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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
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Scholarship 2022
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ANSWER BOOKLET

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Write your answers in this booklet.

Make sure that you have Formulae Booklet S–STATF.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Check that this booklet has pages 2–24 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area ( ). This area may be cut off when the booklet is marked.

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Question	Score
ONE	
TWO	
THREE	
FOUR	
TOTAL	

ASSESSOR'S USE ONLY

Question One:

(a) From the beginning of 1991 to the end of 2019, the amount of nitrogen fertiliser sold in New Zealand has increased from around 60 000 tonnes of nitrogen to around 450 000 tonnes of nitrogen, reflecting a long-term 650% increase in the amount of nitrogen fertiléser sold. With more nitrogen fertiliser sold, the fertility of the soil has increased in many regions of New Zealand, and with increased plant quality more grass can be grown to support larger agricultural herd numbers on a smaller farm area. This likely explains why ~~there has been a decrease~~ the long term trend in the total farm area in New Zealand demonstrates a decrease in farm area from around 15650 000 hectares in 2002 to around 13550 000 hectares in 2019 (this is a long term decrease by around 13%), as farmers no longer need to maintain large farm areas with the use of nitrogen fertiliser to increase productivity. Hence, the number of tonnes of nitrogen sold and the total farm area in New Zealand show an inversely proportional relationship.

The long term trend in the number of sheep in New Zealand has shown a decrease from 70000 000 sheep around the beginning of 1981 to around 28 000 000 sheep ~~2019~~. at the end of 2019, ~~In contrast, the~~ reflecting a long-term decrease by about 60%. The number of beef cattle have also declined long term from around 4 400000 at the beginning of 2000 to 4000 000 beef cattle at the end of 2019, reflecting a long term decrease of 9%; therefore, sheep numbers have declined at a faster rate (nearly 6 times faster) than beef cattle numbers over the same period from 2000 to 2019 inclusive. In contrast, dairy cattle numbers have increased from around 4 200 000 at the start of 2000 to ~~around~~

around 6050 000 at the end of 2019, reflecting ~~an~~ long term increase of around 30%. ~~With~~ The rapid increase in dairy cow numbers in New Zealand ~~[illegible]~~ along with the fall in sheep and beef cattle numbers indicates that farmers may increasingly be switching to dairy farming. As dairy farming has ~~the~~ a higher environmental impact than beef and sheep, the negatively impacts of New Zealand's environment from agriculture may be increasing.

(1b) ~~There~~ Colour is used to illustrate the ~~perce~~ proportion of human modified land cover, with purple reflecting a low proportion ($\leq 25\%$), blue indicating a proportion between 26-50%, green indicating a proportion between 51-75%, and yellow indicating a proportion of human modified land cover in the upstream attachment ~~in excess~~ equal to or greater than 76%. The same colour scale is used to indicate the measured total nitrogen concentrations by percent over ANZG DGV. ~~However~~, for example, just as yellow indicates a high proportion of human modified land cover ($\geq 76\%$), it also indicates areas where the measured total nitrogen concentrations are much higher (200% +) than the ANZG DGV. ~~This allows~~ As colour is used spatially, it is clear that ~~the~~ many of the same ~~regions~~ sites (like those in the Central North Island, around the Waikato) that ~~[illegible]~~ are shown as yellow, with higher human modified land cover in the upstream attachment also ~~[illegible]~~ have nitrogen concentrations that are much higher than ~~[illegible]~~ the ANZG DGV regulations (200%+).

00206

In contrast, sites such as those around Canterbury with low human modified land cover in the upstream catchment (denoted in purple to indicate a proportion less than 25%) also have low nitrogen concentrations, measured in purple to reflect that there are at or below ANZG DGV regulations. Hence, the use of corresponding colour and a spatial component communicate the finding.

(1c)(i) ~~From 1988 to~~ The value of fertiliser imports has increased, long-term, from around 10 million dollars ~~at the 1988~~ ~~to around at~~ in the first quarter of 1988 to approximately 250 million dollars during the 1st quarter of 2021. This reflects an approximately linear trend component. ~~and an overall increase of around~~ ~~2400%~~ The seasonal fluctuations are not particularly consistent, and the magnitude of the seasonal fluctuations increased significantly after 2008; ~~for example,~~ the seasonal peak occurs around the middle of the year (quarter two or three), ~~the ex~~ and the seasonal trough occurs around quarter four. An example is 2011, where in quarter two a very high value of imported fertiliser was recorded (300 million dollars), followed by a ~~very~~ sudden decrease in the value of imported fertiliser to around 75 million dollars in quarter four. ~~The sea~~

(1cii) The Holt-Winters additive model assumes that the size of the seasonal fluctuations remains fairly consistent across the time series. The increasing seasonal fluctuations in the value of imported fertiliser over time (for example the seasonal fluctuations in 2021 were much larger than those in 1995) may mean that a multiplicative Holt-Winters model, which can adjust for increasing seasonal fluctuations, would be more appropriate. ~~As a result~~, the additive model predicts fairly small seasonal fluctuations for 2022, due to this assumption, which ~~is not~~ is an issue because this is unlikely to be an accurate forecast due to the large seasonal fluctuations observed in the past five years. Additionally, there seems to be a particularly large seasonal peak in the last quarter of 2021, reflecting an imported fertiliser rate of 400 million dollars; as this has occurred very recently, it is not known whether ~~these large~~ seasonal fluctuations of ~~these~~ this magnitude can be expected for the future, or whether this was a one-off circumstance, leading to uncertainty in the application of the Holt-Winters additive model. Finally, ~~nitrogen~~ fertiliser may have an uncertain future due to its negative environmental ~~regulations~~ impact, potentially resulting in regulations that limit the ~~value of~~ amount of fertiliser used and hence the value of imported fertiliser. This may mean that the model's forecast could overestimate the value of imported fertiliser for the last quarter of 2022. Finally, the last quarter of 2022 is a whole year after the known data, and according to the Holt-Winters additive model the confidence interval is thus wide, meaning forecasts from the additive model are unlikely to be accurate.

10

15

80 MAX

Q2

(2a)(i) This is a triple comparison study. Each cow received all three treatments because there will be some variation in the amount of methane produced between cows, depending on their diet (feed) or their digestion. As each cow receives all three treatments, the scientists can ~~ensure that only~~ measure the difference in the amount of methane produced at different seaweed additive levels, to confirm that the quantity of seaweed is producing the observed result in each cow, not natural variation in methane production.

(2aii) The researchers could have recorded the amount of methane emission using the individual sensors when the cows received 0% seaweed. ~~This rather could~~ three or more values (over three days, or three weeks for example) could be taken for each cow at 0% emission and an average of these values could be calculated. Then, for each cow, ~~the median~~ multiple values (as above) could be taken when seaweed ~~a~~ was increased to a low additive (0.5%), and an average taken. Finally, an average could be taken for high additive (1% seaweed), for each cow. To compare the impact of the addition of a low amount of seaweed (0.5%) ~~x~~, they could calculate the difference between the ~~median~~ ~~amount of average~~ methane emissions ~~at~~ with 0% and 0.5% seaweed for each cow, then ~~construct a bootstrap distrib~~ average these results for all cows to find a mean difference. A bootstrap distribution could be constructed for the mean difference in the ~~mean~~ methane emissions at 0% seaweed and 0.5% seaweed through repeated resampling with replacement. ~~The difference is the~~ If zero is not contained within the 95% confidence interval (ie both the upper and lower limits are ~~positive~~ negative) ~~in~~, then it can be concluded that there is a significant decrease in methane emissions from the addition of 0.5% seaweed, as 95% of the time the methane emissions after the addition of seaweed are smaller than the control (0% seaweed). The same process could be used to identify a significant decrease for the high additive diet, this time comparing the high additive to the control.

(2b) The identified problem or experimental question is "does the use of AgriSea decrease the nitrogen in cow's urine?" The necessary experimental units would be a collection of ~~farms~~ cows from New Zealand, who would be selected through cluster sampling (for convenience and cost effectiveness for farmer and surveyors). The clusters are individual farms (beef or dairy) who would then be randomly selected to participate. ~~~~ This would be an experiment with two independent groups. ~~~~ ~~farms would be randomly allocated to one of the two groups, and all of the cows on all of the using a block variable, half of the co~~ On each farm, the cattle would ~~first be allocat~~ be randomly allocated into these two groups, so that half of the cattle on each farm receive the treatment (Agrisea fertiliser on their paddock) and the other half ~~receive the~~ ~~~~ are the control (~~regular~~ regular fertiliser on their paddock). The response variable would be the percentage of nitrogen measured

~~and a difference in means would be calculated.~~

in the cow urine. By having multiple farms participate and having half of the cows on each farm receive the treatment and other half be the control, ~~the~~ factors ~~independe~~ unique to individual farms, such as supplementary feed, do not impact the results. A mean percentage of nitrogen in urine could then be calculated across all of the cows in ~~the~~ the treatment and control groups respectively. A rerandomisation test could be then conducted where the results are randomly reassigned (by chance) to the two groups, and a difference in means would be calculated once again. This test would be repeated 1000 times. If the observed difference in means of ~~occurs~~ larger occurs fewer than 100 out of these 1000 trials (the tail proportion is smaller than 10%), then the likelihood of the decrease in urine nitrogen ~~%~~ occurring by chance alone could be considered small. This would indicate a statistically significant difference. If this sample difference is 18%, as Bradley advertises, then his claim is accurate and the connection to the AgriSea is confirmed.

(2c) The median sentiment score, overall, for headlines containing the word "climate" was 0.55, ~~indicating~~ indicating that over 50% of the articles surveyed were at least slightly positive. The median ~~score~~ sentiment score for headlines also ~~articles~~ containing the word "change" was also around 0.55, while ~~for~~ headlines ~~articles~~ that didn't contain the word "change" had a slightly lower sentiment score of around ~~0~~.525. This difference is very small and does not necessarily indicate that the presense of "change" in the headline has an impact on the median sentiment score, particularly as more articles (58%) contained "change" in the headline than those who didn't (42%). The sentiment scores overall range from 0.09 to 0.8, ~~and this range is not significantly different for~~ with an interquartile range of around 0.12; this interquartile range doesn't change dramatically dependending on whether the headline also contains "change", ~~IQR for "yes"~~ with both subgroups having an IQR of about 0.12. This indicates that the ~~subgroups has~~ presense of "change" on the headline does not significantly impact the spread of the sentiment scores. The sentiment scores are slightly ~~A~~ negatively skewed for ~~the~~ all headlines containing "climate" and for those containing ~~the~~ "change" and those without "change" indicating that most articles are ~~A~~ fairly neutral in emotion, or very slightly positive, regardless of the presence of "change." The majority of articles containing climate in the headline are published in September (12%) or April (11%), around the change in season, when ~~A~~ many are inclined to comment on the difference in climate or weather compared to previous years.

Qu 2
80
MAY

00206

(Q3)

(3ai) The sample size of 1097 people aged 18 or over is ~~fairly~~ large, significantly reducing the extent of sampling variability such that ~~as~~ the survey is more likely to accurately reflect the New Zealand adult population, in the census. The accuracy could have been supported using stratified sampling, by dividing the population into groups according to characteristics such as age group and randomly selecting members from each of the strata to participate according to the percentage of each age group (or other demographic) ~~A~~ from the most recent census (ie if there are more people aged 20-30 than 90-100, a greater number of 20-30 year olds are selected to participate in the study). This ensures that the demographic make up of the study represents the NZ adult population at the most recent census, such that it is more likely that the recorded opinions accurately reflect the diversity of opinions on climate change across different age groups. ~~A~~ If this couldn't be achieved, ~~weighted~~ data ~~sample~~ could be weighted to reflect the proportions of each demographic in the most recent census, for the same purpose, to give a more accurate reflection of New Zealand's overall opinions, not just the opinions of one age group / demographic.

(3aii) Margin of error for each year: $1/\sqrt{1097} = 0.03019$
As the surveys are independent, I have used my margin of error as 1.5 x average MoE.
$= 1.5 \times 0.03019 = 0.045285009$.

1S

From 2012 - 2014.

$(0.54 - 0.52) \pm 1.5\left[\frac{1}{\sqrt{1097}}\right] = -0.0252885 - 0.0652885$

From 2014 - 2018.

$(0.64 - 0.54) \pm 1.5\left[\frac{1}{\sqrt{1097}}\right] = 0.054711 - 0.1452885$

From ~~2018~~ 2018 - 2019

$(0.69 - 0.64) \pm 1.5\left[\frac{1}{\sqrt{1097}}\right] = 4.71 \times 10^{-3} - 0.0952885$

From 2019 - 2021

$(0.72 - 0.69) \pm 1.5\left[\frac{1}{\sqrt{1097}}\right] = -0.01529 - 0.07529$

This claim is not entirely accurate. From 2019 to 2021, the confidence interval constructed using the rule of thumb margin of error ~~suggests that~~ has a lower limit for the difference as -0.0153 (4dp), suggesting a decrease in the proportion aggreeing climate change is a problem now may have decreased from 2019 to 2021 by up to 1.53%. Likewise, the CI lower limit for the difference from 2012 to 2014 is -0.0253 (4dp), which also shows the potential for a decrease of up to 2.5%. ~~The remaining~~ From 2014 to 2018 and from 2018 to 2019 the lower limit is positive suggesting that an increase will have occurred (with 95% confidence,) supporting the claim. However due to sampling variability the claim may not be true for difference between 2019 and 2021, or 2012 and 2014.

(3bi)

	U	F	N	D	
1-6	0.065534				0.217
7-8					0.326
9-13					0.457
					1

The total proportion of New Zealanders ~~aged~~ Y1 to 6 who consider climate change a future or urgent problem is 0.492 which is $[0.492 \times 0.217 = 0.106764]$ 0.1068 of the overall proportion.

For Y7-8 → $[(0.526 + 0.211) \times 0.326] = 0.240262]$

For Y9-13 → $[(0.414 + 0.256) \times 0.457 = 0.30619$

Let C = consider climate change a problem.

Consider Y7-8 and Y9-13 "older," O.

$$P(O \mid C) = \frac{P(C \cap O)}{P(C)}$$

$$\cancel{= \frac{[0.240262 + 0.30619]}{0.326 + 0.457}}$$

$$\cancel{= 0.6978952746}$$

$$= \frac{[0.240262 + 0.30619]}{[0.1068 + 0.240262 + 0.30619]} = 0.836556$$

$$P(O' \mid C) = \frac{[0.106764]}{[0.106764 + 0.240262 + 0.30619]} = 0.1634$$

~~New Zealand "older students" as defined as Y7 - Y13, are~~

~~A~~ Of those who consider climate change an urgent or future problem, 84% are "older" (defined as Y7-Y13) which is over 5X the proportion who are "younger" (~~aged~~ Y1-Y6).

If older is defined as Y9-Y13 and younger as Y1-Y8. this particular calculation supports the statement, however it is difficult to define "older" ~~under value~~ which makes it difficult to accurately confirm this statement.

(3bii) The sample may not be representative of students in New Zealand because CensusAtSchool was conducted online; not all students in all schools have access to devices to take online surveys. For example, the ~~[illegible]~~ survey may not reflect the proportions of age groups in lower decile schools, where students are less likely to have access to a device to participate in an online survey. Furthermore, younger students in general (Y1-6) are less likely to have access to their own device to take an online survey ~~[illegible]~~ than an older student (Y9-13, for example), because fewer children are trusted with a device, whereas high school students often require a device to take to school, increasing the proportion who are likely to respond to the survey. This may explain why the proportion of Y9-Y13 respondents (0.457) is over twice the proportion of Y1-Y6 respondents (0.217).

(Q4)

~~(4a) using the calculated proportion of school aged children who attended (from photos), a bootstrap distribution could be constructed by repeatedly resampling~~

(4a) A ~~mean~~ proportion of school aged students in each photo would be calculated to give a range of proportions from a range of different ~~photos~~ photos + ~~proportions~~, forming a sample. ~~Repeated resampling with replacement could then be used to construct a 95% confidence interval~~

From this overall sample a mean or median proportion could be calculated. Repeated ~~resampling~~ resampling with replacement, ~~could form a~~ then calculating the mean or median proportion of school aged ~~stu~~ students in each sample could be used to construct a bootstrap distribution with a 95% confidence interval, by excluding the extreme 2.5% of proportions ~~at~~ at either sides (high extreme and low extreme). This would provide an upper and lower limit ~~at the~~ for a proportion of school-aged students, within a 95% confidence interval.

One limitation of using photos as source data is that ~~the person may appear in multiple photos from the same event, which~~ often photos are taken of those who are the most "active" - yelling the loudest, or carrying the most photographically interesting signs. If school-aged students, ~~are~~ or often more "active" (or vice versa) the true proportional of school aged students may be misrepresented in the photos, skewing the source data.

Another limitation is that statisticans will be estimating ages from photos. Some 18 year olds, for example, may be mistaken for someone older (& thus for not being a school aged child), particularly if they are dressed in a more mature way. Likewise, someone 19 who appears younger may be mistaken in an estimate from a photo as school aged. Age estimates may be particularly difficult if faces in photos are blurred or not frontal, meaning the accuracy of the proportion of school aged children may ~~ss~~ decrease.

(4b)i It is assumed that the crowd density is evenly across the entire occupied area (which may not be accurate, as the outer edges may be more dispersed than the centre of the crowd).
Furthermore, it is assumed that only people are contained within the area, with no physical objects like benches that ~~the~~ the viewer cannot see that cause the area ~~occupied~~ with objects to have fewer people.
Finally, it is assumed that all people take up the same amount of space (occupy the same area). This may not be accurate as a small child will occupy less space than an adult. ~~Thus it is~~
~~As~~

(4bii) The bootstrap distribution suggests that the mean number of people within each square will, 95% of the time, range between 20.4 and 23.2.
~~Using using the lower limit of 20.4:~~

134.164

~~20.4~~ ~~18000~~ Total area $\boxed{?}$ 134.164

18000 / 25 = 720 ~~squar~~ 5x5 squares in area.

using the lower limit of 20.4 :

20.4 x 720 = 14688 people.

upper limit:

23.2 x 720 = 16704

~~As the~~ The number of people is expected to range between 14688 and 16704 people. As the upper limit of 16704 is smaller than 17000, the claim that over 17000 people attended cannot be supported using this confidence interval.

(4c) The poisson distribution assumes that there is no theoretical upper limit in the number of weather-related natural disaster per annum, which is true. ~~However the~~ · A poisson distribution with $\lambda = 4.2$ predicts ~~an proportion of~~ the probability of one weather-related natural disaster to be 0.0629, which would indicate a frequency of 0.0629 x 41 = 2.5789. This ~~pro~~ theoretical ~~probability~~ frequency is relatively close to the ~~actual~~ observed frequency of 3, especially considering it would be rounded up to 3 as it is not possible to have part of an extreme weather event. The poisson distribution would predict a probability of 4 weather-related natural disaster per annum to be 0.1944, to give a frequency of 7.97, rounded to 8 extreme weather event.

this is ~~relatively close~~ an overestimate of the ~~observed~~ frequency of 6. Likewise, as the $p(3) = 0.185$ ~~accord~~ (poisson), giving a frequency of 4.585, rounded to 8. the poisson distribution suggests that the distrubution, when rounded, would be bimodal, with modes of 3 ~~3~~ 4. Overall, the poisson distribution is not a perfect fit, but given the fairly unpredictable nature of weather events this does not prohibit its usage. ~~the poss~~ According to figure 14, the number of weather related extreme natural disasters has increased over time, with 10 in 2015 and 9 in 2016 compared to 4 in 1988, which indicates that the ~~probability~~ mean number per annum of ~~#~~ weather related natural disasters may have changed over time (increased) and may continue changing. As poisson ~~assumes~~ ~~the~~ assumes the mean number per interval length will remain constant, it may not be appropriate moving into the future, as $\lambda = 4.2$ may not be accurate.

MAX

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