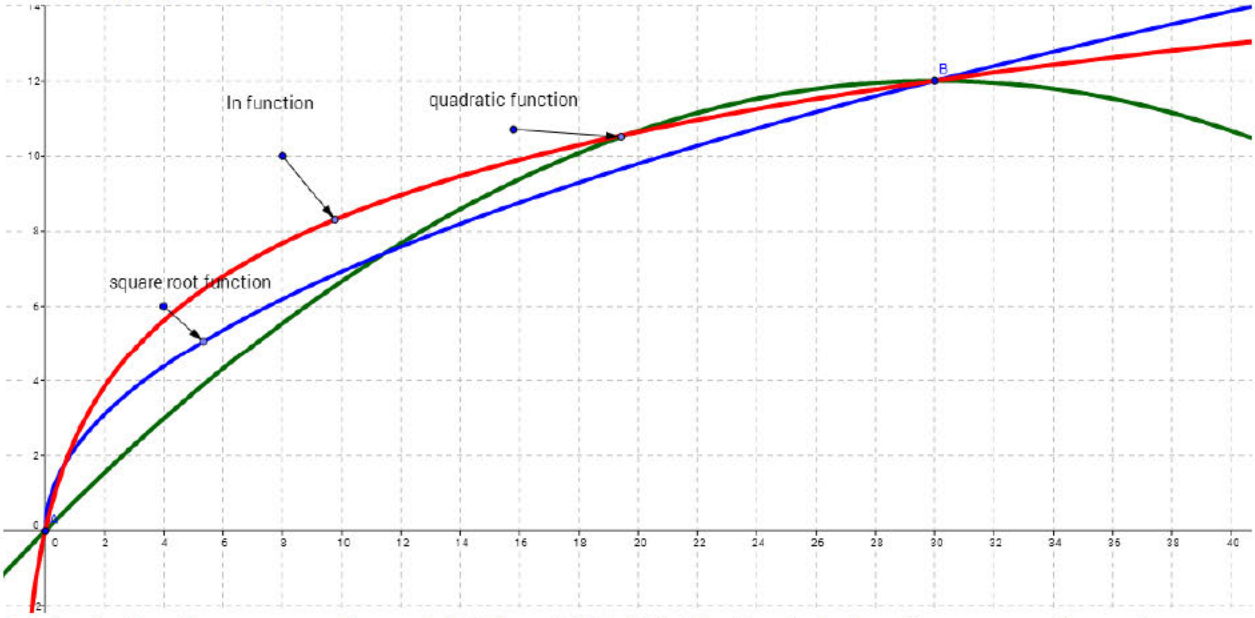
Student 1: Low Excellence
NZQA Intended for teacher use only

Part 1.

I am going to take the bottom left hand point of the bridge as the origin (0,0). The x-axis represents the width of the bridge in metres, and the y-axis the height of the bridge in metres. As the whole bridge is 60m wide and 12m high this means the bridge also passes through (30,12), because the top is in the middle between (0,0) and (60,0)

I am going to fit a ln function, a quadratic function and a square root function for the first half of the bridge.

This is a graph of my functions.



1. The ln function passes through (0,0) and (30,12). The basic ln function passes through (1,0) so I need to translate the basic function 1 to the right and then make sure it goes through (30,12)

$y = k\ln(x+1)$ $12 = k\ln 31$ $k = 3.494$ $y = 3.494\ln(x+1)$

As we only want the function from 0 to 30 the domain must be limited to $0 \le x \le 30$.
If you look at the graph this is a poor model for the bridge in this section. It goes through the right points, but goes up too steeply and is not flat at (30,12)

2. The square root function passes through (0,0) and (30,12).

It will have equation $y = k\sqrt{x}$ because it goes through the origin. I can find k by making it go through (30,12)

$12 = k\sqrt{30}$ $k = \frac{12}{\sqrt{30}} = 2.19(2dp)$ so the function is $y = 2.19\sqrt{x}$

This function has a vertical vertex at (0,0), it passes through the right points but it keeps going up and is not really flat enough at (30,12). It is also far too steep near the origin.

3. The parabola passes through (0,0) and has a vertex at (30,12)

It will have equation $y = -k(x-30)^2 + 12$. I can find k by making it go through (0,0)

$0 = -k(-30)^2 + 12 \qquad k = \frac{-12}{-900} = \frac{4}{300} \qquad y = -\frac{4}{300}(x-30)^2 + 12$

This looks pretty good on the graph.

As we only want all three of the functions from 0 to 30 the domains must all be limited to $0 \le x \le 30$.

Part 2.

To complete the bridge we need to extend the functions so they are symmetrical about the line x=30 and therefore go through (60,0)

The parabola function will do this anyway, because it has symmetry about x=30, so we can have the same function with a different domain

$y = -\frac{4}{300}(x-30)^2 + 12$ for $0 \le x \le 60$

The square root function is more difficult because we want the reflection in x=30 of just the part from 0 to 30.

To get this I can reflect the original function in x=0 to get it the right way round (replace x by –x) and then translate this 60 units to the right to get it in the right place (replace x by x-60).

So the function is a piecewise one.

$y = 2.19\sqrt{x}$ for $0 \le x \le 30$ and $y = 2.19\sqrt{-(x-60)} = 2.19\sqrt{(60-x)}$ for $30 \le x \le 60$

I need to do the same with the ln function, replace x by –x and then x by x-60

$y = 3.494\ln(-(x-60)+1) = 3.494\ln(61-x)$

So the function is piecewise.

$y = 3.494\ln(x+1) \qquad 0 \le x \le 30$ and $y = 3.494\ln(61-x) \qquad 30 \le x \le 60$

Part 3

My original graph suggest the square root function and ln function are not good models and so I am not going to generalise those.

The parabola model will go through (0,0) and have a vertex at $\left(\frac{w}{2}, 12\right)$.

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$y = -k(x - \frac{w}{2})^2 + 12$

$0 = -\frac{kw^2}{4} + 12$

$k = \frac{48}{w^2}$