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ANSWER BOOKLET

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

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Show ALL working. Start your answer to each question on a new page. Carefully number each question.

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Question	Score
ONE	
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1.a.i. The median price of apple phone listings was around $1400, which is higher than the median prices of Oppo and samsung ~~smart~~phone listings which both lie around $600. There was less variation in Oppo phones' prices as evidenced by the narrow interquartile range of around $350, ~~The~~ compared to Samsung and Apple, where the interquartile range was around $600. There may be a relationship between the brand and the colour of ~~smart~~phone listings. 30% of Apple listings were black compared to around 50% of oppo and Samsung listings for instance, and around 28% of ~~apple~~ Apple phones listed were red, compared to around 8% of Oppo and samsung phones. This may have a relationship with price as revealed through subgrouping by colour. Red ~~apple~~ Apple phone listings have a median price of around $1000, compared to the median price of red Oppo and Samsung phones which was around $500. The higher variation in prices of samsung phones compared to Oppo phones may be due to the large interquartile range of around $1200 for black ~~so~~ samsung phones compared to around $200 for black Oppo phones.

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1.a.ii. In order to be able to generalise my descriptions to all Apple, Samsung, and Oppo branded mobile phones listed for sale within New Zealand, there must be evidence to suggest that the sample taken from a New Zealand electronic retailer's website is representative of the wider population. This could be done by comparing the average price of phones at this retailer to the average price of phones sold in New Zealand; if the retailer is more upmarket or budget compared to the wider NZ market, there is evidence to suggest that the data presented is not representative of the wider population. A poll could be carried out to determine the colours of people's recently-purchased smartphones, and this could be compared to the colours of smartphones on sale ~~purchased~~ at this store through constructing a confidence interval. If ~~there is~~ the ~~store's popular colours are not~~ proportions of different phone colours on this retailer's websites are different to what New Zealanders are purchasing, the results cannot be generalised.

1.b.i. For Apple phones listings, the relationship between weight and screen size at this store shows a positive association, with a linear trend. The relationship strength appears ~~weak~~ moderate, with an even, ~~large~~ ~~small~~ moderate amount of scatter above and below the trendline. However, the high correlation coefficient of r=0.89 lies between 0.75 and 1, indicating that the relationship is strong.

For Samsung phones listings, the relationship between weight and screen size at this store shows a positive association, with a linear trend. The relationship strength appears moderate, with more scatter above than below the trendline. The correlation coefficient of r=0.71 which lies between 0.5 and 0.75 supports this. There appears to be an outlier ~~p~~ for a ~~phone~~ 7.6 inch-screen phone listing that weighs around 285g, which is likely lowering the correlation coefficient.

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1.b.ii. Apple: $w = -39.21 + 38.81 \times 7.5$
$= 252g$

Samsung: $w = -16.11 + 31.41 \times 7.5$
$= 219g$

The phone is more likely to be a samsung phone as ~~this~~ the predicted weight of ~~22~~ 219g is closer to the known weight of 225g than the 252g predicted weight of the Apple phone.

1.b.iii. One reservation I would have using these models to decide the brand of this phone is that this phone has a ~~max~~ screen size of 7.5 inches which is beyond the data range, meaning we are extrapolating. There is no evidence to suggest that the trend will continue at larger screen sizes as there is likely an upper limit to the weight of a phone that a customer would tolerate, causing companies to invest in reducing the weight of these larger phones. Another reservation is that the outlier ~~o~~ of the samsung phone is vertically distant from the trendline, and has a ~~meaning the~~ screen size of 7.4 inches which is close. Thus, the prediction may be an underestimate of the actual phone weight. The outlier would also have an effect on the trendline, causing the gradient to increase and the y-intercept to decrease, meaning that if the outlier is an error, ~~it~~ this means that the linear model does not correctly model the correct data, so the prediction could be an overestimate.

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2.a.ii. The long-term trend for the retail sales of electronics over the period 2010 to 2020 is an ~~positive~~ increasing one, with average electronics sales increasing from around ~~$62~~ $620 million in early 2010 to around $1.2 billion ~~$~~ in late 2020. The rate of increased has increased over time with a fast incline in the moving average line beginning in early 2020. ~~The~~ Retail ~~sa~~ sales of electronics show regular seasonal variation over the period, though the effect of this is not pronounced, with a small spike of around $100 million above the trendline on average from October to December, likely as a result of Christmas.

The long-term trend for ~~th~~ accommodation over the period 2010 to mid-2019 is an increasing one, with average accommodation sales increasing from around $660 million in early 2010 to around $1.08 billion in mid-2019. However, the ~~trend~~ moving average line began to decrease from mid-2019 likely due to the Covid-19 pandemic beginning in early 2020, which prevented most travel. The ~~pa~~ trend shows regular seasonal variation aside from a trough from April to June 2020 which was around $400 million below the trendline due to the pandemic. ~~It~~ On average

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accommodation sales peak over the summer months, being around $200 million above the trendline in January to March, with a trough of around $100 million below the trendline from April to September.

In comparison, the long-term trend ~~of both~~ for the retail sales of electronics and accommodation over the period 2010 to ~~2020~~ 2019 has been an increasing one; however, while accommodation sales began to degrease on average in 2020, electronics sales began to increase on average as people needed technology to stay connected during the pandemic. Accommodation sales show stronger seasonality as more people tend to holiday in the summer and less in the winter whereas retail sales of electronics shows less seasonality as electronics are purchased year-round, with only a small peak from october to December in the holiday season.

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2.a.ii. Forecasts should be made using an exponential smoothing model like Holt-Winters which places greater weight on the recent long-term trend and seasonality. This is because recent ~~vari~~ variation is more likely to predict future variation. However, the long-term trend has changed for both retail sales of electronics and accommodation in 2021, with the ~~trend~~ moving average line increasing more quickly for electronics and reversing ~~direc~~ direction and decreasing rapidly for accommodation. ~~He~~ Seasonality also varied significantly in April to June of 2020 for accommodation sales, as already discussed. Hence, ~~the forecast~~ there will be little confidence in the validity of the forecast due to increased uncertainty as to how the time series will behave even into the near future, resulting in wide confidence intervals and forecasts that are not useful.

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2.b. Mobile cellular subscriptions ~~per~~ were ~~around~~ close to 0% from 1960 to around 1990 as ~~nobody~~ very few owned mobile phones. However, around 1990, subscriptions rose exponentially as mobile phones became more commonplace to around 110 per 100 people in 2008 due to people owning multiple phones. ~~Th~~ Subscriptions plateaued until around 2014, from which they rose again.

Fixed telephone subscriptions have remained relatively steady over the period ~~1900~~ 1960 to 2017, reaching a peak of around 50 subscriptions per 100 people in 1999, and gradually decreasing until 2016, before a sharp decline in 2017. This is reflective of consumers switching gradually from landline use to mobile use.

Fixed broadband subscriptions only emerged in 2000 likely when they went to market. This showed signs of ~~expon'l~~ exponential growth until around 2007, from which the number of subscriptions per 100 people began to increase at a constant rate.

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2.c.i. The number of mobile subscriptions per year for Fiji has not increased at a faster rate than New Zealand from 2005 to 2008.

~~ΔSn~~ Let S be the number of subscriptions.

$\Delta S_{NZ} = 4620000 - 3530000$

$= 1090000$

$\Rightarrow 363333$ per year

$\Delta S_{Fiji} = \cancel{395000}\ 600000 - 205000$

$= 395000$

$\Rightarrow 131667$ per year

However, the claim may be referring to the increase in the number of mobile subscriptions per year from 2005 to 2008 as a proportion of the number of ~~pt~~ mobile subscriptions in 2005, in which case the claim is justified

Let P be the change in the number of subscriptions as a proportion of 2005 numbers

$$P_{NZ} = \frac{4620000 - 3530000}{3530000}$$

$= 30.9\%$

$$P_{Fiji} = \frac{600000 - 205000}{205000}$$

$= 193\%$

$P_{Fiji} > P_{NZ}$.

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2.c.ii. In 2009, Fiji had 640 000 mobile phone subscriptions. New Zealand had 4700000.

~~Fiji mode~~

~~F~~ Samoa vs. Fiji model

$$\#\text{Samoa} = \frac{150}{1000} \times \#\text{Fiji}$$

$$= 0.15 \times 640000$$

$$= 96000$$

Samoa vs. NZ model

$$\#\text{Samoa} = \frac{150}{6000-1600} \times \#\text{NZ} - 40000$$

$$= \frac{150}{6000-1600} \times 4700000 - 40000$$

$$= 120000$$

A reasonable estimate may be somewhere between 90000 and 130000 mobile phone subscriptions.

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3.a. The type of experiment is one with two independent groups. The experimental units are university students. The categorical variable is whether or not there were mobile phone notifications (that the units had to read), while the response variable is the number of words correctly recalled. Units are randomly allocated to a control and treatment group, ~~where they~~ and both ~~perfor~~ groups undertake the same test ~~but the treatm~~ under the same conditions except the treatment group performs the test with the intervention ~~of having~~ to read mobile phone notifications.

3.b. Students had to be randomly allocated to the control and treatment groups to ~~ff~~ ensure that any potentially ~~cof~~ confounding variables such as gender, age, degree, were evenly distributed across the two groups. The only ~~dif~~ difference between the two groups should be the intervention so that a causal claim could be made, as this should be the only factor causing a ~~chor~~ difference in the response variable.

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Q 3.c. The mean number of words correctly recalled for students with no mobile phone notifications was 0.85 words higher than the mean number of words correctly recalled for students with mobile phone notifications, suggesting that having mobile notifications decreases ~~mem~~ the amount of words correctly recalled. However, to determine whether this difference is statistically significant to claim causation, a randomisation test was carried out whereby the values were randomly sorted into two groups 1000 times and the difference in means calculated each time, and plotted on a re-randomisation distribution. The ~~distrib~~ re-randomisation distribution has a tail proportion of 0.069 suggesting that the probability of a difference of means 0.85 or greater under chance alone is low (below the threshold of 10%). Hence, there is sufficient evidence to claim that having no mobile phone notifications causes the ~~ave mean~~ number of words correctly recalled to increase, on average.

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3.d. A similar study could be designed to take each student's innate memory ability into account through carrying out a paired comparison. ~~A~~ Each student would have to sit the test twice, once with mobile phone notifications, and once without. ~~Half of~~ The students should be randomly allocated into two groups, one of which carries out the test with notifications first and one of which carries out the test without notifications first, to minimise bias and order effects. ~~A differen~~ Similar tests but with different words should be used ~~t~~ each time to avoid rehearsal effects.

The difference in the number of words correctly recalled ~~ste in~~ with notifications and without notifications should then be calculated. Thus, each student acts as their own control so each student's memory ability is taken into account. ~~A causal~~ There would be evidence to suggest that having no mobile notifications caused an increase in a student's memory ability if 75% of the increases are positive, which can be visualised through an arrows distribution.

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3.e. The claim that "playing video games ~~statistically~~ significantly affects your memory for the better" cannot be supported by the design of this study.

Firstly, an intervention has been applied to ~~some of~~ half of the sample whereby they had mobile phone notifications which they were forced to read. This already means that the results of the randomisation cannot be applied to a wider population.

Secondly, the randomisation from part (c) showed that having no mobile phone notifications caused memory ~~to~~ (or the number of correctly-recalled words) to increase, on average. Since ~~there were~~ units were randomly allocated to control and treatment groups and the proportion of gamers and non-gamers was not controlled across each group, there may simply have been ~~more~~ a higher proportion of gamers in the group that had no mobile notifications, so the intervention applied may explain the result.

Finally, the claim is ~~using~~ actually a form of inferential statistics. If the relationship between the number of words correctly

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recalled and gaming was to be explored, a poll should be carried out instead and the results should undergo bootstrapping instead of randomisation to determine if there is a statistically significant difference in the wider population.

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4.a. One new variable that has been created for the new data set is a student's school level. This has been created through classifying Years 3 to 8 (inclusive) as "Primary or Intermediate" and Years 9 to 13 (inclusive) as "Secondary".

Another new variable that has been created is the ~~i~~ island in New Zealand that the student lives on. This has been created through using the data about the student's region which places them in either the North Island, "Te Ika-a-Māui" or the South Island, "Te Waipounamu". The Chatham Islands and Rakiura may be other possible classifications.

4.b.i. The visualisation reveals that ~~in all Census at School~~ in all years the CensusAtSchool survey has been conducted, a higher proportion of students own mobile phones than do not. The ~~number~~ proportion of students who owned mobile phones increased from 2005 (58%) to 2015, where it peaked at 80.4%. The visualisation also reveals that in all years the survey has been conducted, a much higher proportion of students are from Te Ika-a-Māui rather than Te Waipounamu and this has remained relatively

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consistent at around 76%, with a slight decrease from 2013 onwards to around 73% from around 79%. It also reveals that the proportion of students surveyed from primary/intermediate levels was initially higher at ~~about~~ 58.6% in 2005, but then decreased to only 30.4% in 2015.

4.b.ii. One graphical technique used for the visualisation is visually depicting the difference between the two proportions for each categorical variable in each year through a line. This makes it easier to see how the proportions have changed from year to year as the lines are symmetrical and the eye moves down the page.

Another graphical technique is the use of colour. This ensures that when the majority categorical variable changes such as between 2007 to 2009 when the proportion of secondary school respondents ~~decrease~~ increased from 47.2% to 56.9%, the reader is able to track this change easily and see that while most respondents before 2009 were primary / intermediate students, this changed from 2009 onwards.

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4.b.iii. One reservation I would have with using the sample percentages from the combined ~~to make the given claim~~ sample data set is that selection bias may be occurring. As the survey is conducted online, only ~~students~~ with access to an internet connection are able to respond. Students without an ~~and~~ internet connection, who are part of the wider population of all New Zealand Year 3 to Year 13 secondary students, are ~~m~~ less likely perhaps to own a mobile phone as mobile phones would have less benefit. Thus, the proportion of students owning a mobile phone in this sample is likely different to the wider population, so ~~a gene~~ an inference should not be made.

Another reservation I would have is that the survey appears to have not used stratified sampling as the proportion of secondary students and primary/ intermediate students ~~has fluct~~ in the sample has fluctuated greatly over time, something that is highly unlikely to occur in the wider population. As secondary school students are more likely to ~~do~~ own mobile phones (a lurking variable), the increasing rates of mobile phone ~~owersh~~ ownership over time may just be due to the increasing proportion of secondary

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school students in the sample. Hence, the sample may not be representative of the wider population and a statistical inference should not be made.

4.a $p = \frac{190}{190+186+184+138+98}$

$= 0.248$

$MoE = 1.96\sqrt{\frac{p(1-p)}{n}}$ (95% confidence)

$= 1.96\sqrt{\frac{0.248 \times (1-0.248)}{766}}$

4.c. $p = \frac{190+186+184}{190+186+184+138+98}$

$= 0.692$

As $0.3 \leq p \leq 0.7$, the rule of thumb for the margin of error can be used.

$MoE = \frac{1}{\sqrt{n}}$

$= \frac{1}{\sqrt{766}} = 0.0361$

~~p = 0.692 ± 0~~

CI: 0.692 ± 0.0361

$[0.656, 0.728]$

The proportion of NZ Y3-13 students who own mobile phones who check their phones either sometimes, often, or always for messages or notifications as soon as they wake up could be as low as 65.6% or as high as ~~0.~~ 72.8% with 95% confidence. As the entire confidence interval is above 50%, ~~it is~~ there is sufficient evidence to claim that most ~~stude~~ New Zealand Year ~~13~~ 3 to Year 13 students who own mobile phones either sometimes, often, or ~~always~~ always check their phone for messages or notifications as soon as they wake up.

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5.a.i. The mean size of ~~Android~~ Apple mobile phone apps is 192.72MB higher than the mean size of Android apps in the sample. ~~The~~ Both the distribution of sizes for Android and Apple apps is right-skewed. ~~while the distributio~~

~~Boot~~ Resampling with replacement was carried out ~~for each~~ both the Android and Apple samples and the means of the resamples were calculated. The differences in the means were plotted on a bootstrap distribution taking the middle 95% to ~~construct~~ construct a confidence interval for the difference in means in the wider population [137.64, 261.69]. ~~The entire confi~~ In the wider population, the mean Apple app size is likely to be between 137.64 MB to 261.69MB greater than the mean Android app size. The entire confidence interval is above zero so it can be inferred that Apple apps tend to be bigger in size than Android apps.

5.a.ii. Problem: The ~~larges~~ mean size of the largest mobile app students at a particular school have installed on their phones ~~is it~~ must be known and if the type of phone has an effect on this.

Plan: Take a sample of 30 students or greater and ~~calculate the mean sosizes~~ record the size of their largest app. Inferential statistics can be used to determine if there is a

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difference between Apple and Android phones

Data: Record the largest size app on ~~st~~ random students's phones and whether their phone is an Apple or Android. The Randomness could be ensured by asking every 5th student passing through the school gates for example.

Analysis: the mean and median size of the largest app can be calculated as an average. Resampling with replacement can be performed after calculating the difference in means/medians between the Android appsizes and Apple app sizes is calculated. This will generate a confidence interval for the difference in means/medians (medians should be used if there are many outliers).

Conclusion: The mean/median size of the largest mobile phone app students have in the sample is known. ~~A~~ If the entire bootstrap confidence interval for the difference in means/medians lies above zero, then the type of phone has an effect.

Note that a confidence interval for the mean ~~can~~ in the whole population can also be generated using bootstrapping.

5.b.i. $P(X<60, \sigma=10.6, \mu=52.4) = 0.7633$

$P(X<70, \sigma=10.6, \mu=52.4) = 0.9516$

$P(X>60 \mid X<70) = 1 - \cancel{P(X<60}\ P(X<60 \mid X<70)$

$= 1 - \frac{0.7633}{0.9516} =$

$= \cancel{0.198}\ 0.1979$

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5.b.ii. Another factor that could be taken into account is the day of the week. On weekdays for instance, $P(X<70)$ may be lower as people work more and are on their phones less. Thus, a separate mean and standard deviation for the mean battery charge percentage could be calculated for each day of the week.

5.b.iii. Let C be the event that the phone was charged ~~unplugged~~ using a computer and U be the event that it was unplugged within the first 30mins after the battery had been charged.

$P(U)=0.23 \quad P(C)=0.39$

$P(C|U)=0.6$

$P(C\cap U) = P(U)\cdot P(C|U)$

$= 0.23\times 0.6 = 0.138$

~~P(CA~~ $P(C\cup U) = 0.23+0.39-0.138$

$= 0.482$

$P(C'\cap U') = 1-0.482 = 0.518$

	C	C'	T
U	0.138	0.092	0.23
U'	0.252	0.518	0.77
T	0.39	0.61	1

13.8% of participants charged their phone using a computer and unplugged within 30 mins. 9.2% of participants did not charge using a computer and unplugged within 30 mins. 25.2% of participants charged using a computer and did not unplug within 30 mins. 51.8% of participants neither charged using a computer nor unplugged within 30 mins.

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