No part of the candidate's evidence in this exemplar material may be presented in an external assessment for the purpose of gaining an NZQA qualification or award.

SUPERVISOR'S USE ONLY

S

93201A

932011

Draw a cross through the box (☒) if you have NOT written in this booklet

+

OUTSTANDING SCHOLARSHIP EXEMPLAR

NZQA | Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Scholarship 2023
Statistics

Time allowed: Three hours
Total score: 32

ANSWER BOOKLET

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Write your answers in this booklet.

Make sure that you have Formulae Booklet S–STATF.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Check that this booklet has pages 2–24 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (AREA DO NOT WRITE). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

© New Zealand Qualifications Authority, 2023. All rights reserved.
No part of this publication may be reproduced by any means without the prior permission of the New Zealand Qualifications Authority.

QUESTION ONE

(a) The Sankey diagrams in figure 1 show that for residential, commercial, industrial, and agricultural sectors all of the electronics and lighting end use energy was powered 100% by electricity for 2021.

for both the residential sector and the commercial sectors, the largest proportion of energy used was electricity, ~~both of~~ and in both of these sectors electricity makes up over 50% of the total energy usage for 2021. Alternatively, the industrial sector shows a relatively equal proportion of Natural Gas and Electricity usage, of around 25% each, making them equally the highest used energy sources for this sector. The Agricultural sectors highest proportion of energy use was using diesel fuel, with around 50% of the total energy used in this sector diesel for 2021.

The end use group of Heating/Cooling was the category using the most fuel in the residential, Commercial, and industrial categories; with all three using over 50% of their fuel for Cooling/heating. Alternatively the Agricultural sector used the most energy for the Mobile Motive Power end category, with around 50% of energy, most of which was diesel, being used for Mobile

Motive Power in 2021.

Natural Gas was used in all four sectors, ~~most significantly~~ and made up the highest relative proportion in the industrial sector with around 25%. In the commercial sector it was used around 15%, while in the Residential sector it was used 10% and it made up ~~the lowest proportion~~ of overall energy use in the Agricultural sector, with only around 5% of total energy use.

(b) i) MoE for comparison within one group

$$MoE \approx \frac{1}{\sqrt{3039}} \times 2 = 0.0363$$
$$= 3.63 \text{ p.p}$$

difference between percentages
Heat pump → 60%
Electric → 39%
difference of 21 p.p

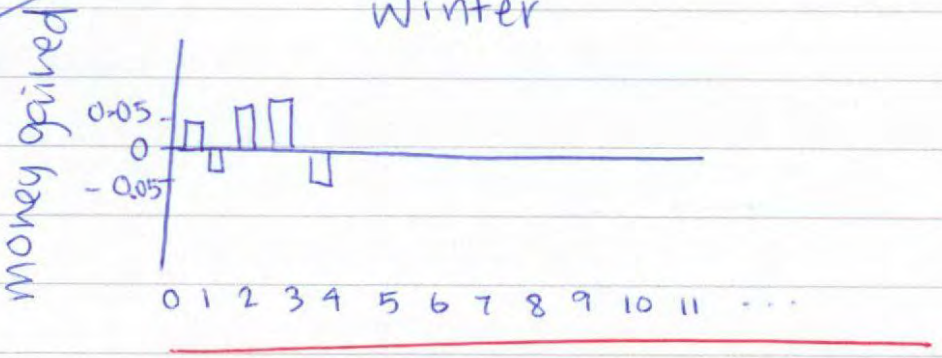
Using a rule of thumb for margin of error to construct a confidence interval, we would expect with 95% confidence for the percentage of New Zealand households who use a heat pump to keep their home warm and dry to be between 17.37 and 24.63

percentage points ~~higher~~ than those who use an electric plug in heater.

ii) Figure 4 shows that both before and after the instillation of a heat pump, the mean hourly cost of electricity was highest during winter. ✓1
Figure 4 also shows that the ~~energy~~ mean hourly ~~usage~~ cost both before and after the heat pump was installed ~~tends~~ to peak ~~at~~ near the end of the day, between the 18th and 20th hour, for all four seasons. ✓1

Figures 3 and 4 show that after the heat pump was installed, the total ~~energy~~ electricity ~~usage~~ cost and mean hourly ~~usage~~ cost tended to be higher in summer than before the heat pump was installed. For example, in January ~~before~~ 2021 the heat pump was installed the total cost of electrity was around $123, while in January 2022 after the heat pump was installed ~~the~~ the toatal electricity cost was around $178, around $55 more.

iii) Figure 4 makes it difficult to compare the electricity usage before and after the heat pump was installed because we cannot see the overall usage (i.e. cost whether they have lost or gained money overall)
To visualise this the differences in cost would need to be graphed. This could be done using a bar chart which shows for each hour of the day in the four respective seasons whether money has been spent or less money has been spent according the mean usage*, like so:



(data not accurate)

this would then display better whether the heat pump is more cost effective and is resulting in less spending each respective hour of the seasons.

*after the heat pump was installed

QUESTION TWO

(a) i) The deviation of 0.57% may have been calculated ~~using~~ by taking the total ~~predicted~~ actual Solar consumption from each month and dividing it by the predicted solar consumption from that same month to calculate the proportion of the predicted solar production that was actually ~~used~~ consumed.
For example using the month of June
900 kWh (actual consumption) ÷ 1600 kWh (predicted production) = 0.5625
= 0.56%
if all of the months were calculated they could be used to find an average deviation, which would likely be around 0.57%

ii) The deviation of 0.57% could have been found by taking the total production of solar (the actual consumption + the exported solar) and the prediction for solar production each month and calculating the percentage of the ~~overall~~ total production each month by which they over or under predicted. ~~For example, calculating the total of all light orange bars then the total of all~~ dark orange ~~and then meet to calculate another total, then finding the~~

* See ~~back~~ page 17

seen

Is H

ii) using a binomial model, as there are two possible outcomes (higher predicted production and not higher predicted production) and these events cannot occur simultaneously (it must be one or the other).

more than half of the time = $p > 50\%$

if we use 0.5 as the probability and 12 as the number of trials, and 7 as x (to represent the seven months where predicted energy production was higher.

$p = 0.806$

as this is a very high probability, there is not sufficient evidence to conclude that more than half of the time the predicted solar energy production will be higher than the actual production.

(Assuming that whether or not the predicted solar energy production is higher, is independent of whether or not it was higher the previous month.)

(b) i) The randomisation test output in Figure 5 shows that ~~the~~ in the sample data students under an incandescent light had an average of 0.85 points ~~differnc~~ higher in ~~the~~ their mean alertness score than the students under an LED light.

The randomisation distribution shows a tail proportion of 0.178, meaning that 178 ~~of the rerandomised simulated data~~ sets showed a difference of ~~this size of~~ greater. As 0.178, or 17.8% is ~~beta~~ higher than the 10% threshold of ~~statistical~~ significance, this ~~da~~ test does not support the claim that students under incandescent light had higher alertness scores than those under LED lights.

ii) in order to take into accound the variation in students ability to ~~take~~ process written information, a similar study could be designed but with the key difference of giving each student a task under LED and incandescent lights. Two ~~sample~~ proofreading tasks of similar difficulty could be prepared and each student could undergo each test seperately, one under LED light and one under incandescent

e.g. Task A and Task B

light. Which test (e.g. A or B) is taken under which light ~~(e.g. LED)~~ should be randomly allocated so that some students complete A under LED light and B under incandescent light and vice versa. This should decrease the possible impact of the differences between test difficulty, as students may find one easier than the other. The results could then be plotted and run through a randomisation test to observe the difference between alertness under each respective light, and whether ~~it~~ the results are statically significant.

QUESTION THREE

(a) i) Treepedia has used the median rather than the mean as the "overall" measure of "greenness" because the mean is more susceptible to the effect of outliers. If there are streets with very high greenness scores in one small part of the city, while most of the city has ~~very~~ much lower greenness scores, the mean may be dragged upwards and therefore be unrepresentative of the actual data.

ii) Vancouver appear to have a higher standard deviation for the Green view Index, as seen in Figure 6, as the data is spread further across the distribution. Much more of the data for Vancouver's green view index lies between 0-15% in comparison to Sydney, which forms a more symmetrical bell shaped curve. The Standard deviation will decrease when more of the data is closer to the mean which, assuming that the mean and medians are similar, it appears to be closer to the mean in Sydney, making Sydney's Standard deviation lower

iii) Sydney → using a normal distribution model

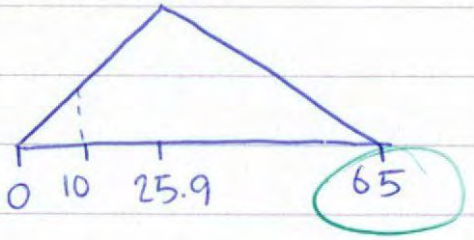
$\mu = 25.9$ (assuming that the median and mean are similar, which appears to be true looking at the model)

$\sigma = 8.33$ (dividing 50 by 6, as most of the data appears to fall within 50% of total or 25% either side of the mean)

lower limit of -10000
upper limit of 10 (as it is 10%)

$p = 0.028$

Vancouver → triangular model



$a = 0$

$b = 65$ (as this is where the data appears to end)

$c = 25.9$ (assuming that the mode and median are similar, as this appears to be the peak of the data).

$p(x < 10)$

$f(10) = \frac{2(10-0)}{(65-0)(25.9-0)} = 0.012$

$x = 10$

$\frac{1}{2} \times 0.012 \times 10 = 0.059$

$p(x < 10) = 0.059$

(b) Using an observational study for this investigation would be more cost effective and take less time than an ~~observational~~ experimental study. If already existing houses were observed, it could be considered a better representation of the city a as whole, as it would represent the city as it actually is, not as it could be. This would also mean that it would be a better indicator of the country as a whole, as it is more representative of actual life ~~that it represents~~

An experimental study would allow for a more controlled experiment. i.e. if two similar houses were found and a matched pairs study design was used and then repeated, we could be more confident about our findings, as there should be less lurking or confounding variables to consider. For example, certain areas may have more or less dark roofed houses, meaning that ~~there~~ we would be lacking data in certain areas ~~an~~. Other confounding variables such as indoor heating/cooling systems could also have an impact, which is something that could be controlled for in an experimental study design.

QUESTION FOUR

(a) images / photographic samples were used to determine how much of each image was a shade of green. The issue with using images as a sampling strategy is that they only capture a moment in time, and can also be impacted by the angle at which the photo is taken (i.e. may not capture some hidden "greenness"). This photo may have been taken before more trees were planted and therefore may not be representative of the greenness that the suburb currently has. This is especially prevalent because the photos were taken from google images, which means they may be outdated (we are lacking this information). By only using the number of "green pixels" in an image, it also means that some of the pixels recorded may be from other green elements such as roofs or signs.

random sampling was used in this study to select a sample of 100 suburbs, as well as to select a random sample of 1000 pixels from each image. Random allocation is good for decreasing the possible effects of sampling errors and therefore providing an unbiased sample. As the images were obtained from google images, it also decreases the possible

impact from the hawthorne effect → i.e. if people in this suburb knew a picture was being taken specifically for this purpose they may have attempted to make it look greener to make themselves seem better.

(b) The Bootstrap confidence interval construction output in figure 8 shows that in the sample data suburbs in Auckland had a mean greenness score 43.31 higher than the mean greenness score for Christchurch on average. The sample data appears to include less suburbs than the in Christchurch than in Auckland, which may have contributed to this result.

The bootstrap distribution confidence interval shows that we would expect the mean greenness score for Auckland suburbs to be between 51.71 lower and 135.14 higher than the mean greenness score for Christchurch suburbs. As this confidence interval contains 0 we cannot make the claim that Auckland suburbs

tend to have a higher mean greenness score than Christchurch suburbs.

(c) The bivariate data graph in Figure 10 shows a weak to moderate positive linear relationship between greenness score and distance to the city centre (km), where as the distance from the city centre increases the greenness score tends to also increase.

The scatter appears to increase as the distance from the city center increases, with much more scatter in the top right of the graph. most of the data is grouped in the bottom left corner of the graph. there is a significant outlier at around 5km from the city centre which has a greenness score of around 630, which is much higher than we would expect based on the rest of the data, and this suburb is in Christchurch. When looking at the two cities of Christchurch and Auckland seperately, Christchurch appears to have a much steeper positive trend than Auckland, indicating that ~~there~~ cities in Christchurch tend to be closer to the city center and tend to have higher green scores relative to their distance from the city centre. This makes sense as Christchurch is a smaller city.

There are some points from Auckland suburbs which are unusually low green-

ness scores for their distance from the city, such as the point at 60km from the city centre which is only 230 in greenness score.

(d) Since Christchurch suburbs tend to be closer to the city center, as it is a smaller city, the sampling method could be changed so that from each city suburbs are picked which are equidistant from the city centre. This means perhaps picking a few suburbs which are around 5km from the city and a few which are around 7km and soon.
This would aslo mean that there would be a more equal split between the cities (i.e. The same sampe size from each) which would make the comparison more feair.

*

Q2 (a)i) for example using June supposing the light orange bar is 1600 and the actual production is 1580, the difference is 20kWh. 20 ÷ ~~1600~~ 1580 (the actual production) is 0.013, or 1.3%. if this was repeated ~~to create a beta~~ relative percentage of deviation for each month it could then be averaged to find an overall deviation between the actual and predicted values.

Outstanding Scholarship

Subject: Statistics

Standard: 93201

Total score: 30

Q	Score	Marker commentary
1	8o	This question provides evidence for 8o because the candidate has gained at least 2 o grades and at least 8 marks overall. The candidate has sufficient comparisons across different diagrams of fuel and fuel use including estimated proportions. They have identified and applied the correct margin of error calculation with an indication of some uncertainty. The candidate has identified differences and similarities in trends before and after heatpump installation across figures 3 and 4 with appropriate numerical evidence. They have not identified a valid difficulty when interpreting figure 4, however, they have identified an appropriate way to visualise the data for easier comparison.
2	7o	This question provides evidence for 7o because the candidate has gained at least 1 o grade and at least 7 marks overall. The candidate has provided a description and example of an appropriate deviation calculation. They have not adequately explained their use of 'average'. The candidate has correctly identified the use of binomial distribution and has given two key features in context. They have not correctly calculated an appropriate probability. The candidate has not correctly interpreted the difference between the two means but has correctly used the tail proportion to reject the claim. The candidate has discussed a paired comparison experiment with random allocation of treatment order including how this may affect results.
3	7o	This question provides evidence for 7o because the candidate has gained at least 1 o grade and at least 7 marks overall. The candidate has demonstrated a sound understanding of measures of central tendency and variation in context. They have identified appropriate distributions and parameters, with one exception, which has resulted in the candidate not gaining an additional mark. They have correctly calculated probabilities from the distributions using their given parameters. The candidate has not demonstrated an understanding of the need for random allocation in an experiment, or that an observational study cannot make a causal claim. They have discussed confounding variables but have not discussed the limitations of causal claims made from an experiment or random selection and the extension of results from an observational study.

4	8o	This question provides evidence for 8o because the candidate has gained at least 2 o grades and at least 8 marks overall. The candidate has identified two of the sampling scenarios and has discussed potential issues with one of these scenarios. They have compared sample means, interpreted a bootstrap confidence interval and have identified that there is not enough evidence to come to a conclusion using this confidence interval. The candidate has described an overall relationship from a bivariate data display and has made a valid comparison between the two cities represented in the display. They have discussed how to change the sampling process to make comparisons fairer between the two cities.