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# OUTSTANDING SCHOLARSHIP EXEMPLAR

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
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# Scholarship 2021
# Calculus

Time allowed: Three hours
Total score: 40

## ANSWER BOOKLET

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Write your answers in this booklet.

Make sure that you have Formulae Booklet S–CALCF.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Answers developed using a CAS calculator require **ALL commands to be shown**. Correct answers only will not be sufficient.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

| Question | Score |
|---|---|
| ONE | |
| TWO | |
| THREE | |
| FOUR | |
| FIVE | |
| TOTAL | |

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Question 1

a) i) From Figure 1 it can be seen that the price distribution for apple phones has a minimum price that is much greater than the minimum price of oppo / Samsung phones, starting at $750 (approximately) vs ~$250 for oppo / samsung. The distribution of Apple phone prices is right skewed with a median of approx $1400. Oppo phones have a distribution that is mostly normal, with the exception of 2 phones priced at close to $2000. The median price of oppo phones is the lowest in the sample at approx ~$550. Samsung phones have a price distribution that is heavily right skewed - most phones offered are under $1000 however the distribution continues until $2700 for the most expensive model. The median price of Samsung phones is slightly higher than oppo at ~$600.

From Figure 2 it can be seen that for all brands Black is the most common colour sold, with both oppo & Samsung offering ~30% of phones in black and Apple offering ~30%. Apple offers equal proportions of Blue, Red, and White phones with approx ~23% each. Oppo and samsung offer relatively similar percentages of Blue / White phones however offer comparatively less than apple for Red phones, only ~9-8% each.

Figure 3 displays the relationship between phone colour and price, by brand. It can be seen that for Apple, their most expensive phone offerings are blue (almost exclusively), while the blue phones only make up cheap / low-priced Samsung phones and mid-priced oppo phones (with the exception of one blue oppo at ~$2000). Samsung only offer their high priced phones ($2000 and above at this retailer) in black, however, black makes up a significant portion (over half) of their low-cost phones.

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Oppo and Samsung only offer 2 and 3 phones respectively in Red, all of which are below $1000.

ii) - The size of this online retailer. A smaller retailer may only carry the most popular / fast selling phones as they are unable to afford to stock all models, and may carry a smaller selection of phone colours than a large retailer who moves inventory more quickly.

- If the quantity of phones carried was representative of Apple, Oppo's, and Samsung's product line-up. The retailer appears to carry a relatively high number of Apple phones compared to Oppo / Samsung. The store may be a mostly Apple-focused store and therefore not necessarily representative of all stores in NZ.

b) i) For Apple phones, the relationship between Screen size and weight in grams is a moderately strong, linear, positive relationship, where for every 1" increase in screen size the weight of a phone increases by 39g on average. Apple phones range in size from 4.5" to just under 7" (~6.9") and range in weight from ~110g to ~225g (approximately). The moderate-strong relationship is indicated by a correlation coefficient of 0.89 (r=0.89). There is moderate amounts of scatter in the data.

- Samsung phones have a moderate, linear, positive relationship between screen size and weight where for every 1" increase in screen size weight increases by 31g on average. There is a moderate amount of scatter about the trend, which is indicated by the correlation coefficient of r = 0.71, indicating

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only a moderate relationship strength between screen size and weight (that is, for any given screen size there are a range of weight's which the phone could ~~pleasably~~ ~~bigger~~ be). There is a notable outlier at approx (7.6, 280) which is significantly larger and heavier than all other phones (for both brands). Samsung phones weigh between 280 and 130g and have screens between 4'' and 7.8''.

ii) Apple prediction: $-39.01 + 38.8 \times 7.5^2 = 251g$
Samsung prediction: $-16.11 + 31.41 \times 7.5 = 219g$
Based on the weight given ~~B~~ of 225g it is more likely to be a ~~Samsung~~ phone, as 219g is closer to 225g than 251g is.

iii) I would be hesitant to use the models given based off the data in Figure 4 as neither graph has phones well represented at 7.5in''. There are no apple phones above 7'' in size - it is unclear whether this is because Apple does not produce this size of phone ~~or~~ or this retailer does not stock apple phones. There is no data above 7'' and hence I would be hesitant to use the model given to predict the weight of an Apple phone with a 7.5'' screen size.

Additionally, for samsung there is only one phone above 7'', which is ~7.4'' and weighs ~280g. As there is only one data point it is not obvious if all samsung phones of this size weigh this much, or if this phone is an outlier. In either case the model does not fit the data for samsung phones over 7'' well, and the actual estimate may be much higher for a Samsung phone of 7.5'', making the phone more likely to be Apple.



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Question 2

a) From Figure 3, showing the change in quarterly Electronics and accomodation sales over time, it can be seen that electronics sales have consistently increased over time (bar 2011 to 2012 when it dropped). Electronics sales remained relatively constant from 2010 to 2014 between $600 to $800 (million) for the trend and peaking at ~800 million in ~~December~~ Q4 2011. Electronics sales then rapidly increased to a peak value of $1350 million (raw value) in Q4 2020. There is a seasonal effect for Electronic Sales, with sales remaining under the trend, and at a minimum/trough in $Q_1$ (Jan - March), ~~and~~ from $Q_1$ to $Q_3$ and then peaking above the trend annually in $Q_4$, with an average seasonal ~~effect of~~ difference of +100 million in $Q_4$.

Accomodation Sales increased year on year from 2010 until 2019, with the trend curve increasing from ~$600 million in Q1 2010 to ~$1050 in $Q_1$ 2019. The raw data peaked at $1300 million in $Q_1$ 2019. At the start of 2020 ~~End of 2019~~ the accomodation sales plummeted to a raw all-time-low (since 2010) of ~$550 million before increasing to $800 and $950 million in $Q_2$ and $Q_3$ of 2020 respectively. This was caused by the global lock-down due to the covid-19 pandemic where visitors were unable to travel to NZ. It can be seen that the seasonal difference is +200 million above the trend in $Q_1$, later falling to ~-100 million in $Q_2$ & $Q_3$. It is ~+50 million above the trend in $Q_4$ on average.

ii) As the data for both graphs is influenced by covid-19, with electronic sales spiking due to working/studying from home and accomodation sales decreasing



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sharply due to visitors being unable to travel to NZ it is unclear if a 'staircase' effect will be observed where sales will spike/dip sharply and then resume pre-2020 growth levels, or rapidly rebalance to the same pre-2020 levels/values once lockdowns lift globally. As the most recent data in Figure 5 is heavily influenced by recent events, it is not necessarily suitable to use it to forecast sales in 2021.

b) In 1960, fixed telephone subscriptions were by far the most popular, at approx 20 subscriptions per 100 people, and Mobile subscriptions were at almost 0/100 per 100 people. Fixed telephone subscriptions increased from 20/100 people in 1960 to a peak of ~48/100 people in 2000 before declining to ~ ~~30~~ per 100 people in 2017 (the rate of decline is more rapid since ~2015). Mobile cellular subscriptions remained close to 0/100 people until 1990, where they began to grow at a very fast rate, increasing to almost 140 per 100 people in 2017, with the exception of the slight decline from 2010-2011 from 110-105 subscriptions/100 people. Mobile cellular subscription remain the most popular (highest rate of subscription since overtaking fixed telephone in 2000. Fixed broadband started being offered in 2000, increasing often rapidly from 2000-2007, ~~before~~ from close to 0/100 people to 20 per 100 people in that time period. Fixed broadband ~~remain~~ then increased at a more steady rate from 2007 to 2017 increasing from 20 per 100 people to ~35 per 100 people.





c) i) Fiji: $\frac{620000-225000}{4}$ = ~~98750~~ 98750 per year increase

NZ: $\frac{4620000-3530000}{4}$ = 272500 per year increase

The claim is not justified. NZ increased at a rate of 272500 subscriptions a year on average from 2005-2008 while Fiji increased at a rate of 98750 per year on average. Fiji had a greater proportional change but not a faster rate of increase.

ii) • Based on Samoa from 2010-2017:
$\frac{124211-90000}{8}$ = 4276 / year. 90,000-4276 = 85723 in 2009.

• Based on ~~NZ~~ Fiji: $\frac{150}{1000}$ = 0.15. Approx 1:0.15 ratio of phone subscriptions.
Fiji 2009: 640,000. 640,000 × 0.15 = 96000

• Based on NZ: From 2007-2010, $\frac{40}{4700-4700}$ = 0.08
NZ 2009: 4700,000. 4700000 × 0.08 = 376,000

Estimated value of 86000 using the trend from 2010-2017 for Samoa, applied to 2009. Estimates from NZ/Fiji are too high.

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Question 3

a) ~~The treatment group~~ The experimental study sought to measure the impact of the explanatory variable, mobile phone notifications, on the response variable, number of words recalled from a memorised list. The control group was given the list to memorise with no notifications while the treatment group had to read phone notifications. Random allocation was used to minimise the differences / balance individual characteristics between the two groups.

b) Random allocation ensures that each student has the same chance of being assigned to either of the two groups, and therefore the two groups have the greatest chance of being characteristically similar. This helps to minimise the impact of individual variances (such as in this case, individual differences in memory skill between participants) on the overall outcome of the experiment.

c) The students in the sample had a difference in mean words recollected of 0.85, where the mean of the group with no mobilephones was 0.85 words higher. From the re-randomisation distribution we can see that when chance is acting alone a difference of 0.89 or greater occurred 69 times out of 1000 trials or 6.9% of the time. This is greater than the 5% or less conventionally required to establish that chance acting alone could not have caused the difference observed and hence the ~~call~~ claim cannot be made that the lack of mobile notifications alone caused the shift and other factors may have influenced the trial.





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d) The students could again be assigned into two groups randomly. The students would then take a pre-test where no other variables are changed (no treatment variable / no phones). The treatment group would then be assigned their phones with notifications and ~~the~~ instructions to read them and the test repeated for both groups ~~after the difference~~ with a similar in difficulty ~~as~~ list to memorise to the first list. The difference in each participants score for groups 1 and 2 (control and treatment) could then be calculated and a mean value calculated for each group. The differences in each participant's score could be plotted and ~~bootstrapped~~ re-randomised to observe the likelihood of the shift occuring by chance alone and then a claim can be made / not made of the memory of group 2 is sufficiently worse (the mean change is sufficiently greater than group 1). Therefore each participant's pretest acts as their own control. (Alternatively a similar study can be done with just 1 group and then the mean change ~~change~~ bootstrapped to find a 95% confidence interval for the shift. If ~~a~~ confidence interval does not overlap 0 claim can be made).

e) The study is an observational study, as they cannot randomly group participants into 'gamer' and 'non-gamer' groups and subsequently force them to play games for years, as that would be both unethical and impractical. The study would have observed that gamers recalled 1.2 words on average and the bootstrap distribution / trial proportion supports that the shift could not have occurred by chance alone (as 3.2% < 5%.), However, a causal



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claim cannot be made as there was no treatment and control grouping or random allocation. Hence the headline is not supported by the study ~~and would~~ as it implies that playing video games leads to better memory, however the study only identified a correlation. The headline would be more suitably re-worded to 'Playing games linked to better memory' or 'Gamers are more likely to have better memory than non-gamers'.



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Question 4

a) 'school_level' and 'Island'. School level, with the options being 'secondary' or 'primary or intermediate', would have been created based off the 'year level' variable with students year 8 and below being assigned 'primary or intermediate' and students Year 9 and above being assigned 'secondary'. 'Island', with the options being North or South, would have been ~~taken~~ created based on the 'Region' variable, by identifying which of NZ's 2 main Islands the students were from based on their region.

b) i) From Figure 9 we can see that mobile phone ownership increased from 2003 (no: 42%, yes 58%) to 2009 (no: 24%; yes: 75.3%) and then remained at a relatively constant proportion (roughly 20:80, no:yes) since then. The proportion of students living on the South Island has increased from 2003 to 2019 increasing from 18.5% to 26.7% during that time. The students responding living on the north island have ~~decreased~~ decreased in proportion from 81.5% in 2003 to 73.3% in 2019. The proportion of secondary primary/intermediate students was initially at a ratio of 41.4 : 58.6 (secondary : Primary/intermediate) in 2003, ~~decreasing~~ increasing to 47.2 : 52.8 in 2007 and then ~~decreasing~~ increasing rapidly to ~~30.4 : 69.6~~ 69.6 : 30.4 in 2015. It then ~~increased~~ decreased again to 43.3 : 56.7 in 2019.

ii) Figure 9 uses one y-axis scale of year that the survey was taken, and 3 x-axis categories which contain values

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for each year, as to compare ~~them~~ visually against one another for the same year, and compare the change in one category/variable over multiple years in the same graph. ~~The graph a~~

- ~~The~~ figure 9 also uses percentage ratios on a linear scale a set distance apart from one another, as to allow a visual comparison/contrasting of one percentage ratio against another (e.g. the 84.2 : 58% ~~yes~~ no/yes ~~ra~~ dots in 2003 are much closer together than the 19.6 : 80.4% no/yes dots in 2019 for mobile phone ownership)

iii) - The proportion of secondary school students who own mobile phones is likely much greater than primary/intermediate school students. As the proportion of secondary to primary/intermediate students fluctuates a great amount over the years the sample was taken from the relative proportion would likely fluctuate due to this factor, as well as the actual change in proportion.

- Additionally, the survey was conducted online, which may bias the results away from very young students (year 1-3 for example) who may not know how to complete it. This may cause the sample to not be representative of all NZ school students, as older students may have had a higher rate of completion.

c) $n = 190 + 152 + 184 + 136 + 98 = 760$

$P(S,A,O) = \frac{190+152+184}{760} = 0.6973$

~~Sample size~~ $n = 760$, $MOE = \frac{1}{\sqrt{n}} = \pm 0.036327$

95% CI: $0.6973 \pm 0.036272 = 0.661 - 0.734$

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~~We can say~~ We can be reasonably sure that based on a ~~people~~ sample size of 760, the proportion of NZ year 3-13 students who either sometimes, often, or always check their phone for messages or notifications when they ~~wake~~ wake up is between 0.661 and 0.734. Therefore the claim can be made that most (>50%) NZ students sometimes, often, or always check their phone after waking up as the lower limit of the confidence interval does not overlap ~~the~~ 50%.



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Question 8

a)i) From the sample distribution in Figure 10 it can be seen that the ~~actual~~ size of Android apps are all less than 100 MB, with a mean of ~10mb. The android distribution is right skewed. The distribution of the size of Apple apps is ~~all~~ spread over a much greater range, from ~1 MB to almost 800 MB (~780MB). The mean of the ~~and~~ Apple apps size is ~200 MB. There is ~~a mean~~ difference of 192.72 MB in means between Apple and Android app download size in the sample. From the bootstrapped distribution it can be seen tha 95% of the bootstrapped distributions had a mean difference of between 137.64 and 126.69. 100% of the bootstrapped samples had a ~~mean~~ mean difference greater than 100 MB. Therefore it can be concluded that the mean size of newly released Apple apps was greater than the mean size of newly released Android apps in the sample.

ii) Problem: To investigate ~~the~~ the average size of largest phone app at their school and investigate if there is a relationship between the ~~avg.~~ size of the largest app and the type of phone (Android / Apple).

Plan: Gather a representative random sample of students *¹ possessing both Android / Apple phones from All year levels of the school. Record the phone type (android / Apple) and size of the largest app for each student.

Data: The ~~mean~~ median size of the largest app can be calculated by summing all app size (in MB) and dividing by number of data points. The student could also then ~~plot~~ construct two ~~graphs with the~~ distribution, of largest app size (in MB) and phone type.

*¹ Potentially randomly sampling from the school roll.



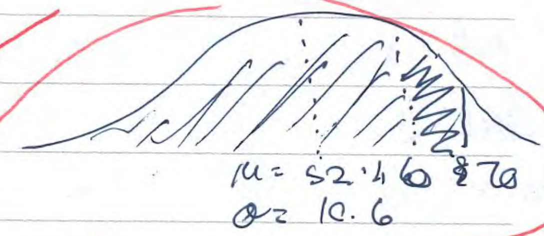


Analysis: The difference in means for the largest app size can be calculated and recorded. A bootstrap confidence interval could then be constructed by resampling with replacement 1000 times from the initial ~~population~~ sample and recording a mean difference each time. ~~If 95% of~~

Conclusion: If 95% of the confidence interval ~~data~~ is greater than 0 then a claim can be made that the one type of phone has a greater mean size of largest app than the other. If the confidence interval overlaps 0 no claim can be made.

b) i) $p(60 < x < 70) = 0.1882$

$p(x < 70) = 0.9513$.




$P(60 < x \mid x < 70) = \frac{0.1882}{0.9513}$ ~~$\frac{0.1978}{0.198}$~~ $0.1978$

ii) The probability of the battery being dead (0%) ~~is~~ a hard cutoff as a battery cannot be less than 0% charged. This may slight ~~hence~~ change the $p(60 < x \mid x < 70)$ as the normal dist assumes all p are ~~pos~~

iii) ~~Cont. on next page for space.~~

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iii) Computer / outlet , <30 , >30m

| | Computer | Outlet | P |
|---|---|---|---|
| <30 | 0.138 | ~~0.100~~ 0.092 | 0.23 |
| ≥30 | 0.252 | 0.618 | 0.77 |
| P | 0.39 | 0.61 | 1 |

$P(<30|C) = 0.354$

$P(<30|O) = 0.151$

$RR = \frac{0.354}{0.151} = 2.34$

Participants in this study used a computer to charge their mobile phones 39 % of the time, and an outlet 61 % of of the time. Participants who used a computer were ~~2.34~~ 2.34 times as likely to ~~charge~~ unplug their phone within 30 minutes of ~~them~~ reaching 100 % charge than participants who ~~had~~ used an outlet. 84.9 % of outlet users took more than 30 minutes to charge their phone, ~~while 64.6~~ after reaching 100 % while only 64.6 % of computer charger users took more than 30 minuts charged their phones. If a phone took more than 30 minuts to be unplugged it was twice as likely that it was ~~a~~ charged from a outlet (67.2 % chance) than a computer (32.7 % chance)

$P(>30|O) = 0.849$

$P(>30|C) = \frac{0.252}{0.39} = 0.646$

$P(C|>30) = \frac{0.252}{0.77} = 0.327$

$P(O|>30) = 0.672$

$RR = \frac{0.672}{0.327} = 2.06$





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**Annotated Outstanding Scholarship Exemplar Template**

| *Subject* | Statistics | | *Standard* | 93201 | *Total score* | 34 |
|---|---|---|---|---|---|---|
| ***Q*** | ***Grade score*** | ***Annotation*** | | | | |
| 1 | 6 | For all of question 1 where required specific numerical evidence is present. The two Outstanding marks were missing in this candidate's response as in Qu 1 (a) (ii) there was only one additional piece of information supplied that would be required before generalising. The second Outstanding mark is missing as there was not enough of a link made to the modelling aspect of the Samsung scatter graph in Figure 4, such as non linear model would be better or spotting that without the one pone at 7.5 inches the gradient of the trend line would be impacted. | | | | |
| 2 | 6 | As in Qu 1 there is high quality reference to key features of the data sets with specific numerical evidence. The candidate has reference a comparison in mobile phone subscriptions between New Zealand and Fiji through an absolute approach, where a relative approach was required. The candidate should have used a process to assume a linear step increase in the missing years taking the 2007 and 2010 figures as the start and end values for an estimation for 2009 Samoan mobile phone numbers. | | | | |
| 3 | 7 | There is excellent knowledge overall of experimental design principles throughout question 3, using correct vocabulary commensurate with Level 8 on the NZ Curriculum. In Qu 3 (d) reference as made toa bootstrapping process where the process should be a re-randomisation and so this was not sufficient for the Outstanding mark. In question 3 (e) the candidate superbly spotted and explained about the observational nature of the study and how a causal claim therefore cannot be made. | | | | |
| 4 | 8 | The high quality descriptions and analysis using numerical evidence continue in this question. A minor error has been ignored in adding the total of the number of students in the survey for question 4 (c) – 760 instead of 766. It may be useful to note that this type and level of error is at about the maximum that would be allowed at Scholarship level for continuation and consistency. The Performance Standard does require "precision and clarity of ideas". The follow up calculation for the margin of error, associated confidence interval and interpretation of the claim made is dependent on this total being correct. The 2$^{nd}$ mark in Qu 4 (b) (iii), at Outstanding, could have made a clearer link to the impact of ownership on mobile phones for Yr 3 to 13 students, but it is implied and a follow on from the first point made. | | | | |
| 5 | 7 | This question has been answered to a high level and it was only important vocabulary/conceptual ideas in 3 (a) (i) and 3 (a) (ii) slightly awry that prevented a full 9 marks being awarded. The probability calculations in question 3 (b) are logically and clearly developed and the relative comparison for the final question done to a very good standard, including a high quality explanation about what the 2.34 times as likely meant, in context. In Qu 3 (a) (i) "mean difference" has been stated instead of the correct difference in means. In 3 (a) (ii) the level of analysis required for the design of the PPDAC study should have made more reference to an exploratory data analysis and initial examination of summary statistics. | | | | |