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Draw a cross through the box (☒) if you have NOT written in this booklet

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TOP SCHOLAR

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Scholarship 2023
Physics

Time allowed: Three hours
Total score: 32

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should answer ALL the questions in this booklet.

For all 'describe' or 'explain' questions, the answers should be written or drawn clearly with all logic fully explained.

For all numerical answers, full working must be shown and the answer must be rounded to the correct number of significant figures and given with the correct SI unit.

Formulae you may find useful are given on page 3.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–24 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (AREA DO NOT WRITE). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

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The assessment starts on page 4.

The formulae below may be of use to you.

$$v_f = v_i + at$$
$$d = v_i t + \frac{1}{2}at^2$$
$$d = \frac{v_i + v_f}{2}t$$
$$v_f^{\,2} = v_i^{\,2} + 2ad$$
$$F_g = \frac{GMm}{r^2}$$
$$F_c = \frac{mv^2}{r}$$
$$\Delta p = F\Delta t$$
$$\omega = 2\pi f$$
$$d = r\theta$$
$$v = r\omega$$
$$a = r\alpha$$
$$W = Fd$$
$$F_{net} = ma$$
$$p = mv$$
$$x_{COM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$
$$\omega = \frac{\Delta\theta}{\Delta t}$$
$$\alpha = \frac{\Delta\omega}{\Delta t}$$
$$L = I\omega$$
$$L = mvr$$
$$\tau = I\alpha$$
$$\tau = Fr$$
$$E_{K(ROT)} = \tfrac{1}{2}I\omega^2$$
$$E_{K(LIN)} = \tfrac{1}{2}mv^2$$
$$\Delta E_p = mg\Delta h$$
$$\omega_f = \omega_i + \alpha t$$
$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$
$$\theta = \frac{(\omega_i + \omega_f)}{2}t$$
$$\theta = \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$E_p = \tfrac{1}{2}ky^2$$
$$F = -ky$$
$$a = -\omega^2 y$$
$$y = A\sin\omega t \qquad y = A\cos\omega t$$
$$v = A\omega\cos\omega t \qquad v = -A\omega\sin\omega t$$
$$a = -A\omega^2\sin\omega t \qquad a = -A\omega^2\cos\omega t$$
$$\Delta E = Vq$$
$$P = VI$$
$$V = Ed$$
$$Q = CV$$
$$C_T = C_1 + C_2$$
$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$
$$E = \tfrac{1}{2}QV$$
$$C = \frac{\varepsilon_o \varepsilon_r A}{d}$$
$$\tau = RC$$
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$R_T = R_1 + R_2$$
$$V = IR$$
$$F = BIL$$
$$V = BvL$$
$$F = Bqv$$
$$F = Eq$$
$$E = \frac{V}{d}$$

$$\phi = BA$$
$$\varepsilon = -\frac{\Delta\phi}{\Delta t}$$
$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$
$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$
$$E = \tfrac{1}{2}LI^2$$
$$\tau = \frac{L}{R}$$
$$I = I_{MAX}\sin\omega t$$
$$V = V_{MAX}\sin\omega t$$
$$I_{MAX} = \sqrt{2}\,I_{rms}$$
$$V_{MAX} = \sqrt{2}\,V_{rms}$$
$$X_C = \frac{1}{\omega C}$$
$$X_L = \omega L$$
$$V = IZ$$
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$
$$v = f\lambda$$
$$f = \frac{1}{T}$$
$$n\lambda = \frac{dx}{L}$$
$$n\lambda = d\sin\theta$$
$$f' = f\frac{V_W}{V_W \pm V_S}$$
$$E = hf$$
$$hf = \phi + E_K$$
$$E = \Delta mc^2$$
$$\frac{1}{\lambda} = R\left(\frac{1}{S^2} - \frac{1}{L^2}\right)$$
$$E_n = -\frac{hcR}{n^2}$$

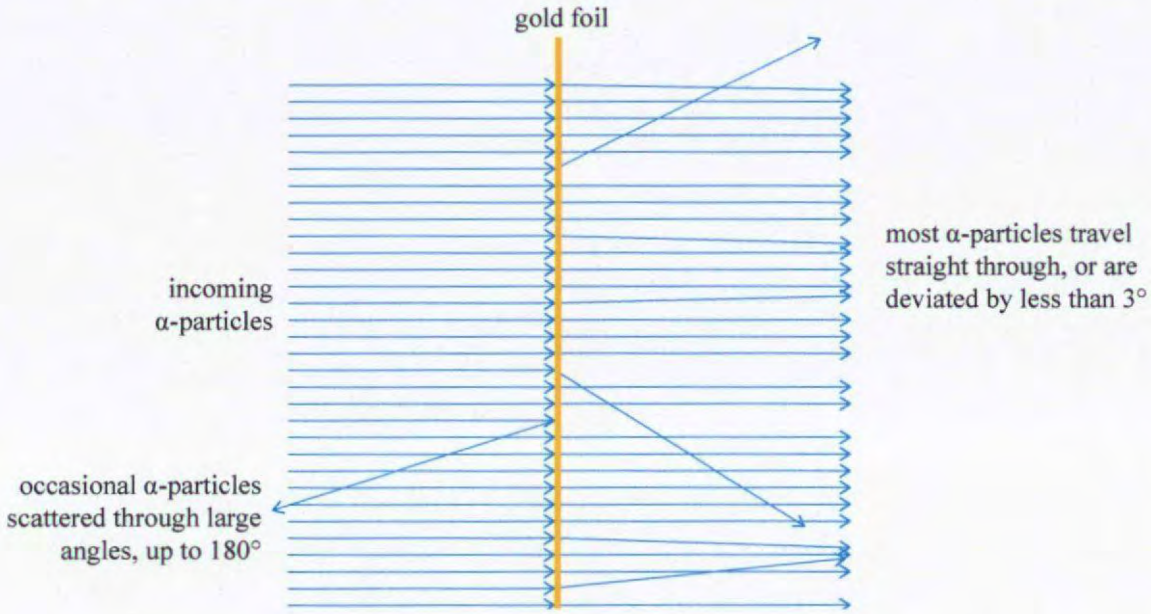
QUESTION ONE: THE DISCOVERIES OF ERNEST RUTHERFORD

Atomic number of gold = 79
Charge of an electron = -1.60×10^{-19} C

Ernest Rutherford won a Nobel Prize in 1908 for work on understanding radioactive decay and for discovering α-particles. Later, he and his fellow researchers used α-particles in two famous experiments.

Experiment 1: Scattering of alpha particles by gold foil

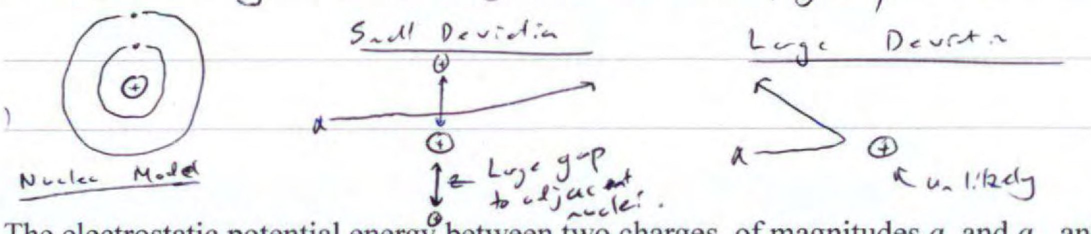
When Rutherford fired α-particles at a thin foil of gold, he observed that most went straight through or deviated by less than 3 degrees. However, the researchers were surprised to see occasional α-particles were scattered through large angles, some even returning in the direction from which they had come.



(a) Explain how these results were consistent with the model of the atom that Rutherford proposed.

Firstly, the majority of α particles were not deflected. This implies they did not come near enough to a charged particle to feel any significant electrostatic forces of attraction or repulsion. This is consistent with the Rutherford atomic model, where most of the atom is in fact empty, uncharged space. Some small deviation may occur due to weak electrostatic forces of attraction/repulsion, however these are slight due to the distance between the α particles and the charged ~~pt~~ ~~of the~~ nuclei. Nevertheless, some particles were still deviated by large angles. This implies they felt strong repulsive forces. The fact that this occurred to only some particles supported the theory that there existed a small

region of large positive charge in each atom.
Positive, as it repelled the positive α-particles, and small due to the low proportion of reflection. This was termed the 'nuclear model' with a positive nucleus made of protons (positive) surrounded by small orbiting electrons of empty space.



(b) The electrostatic potential energy between two charges, of magnitudes q_1 and q_2, and separated by distance r, is given by $E_p = \frac{kq_1q_2}{r}$, where $k = 8.99 \times 10^9$ N m^2 C^{-2}.

An α-particle of mass m, velocity v, and charge $2e$, travels directly towards a nucleus that remains stationary at all times. The charge on the stationary nucleus is Ze, where Z is the atomic number of the stationary nucleus, and e is the charge of an electron.

(i) Show that the distance of closest approach, D, is given by:

$$D = \frac{4kZe^2}{mv^2} \qquad \frac{1}{2}mv^2 = \frac{k \cdot 2e \cdot Ze}{D}$$

Explain your reasoning.

The α-particle begins with maximum kinetic energy, and as it approaches the nucleus, loses this energy due to the electrostatic force of repulsion felt between the two positive charges. The closest approach to the nucleus occurs when all kinetic energy of the α-particle is converted to electric potential energy - it is at the 'highest' point on the 'electric potential hill'. As $D \propto \frac{1}{E_p}$, ~~maximum~~ minimum D occurs at maximum E_p.

$\therefore$ ~~$E_K =$~~ $E_K = E_p$

$$\frac{1}{2}m \cdot v^2 = \frac{kq_1q_2}{r}$$

$$\frac{1}{2}mv^2 = \frac{k \cdot 2e \cdot Ze}{D}$$

$$D = \frac{2kZe^2}{\frac{1}{2}mv^2}$$

$$D = \frac{4kZe^2}{mv^2}$$

(ii) Calculate the distance of closest approach of a 4.78 MeV α-particle travelling directly towards a gold nucleus, which is fixed in position.

$E_K = E_p$

$4.78\times10^{6}\times1.6\times10^{-19} = \frac{2kZe^2}{D}$

$D = \frac{2kZe^2}{4.78\times10^{6}\times1.6\times10^{-19}}$

NB: conversion of eV → J. 1eV = 1.60×10^{-19} J

$D = \frac{2\times8.99\times10^{9}\times79\times(+1.60\times10^{-19})^2}{4.78\times10^{6}\times1.60\times10^{-19}}$

$= 4.7545\ldots\times10^{-14}$ m

$D = 4.75\times10^{-14}$ m (3 S.F.)

(c) If a nucleus of charge *Ze* were free to move, as would occur if it were in the gaseous state for example, would the distance of closest approach be the same, greater, or less than given by the equation in part (b)(i)?

Explain your answer using physical principles.

No calculation is required.

As the α particle moved towards the nucleus, the electrostatic forces of repulsion would cause the nucleus to experience a force (equal in magnitude to that on the α-particle, and in the opposite direction, according to Newton's 3rd Law). that causes it to accelerate away from the α particle. This means that, once the α-particle has lost all of its kinetic energy, if indeed it does. If both particles retain some kinetic energy, then the same conclusion is reached, with a larger D due to decreased Ep. and is stationary, the nucleus will still have ~~a~~ ~~velo~~ some velocity, and therefore some amount of kinetic energy. As the amount of initial kinetic energy was the same (same v for the α-particle), there must be less ~~electrical~~ electrostatic potential energy in the system, as a consequence of the law of conservation of energy. Therefore, the distance D must be greater, as D is inversely proportional to electrostatic potential energy.

(d) **Experiment 2: Bombardment of nitrogen gas by high-energy alpha particles**

Rutherford and his fellow researchers fired high-energy, 7.70 MeV, α-particles at a container of nitrogen gas and were surprised to see that protons, ${}^{1}_{1}\mathrm{H}$, were emitted. At the time, the researchers knew that a nuclear reaction had occurred, but they did not know what the reaction was. Two possible nuclear reactions are:

$${}^{14}_{7}\mathrm{N} + {}^{4}_{2}\alpha \rightarrow {}^{17}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H} \qquad \text{Reaction 1}$$

$${}^{14}_{7}\mathrm{N} + {}^{4}_{2}\alpha \rightarrow {}^{13}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} + {}^{4}_{2}\alpha \qquad \text{Reaction 2}$$

as atomic number increases.

(i) Using your knowledge of binding energy per nucleon, explain which reaction, Reaction 1 or Reaction 2, is more likely.

For nuclei up to around atomic number 28 (Iron), the binding energy per nucleon generally increases. The binding energy per nucleon is defined as the energy required to separate all the nucleons in a nucleus to infinity, per nucleon in the original nucleus. Therefore, an increase in the binding energy per nucleon from reactants to products represents a net release of energy, and a feasible reaction. In reaction two, there is an α-particle in both the reactants and products, which can be disregarded, along with the proton, as it is an isolated nucleon and has zero binding energy. SEE PAGE 17

such that they are isolated and feel minimal forces of attraction/repulsion. (essentially zero)

(ii) Explain why it was necessary to use high-energy α-particles for this experiment.

Both reactions require a positively charged α-particle to collide with an even more positively charged ${}^{14}_{7}\mathrm{N}$ nucleus. This will result in a high forces of electrostatic repulsion which must be overcome. There is a minimum energy of α-particle, below which no collision and therefore no reaction can occur, a kind of 'activation energy'. ~~There also must be sufficient energy supplied to overcome the strong nuclear force holding binding the proton to the nucleus, which must be over which must be overcome before the electrostatic forces of repulsion become significant in expelling the proton.~~ Although there is a net release of energy, there must still be an initial input before the energy can be released.

QUESTION TWO: AXE THROWING

Acceleration due to gravity = 9.81 m s^{-2}

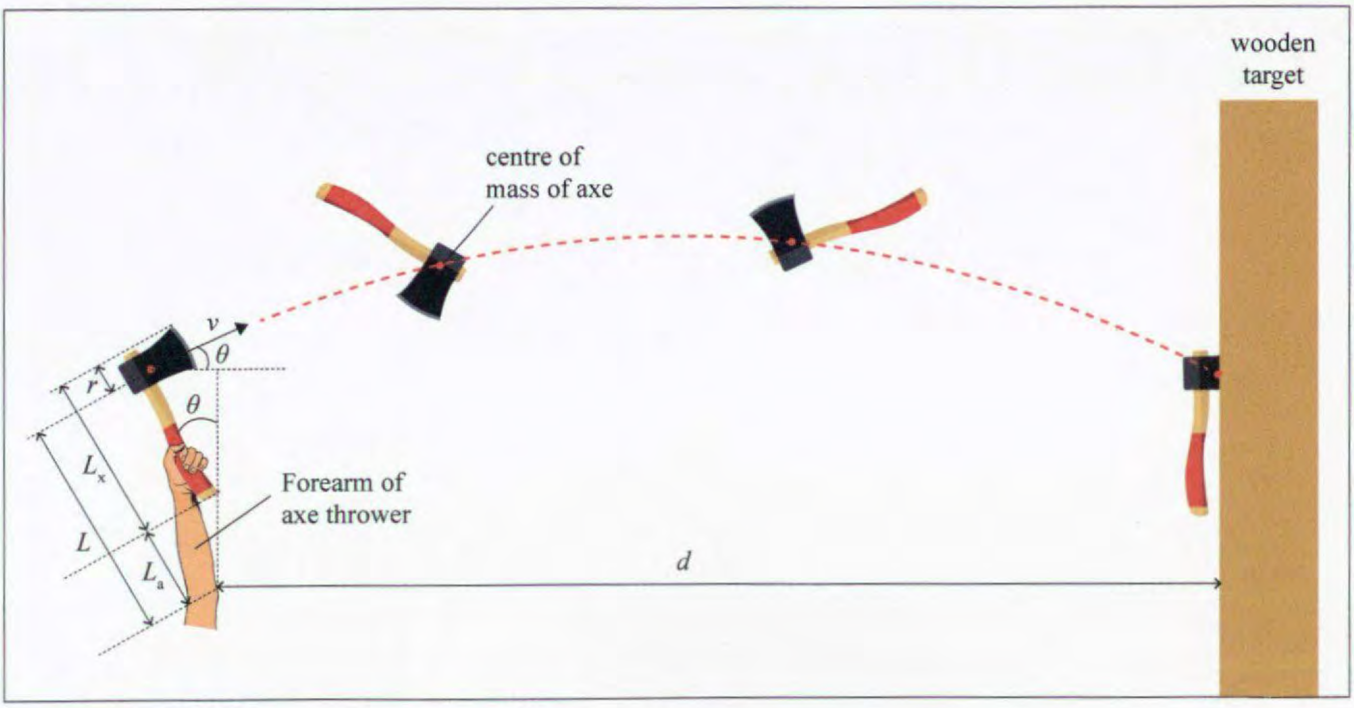
Source: www.sydney.com/destinations/sydney/sydney-west/penrith/attractions/throw-axe

Axe throwing is a traditional sport that has become more popular recently. It involves throwing an axe at a wooden target. The path of the axe can be described with the physics of projectile motion and of rotational motion. If the axe is thrown correctly, it rotates after it is thrown so that it is vertical as it reaches the target, allowing the blade to stick in the target.

Although everybody will throw the axe in a slightly different way, we can describe the throw as follows.

- The axe is held so that the forearm and the axe handle form a straight line, as shown in the diagram below.
- The throw is made by keeping the upper arm still and swinging the forearm from the elbow.

The axe is released at an angle θ, so that its centre of mass has a velocity, v. The axe is thrown from the same height as the target. The axe completes just over one full rotation as it travels from the release point to the target. The centre of mass of the axe finishes up level with the surface of the target, as shown in the diagram below.



The velocity of the centre of mass at release is v.

r = distance from the end of the axe to centre of mass

L_x = total length of axe

L_a = length of axe thrower's forearm

The length from the centre of mass to the elbow is, $L = L_a + L_x - r$

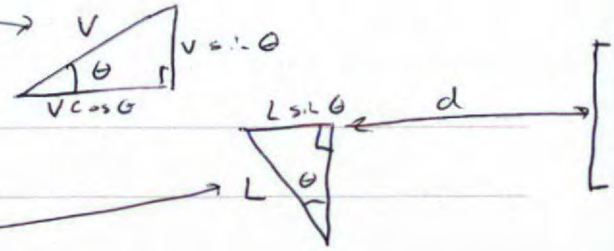
d = distance of axe thrower's elbow from the target

(a) An analysis of the projectile motion of the axe can be used to show that the time of flight of the axe, from the time it is released to when it strikes the target at exactly the same height, is $t = \dfrac{2v\sin\theta}{g}$.

Show that the initial velocity, v, required for the axe to strike the target successfully is given by:

$$v = \sqrt{\frac{g(d + L\sin\theta)}{2\sin\theta\cos\theta}}$$

Clearly show your working.



$V_x = V\cos\theta$ $\qquad t = \dfrac{s_x}{v_x} = \dfrac{d + L\sin\theta}{V\cos\theta}$

$s_x = d + L\sin\theta$

Velocity/displacement in the horizontal direction.

$\dfrac{2V\sin\theta}{g} = \dfrac{d + L\sin\theta}{V\cos\theta}$ $\quad$ * As time taken to travel s_x = time taken to strike target at same height.

$2V^2\sin\theta\cos\theta = g(d + L\sin\theta)$

$V^2 = \dfrac{g(d + L\sin\theta)}{2\sin\theta\cos\theta}$

$V = \sqrt{\dfrac{g(d + L\sin\theta)}{2\sin\theta\cos\theta}}$

Assuming air resistance to be negligible, only force on axe once released is the weight force due to gravity.

(b) The angular velocity of the axe is given by $\omega = \dfrac{v}{L}$. For a successful throw that ends up with the axe rotating and sticking in the target, as shown in the diagram opposite, show that the ratio of $\dfrac{d}{L}$ is given by:

$$\frac{d}{L} = (\theta + 2\pi)\cos\theta - \sin\theta$$

$\omega = \dfrac{\Delta\theta}{\Delta t}$ $\quad$ (total angle of rotation = 2π (complete rotation) + θ (initial angle)) relative to angle once in wood

$\omega = \dfrac{2\pi + \theta}{t}$

$\omega t = 2\pi + \theta$ $\quad$ (substitute in $\omega = \frac{V}{L}$ & $t = \frac{d + L\sin\theta}{\cos\theta}$)

$\dfrac{V}{L} \times \left(\dfrac{d + L\sin\theta}{V\cos\theta}\right) = 2\pi + \theta$

$\dfrac{d + L\sin\theta}{L\cos\theta} = 2\pi + \theta$

$\dfrac{d}{L} + \sin\theta = (2\pi + \theta)\cos\theta$

$\dfrac{d}{L} = (2\pi + \theta)\cos\theta - \sin\theta$

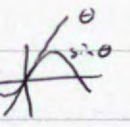
(c) In axe throwing, the angle θ is usually small.

Derive a simplified form of the equation in part (b), for a small angle θ.

Clearly show your working and state any assumptions made.

As $\theta \to 0$, $\cos\theta \to 1$

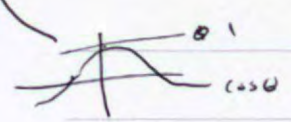
As $\theta \to 0$, $\sin\theta \to \theta$



$\frac{d}{L} = (2\pi + \theta)\cos\theta - \sin\theta$

$= (2\pi + \theta) \times 1 - \theta$

$= 2\pi$



$$\frac{d}{L} = 2\pi$$

~~This is logically true, as when θ is small there is very little difference between the release angle of the axe when it collides with the wood as it only needs to complete the single full rotation.~~

(d) Axe throwers have limited scope to vary their angle of release, and can throw from any distance, provided they stay behind a line marked on the ground.

(i) Mika throws an axe with a larger total length, L_x.

i.e. stand further back.

What other adjustment can she make to ensure that her throw still hits the target successfully?

Assume r remains constant, an increase in L_x results in an increase in L. By increasing d, she will be able to keep the ratio $\frac{d}{L}$ the same, and continue to throw with the same angle of release. Due to an increase in d & L, she would also have to increase her throw speed velocity, V to ensure she hits the target, ~~again assum~~ with the same Θ.

(ii) Giving reasons, explain which aspects of the axe's flight would change, and which would stay the same, if Mika were throwing an axe on the Moon, where the acceleration due to gravity is less than on Earth.

acceleration due to gravity. than on earth.

On the moon, g is approximately 6x smaller. In this case, she should throw the axe at the same angle Θ, as the optimal angle is independent of g provided other variables (d, L, L_x, h_a, r) remained constant. She would, however, have to throw the axe slower, at approximately $\frac{1}{\sqrt{6}}$ / 0.408... the original speed on earth. Otherwise, she would overshoot the target. If she did throw at the same speed, the flight would retain a parabolic shape, however it would reach a higher peak, and take a larger distance d to return to it its original height. w would remain the same, however more rotations would be completed due to spending a longer time in flight.

and a correspondingly longer time

the target height

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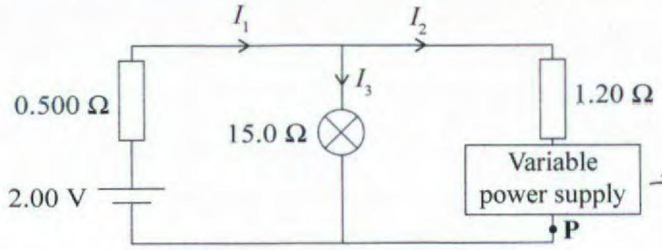
QUESTION THREE: DC AND DOPPLER

A circuit is set up with two power supplies. One supplies a constant EMF of 2.00 V, the other is a variable power supply that can provide a continuous range of EMFs.

$I_1 + I_2 = I_3$



$2 = 0.5I_1 + 15I_3$

$\mathcal{E} = 1.2I_2 + 15I_3$

$I_1 = \frac{2 - 15I_3}{0.5}$

$I_2 = \frac{\mathcal{E} - 15I_3}{1.2}$

$\frac{2 - 15I_3}{0.5} + \frac{\mathcal{E} - 15I_3}{1.2} = I_3$

(a) In addition to Ohm's Law, describe two other key circuit rules that could be applied to determine currents and potential differences in a circuit like this, and state the fundamental physics principles these rules are based on.

Kirchoff's first law states that the sum of currents entering a junction (the net current at a junction is 0A,) is equal to the sum of currents leaving it, due to the law of conservation of charge. Kirchoff's second law states that the sum of any emfs in a closed loop is equal to the sum of potential differences. The net potential difference is zero (if direction is accounted for). A result of the law of conservation of energy.

(b) The orientation and EMF of the variable power supply are adjusted until no current flows through the 15 Ω lamp, and it does not light up.

(i) Calculate the EMF of the variable power supply when the lamp does not light up, and clearly state whether point **P** shown on the diagram is the positive or negative end of the variable power supply.

(As the current is in the opposite direction.)

Kirchoff II: $2.00 = 0.500I_1 + 15.0I_3$ $\quad \mathcal{E} = -15.0I_3 + 1.20I_2$ (Assuming P is positive)

$\cancel{I_1 = I_2 + I_3}$ $\quad \frac{\mathcal{E} + 15I_3}{1.2} = I_2$

$\frac{2 - 15I_3}{0.5} = I_1$

As the lamp does not light up, $I_3 = 0.00\text{ A}$.

Kirchoff I: $I_1 = I_2 + I_3$

$2.4 = 0.5\mathcal{E}$

$\mathcal{E} = 4.80\text{V}$

$\frac{2 - 15I_3}{0.5} = \frac{\mathcal{E} + 15I_3}{1.2} + I_3$

As $\mathcal{E}$ is positive, assumption was correct.

$1.2(2 - 15I_3) = 0.5(\mathcal{E} + 15I_3) + 0.6I_3$

$\mathcal{E} = 4.80\text{ V}$ D.C.

$2.4 - 18I_3 = 0.5\mathcal{E} + 7.5I_3 + 0.6I_3$

P is the positive terminal.

(ii) With the variable power supply still set so the 15.0 Ω lamp does not light up, the lamp is replaced by another lamp with a lower resistance.

Explain whether the new lamp with lower resistance will light up or not.

It will not light. $\mathcal{E}$ is independent of the resistance of the light bulb. If I_1 goes through R_1, I_2 through R_2, and I_3 through R_3, then

$R_2(2 - R_3I_3) = R_1(\mathcal{E} + R_3I_3) + R_1R_2I_3$, $I_3 = 0$ See Page 19

$2R_2 = R_1\mathcal{E}$ which is independent of R_3.

$\mathcal{E} = \frac{2.00 \times R_2}{R_1}$

$f' = f \frac{V_w}{V_w \pm V_s}$

(c) A moving car, with a horn emitting sound with frequency, f, starts from rest and accelerates with constant acceleration, a, towards a stationary observer a distance, d, away.

$v^2 = 2as.$

Show that the observer will eventually hear a frequency of $2f$ only if $d > \frac{v_w^{\ 2}}{8a}$, where v_w is the speed of sound.

Maximum frequency of sound occurs at maximum speed towards observer – which occurs after the longest possible distance travelled, i.e. the full distance d.

$(V_{s(max)})^2 = 0^2 + 2 \times a \times d$

$V_s = \sqrt{2ad}$

$V_w - V_s$ used as source moving towards the observer.

$f \times \frac{V_w}{V_w - \sqrt{2ad}} > 2f$

$\frac{V_w}{V_w - \sqrt{2ad}} > 2$

$V_w > 2V_w - 2\sqrt{2ad}$

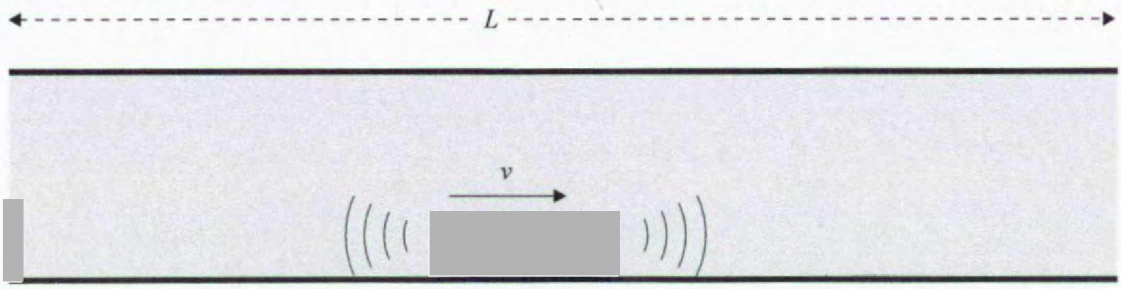
$2\sqrt{2ad} > V_w$

$8ad > V_w^2$

$d > \frac{V_w^2}{8a}$

(d) A car travels through a tunnel at a constant speed. The car horn emits sound at a constant single frequency. When sound reaches one of the open ends of the tunnel it is reflected and travels back along the tunnel in the other direction.

The size of the car is small compared to the diameter of the tunnel, so that the presence of the car does not affect the sound travelling through the tunnel.



Observer

Sources: https://signalvnoise.com/posts/920-car-design-the-side-crease-is-in
https://www.istockphoto.com/photo/casual-man-side-view-gm183765770-15426060

$v = f\lambda \quad f = \frac{v}{\lambda}$

An observer standing at one of the open ends of the tunnel will hear two distinct frequencies from the car horn.

(i) Explain why the observer hears two distinct frequencies from the car horn.

For the sound waves travelling directly from the car to the observer, a frequency lower than the horn's frequency will be heard, as a result of the doppler shift, and the relative movement of the source away from the observer. The sound waves emitted from the front of the car will be doppler shifted to have an increased frequency, as the car is moving toward the 'observer' – the end of the tunnel.

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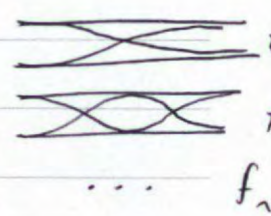
(ii) The car travels through the tunnel at a constant speed, *v*, while the horn emits sound at a constant frequency, *f*, so that both the 20th and 21st harmonics resonate in the tunnel. These harmonics cause a beat frequency of 4.76 Hz at the end of the tunnel.

Speed of sound = 343 m s^{-1}

Calculate the speed of the car, *v*, AND the frequency of the horn, *f*.

The 20th harmonic will be relevant for the wave with a ~~dop~~ downwards doppler shifted frequency, the 21st for the upward shifted.

For the first harmonic (fundamental frequency), $L = \frac{\lambda}{2}$, $\lambda = 2L$, $f_1 = \frac{343}{2L}$



$f_{20} = \frac{343}{2L} \times 20 = \frac{3430}{L}$ $\qquad f_{21} = \frac{343}{2L} \times 21 = \frac{7203}{2L}$

$f_{beats} = f_1 - f_2$

$4.76 = \frac{7203}{2L} - \frac{3430}{L}$

$4.76 = \frac{343}{2L}$

$9.52L = 343$

$L = 36.029\ldots$ m

$\boxed{f_{20} = 95.2\text{ Hz} \qquad f_{21} = 99.96\text{ Hz}}$

$f' = f \cdot \frac{v_w}{v_w \pm v_s}$

$95.2 = f \times \frac{343}{343 + v}$ $\qquad 99.96 = f \times \frac{343}{343 - v}$

$f = 95.2 \times \frac{343 + v}{343}$ $\qquad f = 99.96 \times \frac{343 - v}{343}$

$95.2 \times \frac{343 + v}{343} = 99.96 \times \frac{343 - v}{343}$

$32653.6 + 95.2v = 34286.28 - 99.96v$

$195.16v = 1632.68$

$v = 8.365\ldots$ m s^{-1}

$v = 8.37$ m s^{-1} (3 s.f.)

$f = 99.96 \times \frac{343 - 8.37\ldots}{343}$

$= 97.5195\ldots$

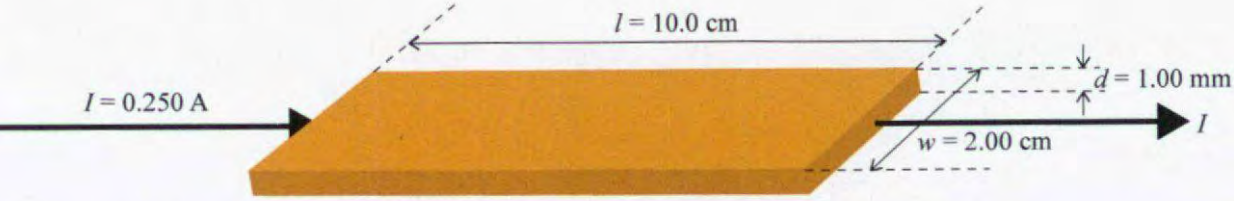
$= 97.5$ Hz (3 s.f.)

QUESTION FOUR: HALL EFFECT

Charge of an electron = -1.60×10^{-19} C

When charge flows through a conductive material, e.g. a metal, only some of the electrons are free to move. A conductor has a fixed number of free electrons per unit volume, n.

For copper metal, $n = 8.49 \times 10^{28}$ electrons m^{-3}



(a) (i) A piece of copper metal 10.0 cm long, 2.00 cm wide, and 1.00 mm thick has a current of 0.250 A flowing through it.

By first calculating the amount of free charge in the piece of copper, determine the average speed of a free electron as it flows through the piece of copper.

N is the total number of free charge carriers.

$N = n \times V$

$= 8.49 \times 10^{28} \times 10 \times 10^{-2} \times 2 \times 10^{-2} \times 1 \times 10^{-3}$

$= 1.698 \times 10^{23}$ electron

Q_T is the total free charge

$Q_T = N \times e$

$= 1.698 \times 10^{23} \times 1.6 \times 10^{-19}$

$= 27168$ C $\boxed{\text{See Page 18}}$

charge per unit length $= \frac{27168}{10 \times 10^{-2}} = 271680$ Cm^{-1}

~~I = Coulomb per second~~

~~$v = \frac{I}{\text{charge per unit length}} = \frac{0.25}{271680}$~~

~~$= 9.202... \times 10^{-7}$ ms^{-1}~~

~~$= 9.20 \times 10^{-7}$ ms^{-1} (3 s.f.)~~

$\boxed{\text{See Page 18}}$

$v = \frac{I}{ne}$

(ii) The current flowing through a conductor is given by the relationship:

$I = neAv_d$

where e is the charge of an electron, A is the cross-sectional area of the conductor, and v_d is the average drift velocity of a free electron.

Show that the relationship above is dimensionally consistent.

I : Cs^{-1} $I = neAv_d$

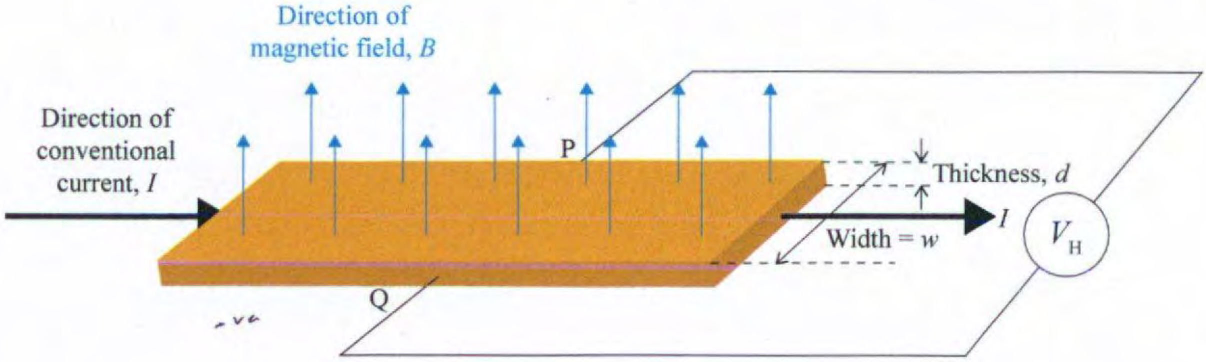
n : m^{-3} Cs^{-1} = m^{-3} × C × m^2 × ms^{-1}

e : C   Cs^{-1} = Cs^{-1}

A : m^2 Same units in each term, therefore it is dimensionally consistent.

v_d : ms^{-1}

When charge flows through a conductor which is inside a uniform magnetic field, a sideways force is exerted on the moving electrons that pushes them to one side of the conductor. This makes one side of the conductor positively charged and the opposite side negatively charged. This separation of charge produces a potential difference, known as a Hall Voltage, V_H, across the two sides of the conductor.



(b) By considering the magnetic force acting on an electron moving through the conductor, state which side of the conductor, P or Q, is positively charged.

Your answer must include a description of how you made your selection.

As conventional current is moving from left to right, the electrons are actually moving from right to left (conventional current presumes the movement of positive charges). According to Fleming's Left Hand Rule for the force on a moving particle in a magnetic field, the force on a negatively charged particle moving from right to left is towards Q, making P positively charged due to a depletion of electrons. There is a relatively higher number of protons at P, making it positive.

(c) (i) When a steady current is flowing, the sideways forces acting on an electron moving through the conductor are balanced.

Explain the origin of the force that balances the magnetic force on a moving electron.

As electrons move to Q end, an imbalance of charge is created. Q is more negatively charged, and P more positively. This results in an electric field, from P to Q, which exerts a force on electrons, towards P. If the electric field is weaker than the magnetic field, electrons will be deflected toward Q, increasing the electric field strength. If the electric field is stronger, electrons would be deflected towards P, reducing the strength of the electric field.
Overall, an equilibrium is reached when, on average each electron is undeflected, as electric forces balance magnetic forces.

Question Four continues on the following page.

(ii) By considering the sideways forces acting on a moving electron, show that the Hall Voltage, V_H, is given by the expression:

$$V_H = \frac{BI}{nde}$$

$E = \frac{V_H}{w}$ (Electric field strength is equal to the potential gradient.)

$F_E = Eq$
$= \frac{V_H e}{w}$

$F_B = Bqv_d$ $\quad$ $I = neAv_d$
$= B \cdot e \cdot \frac{I}{neA}$

$F_B = F_E$ $\quad$ Magnitudes are equal, as the forces are in opposite directions.

$\frac{V_H e}{w} = \frac{BeI}{neA}$ $\quad$ ~~A =~~

$V_H = \frac{BIw}{neA}$ $\quad$ $A = w \times d$

$\cancel{V_H = \frac{BI}{ne}}$

$V_H = \frac{BI}{nde}$

(d) Measuring the Hall Voltage is a commonly used method for determining the strength of a magnetic field.

Describe the conditions necessary to achieve the most precise measurement of the strength of a magnetic field, and any practical limitations to achieving these conditions.

B obviously cannot be changed ~~by experiment~~, as it is the measured quantity. Nor can e, as it is a fundamental quantity. To measure B most precisely, we would want the maximum possible value for V_H, so the absolute error due to measurement uncertainty of our equipment corresponds to a smaller percentage error. Increasing the current will help, but could lead to overheating of the slice. Especially considering the low thickness, the resistance will be high, leading to large energy dissipation. The slice could melt if the current was too high. Materials with low n should also be selected. Semi-conductors are often used due to their low n, however again, if the resistance is too high, the current will have to drop. There are similar considerations with d, which should be as low as possible to retain some structural integrity to prevent damage. w should be large so as to reduce resistance

 See Page 18

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

2 d i

$\boxed{\text{cont.}}$ Therefore, due to the decrease in atomic number from the ~~product ($^{13}_{6}C$)~~ reactant ($^{14}_{7}N$) to the product ($^{13}_{6}C$), there was likely a decrease in the binding energy per nucleon, and due to the decrease in nucleon number, almost certainly a decrease in total binding energy, making this reaction highly unfeasible. By contrast, in reaction one there is an increase in atomic number from $^{14}_{7}N$ to $^{17}_{8}O$, resulting in an increase in binding energy per nucleon, and a much more feasible reaction. Reaction One is more likely.//

Seen

3 d i

$\boxed{\text{cont.}}$ These sound waves will retain the higher frequency after being reflected by the end of the tunnel, and will be heard by the observer at a frequency higher than the car's horn, as if the car was driving towards him at the same speed it is travelling towards the end of the tunnel at. //

Seen

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

while not affecting V_H

4 d

I must also [cont.]. l should also be short, again to reduce resistance. Together, these will allow the maximum current to flow. The magnetic field must also be perpendicular to the current for maximal of Hall Voltage - ~~the conductor should be positioned (rotated) so a maximum value of V_H is obtained~~

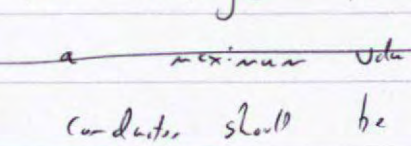
The conductor should be positioned (rotated) so a maximum value of V_H is obtained. Other values of V_H would lead to ~~inaccurate~~ inaccurate ~~estimates~~ values for of B. // seen

Overall, these changes maximise the value of V_H, minimises the percentage error, and therefore leading to maximum precision.

Simply using more precise voltmeters/ammeters/micrometers for d/data for n will also help improve precision //

4 d i

Further Explanation



$Cur = \frac{Q_T}{\ldots}$

$I = \frac{Q_T}{l/v_d}$ $I = \frac{Q_T v_d}{l}$

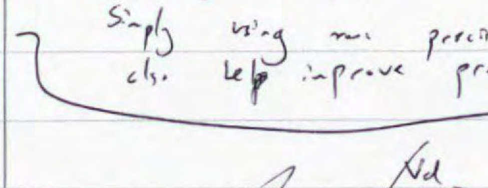
$v_d = \frac{I}{Q_T} l$

Charge per unit length = 271680 C m⁻¹ (total charge / Length)

Current = Charge per unit length × V_d

$V_d = \frac{I}{\text{Charge per unit length}}$

$= 9.20 \times 10^{-7}\ m s^{-1}$ (3 s.f.)



$$\text{Current} = \frac{\text{total charge}}{\text{Time taken to move length } l}$$

$$I = \frac{Q_T}{l/V_d}$$

$$I = \frac{Q_T \cdot V_d}{l}$$ seen

$$V_d = \frac{I \times l}{Q_T}$$

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

$V_d = \cancel{0.25 \times 10^{-2}}$

$V_d = \frac{0.25 \times 10 \times 10^{-2}}{27168}$ seen

$= 9.2020\ldots \times 10^{-7}$

$= 9.20 \times 10^{-7}\ m\,s^{-1}$ (3 S.F.) //

3

b

ii $\boxed{\text{Co.t.}}$ In this circuit, the potential difference ~~between~~ across the lamp is 0V. Therefore, no matter the resistance, no current will flow // seen

2

d

ii $\boxed{\text{Co.t.}}$ It would still have a parabolic slope as there is still no force in the horizontal direction, and a constant magnitude of acceleration in the vertical direction (due to gravity). time of flight would be larger ($t = \frac{2u\sin\theta}{g}$, g inv decrease, t increase). Therefore a greater horizontal distance is covered. (Same v_x, larger t, large s_x/range) ///

seen

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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