Gm_1}$$



where R is the distance between the centre of masses of the two objects, and m_1 is the mass of the more massive body.

(b) One assumption of the relationship above is that m_1 is much greater than m_2.

(i) Explain why it is necessary to assume that m_1 is much greater than m_2 to derive this relationship.

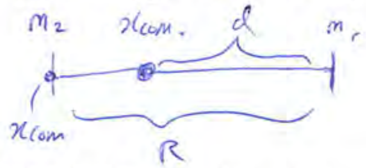
The relationship is based off F_g providing F_c for circular motion about centre of m_1. This assumes x_{com} of the system is sufficiently close to com of m_1 $\therefore m_1 >> m_2$.

(ii) State another key assumption of this relationship.

Assume that the bodies in orbit are such that m_2 is in circular motion around m_1, with constant R value. (no eccentricity).
The centripetal force is solely provided by F_g between m_1 and m_2.

(c) In the case that m_1 is **not** much larger than m_2, show that the period of the orbit is given by the relationship:

$$T^2 = \frac{4\pi^2 R^3}{G(m_1 + m_2)}$$



Any other assumptions made for the relationship given on page 12 are still valid.

$F_g = \frac{G m_1 m_2}{R^2}$, $F_c = m_1 \omega^2 \cdot d.$

but we know that $d = \frac{\Sigma m x}{\Sigma m} = \frac{m_2 \cdot R + m_1(0)}{m_2 + m_1}$

$\Rightarrow F_c = m_1 \cdot \omega^2 \cdot \left(\frac{m_2 R}{m_2 + m_1}\right).$

Since $F_g = F_c$,

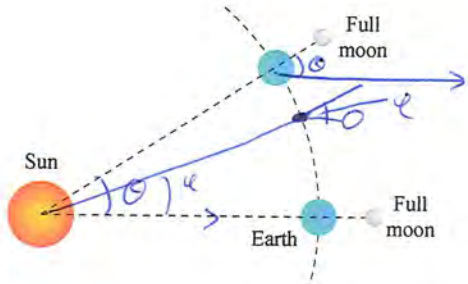
$\frac{G m_1 m_2}{R^2} = \frac{(m_1 m_2 \cdot R)\,\omega^2}{(m_2 + m_1)}$

$\omega^2 = \frac{G}{R^2} \cdot (m_2 + m_1).$ $\left[\omega^2 = \frac{4\pi^2}{T^2}\right].$

$\Rightarrow \frac{T^2}{4\pi^2} = \frac{R^2}{G(m_2 + m_1)}$

$T^2 = \frac{4\pi^2 R^2}{G(m_1 + m_2)}$ //.

(d) The phases of the Moon are caused by the relative positions of the Sun, Earth, and Moon. A full moon occurs when the Sun, Earth, and Moon are directly aligned. The Moon takes 27.3 days to complete one 360° orbit around the Earth. However, because the Moon must orbit more than 360° to return to a direct alignment with the Sun and Earth (as shown in the diagram on the right), the time between one full moon and the next is more than 27.3 days.



Show that the time between one full moon and the next is 29.5 days.

in 27.3 days, φ rotation travelled, = ~~27.3/365.25~~ = ~~0.0747 radians~~.

$\varphi = \frac{27.3}{365.25} \times 360° = 0.0747°$

Let θ be the rotation for full moon. Then $t_\theta = \frac{\theta°}{360°} \times 365.25$.

But also, $t_\theta = 27.3 + \frac{\theta}{360°} \times 27.3.$

Hence, $27.3 + \frac{27.3\theta}{360} = \frac{365.25\,\theta}{360}.$

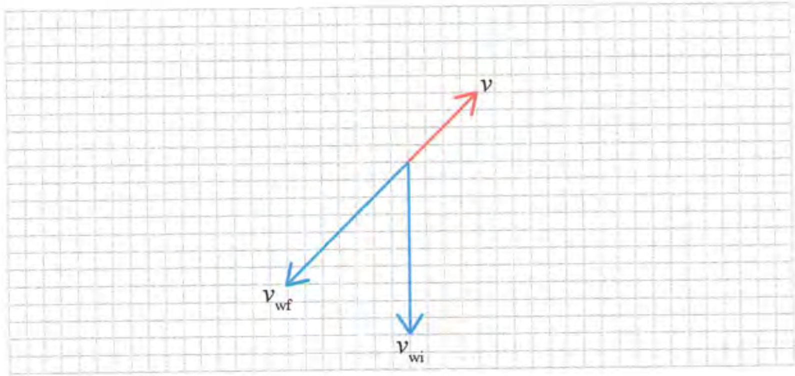
$9828 + 27.3\theta = 365.25\,\theta.$

$9828 = 337.95\,\theta.$

[contd at back]. ~~$\theta = 0.0347°$ (3sf) $\Rightarrow t_\theta = \frac{0.0347}{360} \times 365.25 =$~~

SPARE DIAGRAMS

If you need to redraw your response to Question Two (a)(ii), use the diagram below. Make sure it is clear which answer you want marked.



Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

2a(ii)

$\overrightarrow{\Delta V_w} = \overrightarrow{V_{wf}} - \overrightarrow{V_{wi}}$

$= \begin{pmatrix} -\frac{10}{\sqrt{2}} - 0 \\ -\frac{10}{\sqrt{2}} + 10 \end{pmatrix}$

$\overrightarrow{\Delta V_w} = \begin{pmatrix} -\frac{10}{\sqrt{2}} \\ 10 - \frac{10}{\sqrt{2}} \end{pmatrix} = 10 \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 1 - \frac{1}{\sqrt{2}} \end{pmatrix}$, then $\overrightarrow{\Delta p_w} = m_w \cdot 10 \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 1 - \frac{1}{\sqrt{2}} \end{pmatrix}$.

But by conservation of momentum, $\overrightarrow{\Delta p_{walker}} = m_w \cdot 10 \begin{pmatrix} +\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} - 1 \end{pmatrix}$.

Direction of of force, $\vec{F} = \frac{d(\overrightarrow{\Delta p})}{dt}$,

$\therefore$ direction of force $= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} - 1 \end{pmatrix}$.

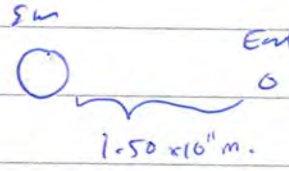
since $|F_x| > |F_y|$, $\frac{1}{\sqrt{2}} > \frac{1}{\sqrt{2}} - 1$, $0.70 > 0.29...$

then a NE component of F is positive, and in direction of walker.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

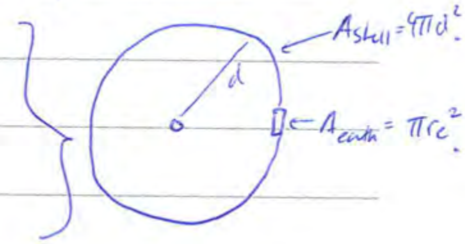
1c)



$\Delta E = \Delta mc^2 = (4.30\times10^{9})(3.00\times10^{8})^2$

$\Delta E = 3.87\times10^{26}$ J. per second.

But Earth only ~~can~~ absorbs a certain proportion,



$\text{proportion} = \dfrac{\pi(r_e)^2}{4\pi d^2} = \dfrac{r_e^2}{4d^2}$

$= \dfrac{(6.37\times10^{6})^2}{4(1.50\times10^{11})^2}$

$= 4.51\times10^{-10}$ (3sf)

$\therefore E_{\text{absorbed by Earth}} = (4.51\times10^{-10})(3.87\times10^{26})$

$= 1.74\times10^{17}$ J per second (3sf)

But $E = pc$. $\therefore$ $p = \dfrac{E}{c} = \dfrac{1.74\times10^{17}}{3.00\times10^{8}}$

$\Delta p = 5.82\times10^{8}$ kgms^{-1}, every second.

But $F = \dfrac{\Delta p}{\Delta t}$.

$\therefore$ $F = 5.82\times10^{8}$ N (3sf).

3ai)

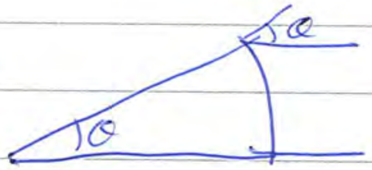
$mg\sin\theta = \dfrac{B^2 v L^2\cos^2\theta}{R}$

$v = Rmg\sin\theta \times \dfrac{1}{B^2L^2\cos^2\theta}$

$v = \dfrac{mgR}{B^2L^2} \times \dfrac{\sin\theta}{\cos^2\theta}$.

$v = \dfrac{mgR\tan\theta}{B^2L^2\cos\theta}$ as required.

4d)



let $T_{\text{full moon}} = T$.

$T = \left(\dfrac{\theta}{360}\right)\times 365.25$

But also $T = 27.3 + \left(\dfrac{\theta}{360}\right)(27.3)$.

$\therefore$ ~~$\left(\dfrac{\theta}{360}\right)\times365.25 = 27.3 + \left(\dfrac{\theta}{360}\right)27.3$~~.

$\theta = (3.44\times10^{-3})^\circ$ (3sf).

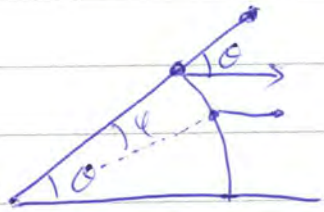
$\Rightarrow t_\theta = \dfrac{3.44\times10^{-3}}{360}\times 365.25 =$

00940

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

$T = \left(\frac{\theta}{360}\right) 365.25$ but $T = 27.3 + \left(\frac{\theta}{360}\right) 27.3$.



$$\left(\frac{\theta}{360}\right) 365.25 = \left(\frac{\theta}{360}\right)(27.3) + 27.3$$
$$(365.25 - 27.3)\theta = 9828.$$
$$337.25\,\theta = 9828.$$
$$\theta = 29.08^\circ \ldots$$

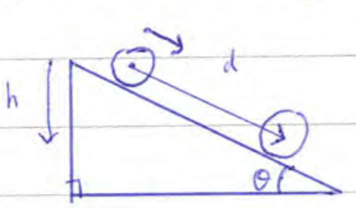
$$\therefore T = \left(\frac{29.08}{360}\right) \times 365.25$$
$$T = 29.50532\ldots$$
$$T = 29.5 \text{ days (3sf) as required.}$$

1(c) $p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{550\times10^{-9}} = 1.21\times10^{-27}$ Ns (3sf) per photon.

$E = h\cdot\frac{c}{\lambda} = (6.63\times10^{-34}) \times \frac{3.00\times10^{8}}{550\times10^{-9}} = 3.62\times10^{-19}$ J (3sf) per photon.

1(c)(i)

3d). Although the equation is still valid, the roller may have to roll further to convert enough GPE into KE(rot), for the constant velocity state.



As v is constant, both $KE_{(rot)}$ and KE(trans) are constant, which requires $\Delta GPE = \Delta E$ (resistor).

But this is identical to original derivation.

$$mg.d\sin\theta = \Delta t \times P = \Delta t \cdot IV = \Delta t \cdot I^2R$$
$$mg.v\sin\theta = \left(\frac{BLv\cos\theta}{R}\right)^2 \times R = \frac{B^2L^2v^2\cos^2\theta}{R}$$
$$v = \frac{mgR\sin\theta}{B^2L^2\cos^2\theta} = \frac{mgR\tan\theta}{B^2L^2\cos\theta}$$

$\frac{\Delta d}{\Delta t} = v$.

Same equation. ∴ remains valid.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

3b)

① $v = \dfrac{mgR\tan\theta}{B^2L^2\cos\theta} = \dfrac{(5.00\times10^{-3})(9.81)(10.0)\times\tan(25.0°)}{(2.00)^2(0.500)^2\times\cos(25.0°)}$.

$V_{①} = 0.252368\ldots\ ms^{-1}$

② $v = \dfrac{mgR}{B^2L^2}\left(\theta \times \frac{5}{6}\theta^3\right) = \dfrac{(5.00\times10^{-3})(9.81)(10.0)}{(2.00)^2(0.500)^2}\times\left(0.436+\frac{5}{6}(0.436)^3\right)$

$V_{②} = 0.247735\ldots\ ms^{-1}$.

"Accuracy" can be considered by the error percentage,

$\frac{\Delta V}{V} = \left(\frac{0.252-0.248}{0.252}\right)\times 100\%$

$\frac{\Delta V}{V} = 1.84\%$ (3sf)

This is a very low error percentage, so with $\theta = 25.0°$, Equation #2 is sufficiently accurate to approximate V, compared to equation #1. Since $\left(\frac{mgR}{B^2L^2}\right)$ is a shared factor, only "$\theta + \frac{5}{6}\theta^3$" and "$\frac{\tan\theta}{\cos\theta}$" needed to be compared really.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Extra space if required.
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QUESTION NUMBER

93103