National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 2

This exemplar supports assessment against:

Achievement Standard 91257

Apply graphical methods in solving problems

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to apply graphical methods, using extended abstract thinking, in solving problems. This involves one or more of: devising a strategy to investigate a situation, identifying relevant concepts in context, developing a chain of logical reasoning, or proof, forming a generalisation, and also using correct mathematical statements, or communicating mathematical insight. This student's evidence is a response to the TKI task 'Bridges'. The student has identified relevant concepts in context and formed a generalisation for the parabolic model (1). Correct mathematical statements have been used in the response. For a more secure Excellence, the student could complete the generalisation for the quadratic model and discuss its features/properties. The student could also consider the generalisation for the other models.

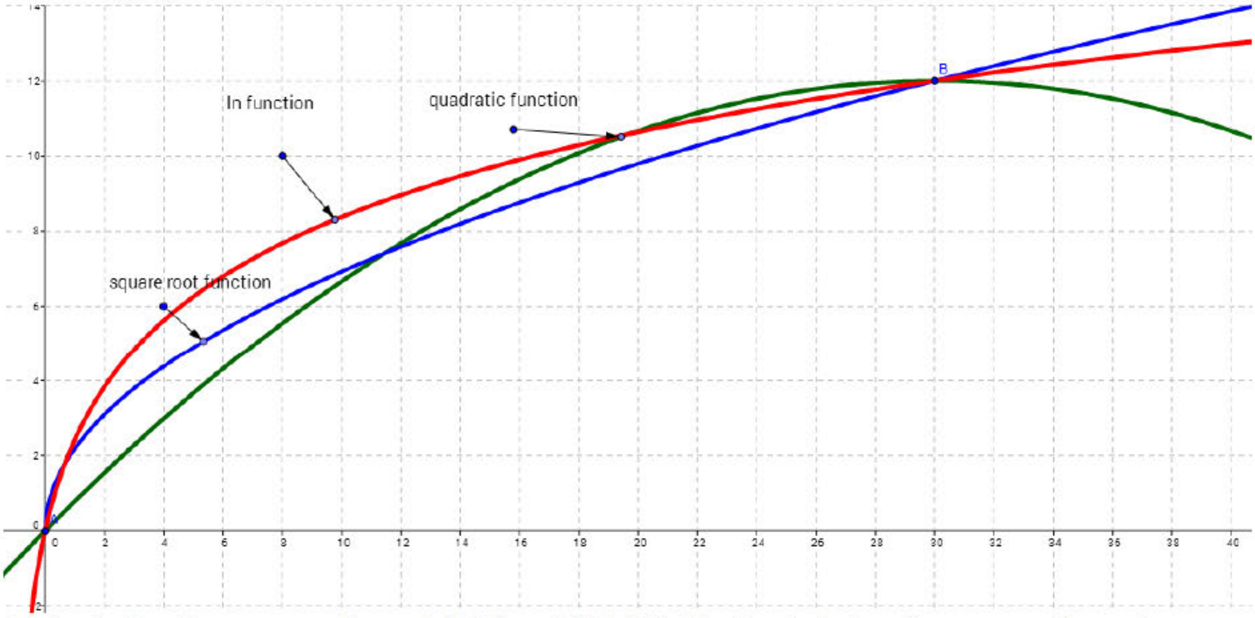
Student 1: Low Excellence
NZQA Intended for teacher use only

Part 1.

I am going to take the bottom left hand point of the bridge as the origin (0,0). The x-axis represents the width of the bridge in metres, and the y-axis the height of the bridge in metres. As the whole bridge is 60m wide and 12m high this means the bridge also passes through (30,12), because the top is in the middle between (0,0) and (60,0)

I am going to fit a ln function, a quadratic function and a square root function for the first half of the bridge.

This is a graph of my functions.



1. The ln function passes through (0,0) and (30,12). The basic ln function passes through (1,0) so I need to translate the basic function 1 to the right and then make sure it goes through (30,12)

$y = k\ln(x+1)$ $12 = k\ln 31$ $k = 3.494$ $y = 3.494\ln(x+1)$

As we only want the function from 0 to 30 the domain must be limited to $0 \le x \le 30$.
If you look at the graph this is a poor model for the bridge in this section. It goes through the right points, but goes up too steeply and is not flat at (30,12)

2. The square root function passes through (0,0) and (30,12).

It will have equation $y = k\sqrt{x}$ because it goes through the origin. I can find k by making it go through (30,12)

$12 = k\sqrt{30}$ $k = \frac{12}{\sqrt{30}} = 2.19(2dp)$ so the function is $y = 2.19\sqrt{x}$

This function has a vertical vertex at (0,0), it passes through the right points but it keeps going up and is not really flat enough at (30,12). It is also far too steep near the origin.

3. The parabola passes through (0,0) and has a vertex at (30,12)

It will have equation $y=-k(x-30)^2+12$. I can find k by making it go through (0,0)

$0=-k(-30)^2+12 \qquad k=\frac{-12}{-900}=\frac{4}{300} \qquad y=-\frac{4}{300}(x-30)^2+12$

This looks pretty good on the graph.

As we only want all three of the functions from 0 to 30 the domains must all be limited to $0 \le x \le 30$.

Part 2.

To complete the bridge we need to extend the functions so they are symmetrical about the line x=30 and therefore go through (60,0)

The parabola function will do this anyway, because it has symmetry about x=30, so we can have the same function with a different domain

$y=-\frac{4}{300}(x-30)^2+12$ for $0 \le x \le 60$

The square root function is more difficult because we want the reflection in x=30 of just the part from 0 to 30.

To get this I can reflect the original function in x=0 to get it the right way round (replace x by –x) and then translate this 60 units to the right to get it in the right place (replace x by x-60).

So the function is a piecewise one.

$y=2.19\sqrt{x}$ for $0 \le x \le 30$ and $y=2.19\sqrt{-(x-60)}=2.19\sqrt{(60-x)}$ for $30 \le x \le 60$

I need to do the same with the ln function, replace x by –x and then x by x-60

$y=3.494\ln(-(x-60)+1)=3.494\ln(61-x)$

So the function is piecewise.

$y=3.494\ln(x+1) \qquad 0 \le x \le 30$ and $y=3.494\ln(61-x) \qquad 30 \le x \le 60$

Part 3

My original graph suggest the square root function and ln function are not good models and so I am not going to generalise those.

The parabola model will go through (0,0) and have a vertex at $\left(\frac{w}{2},12\right)$.

1

$y=-k(x-\frac{w}{2})^2+12$

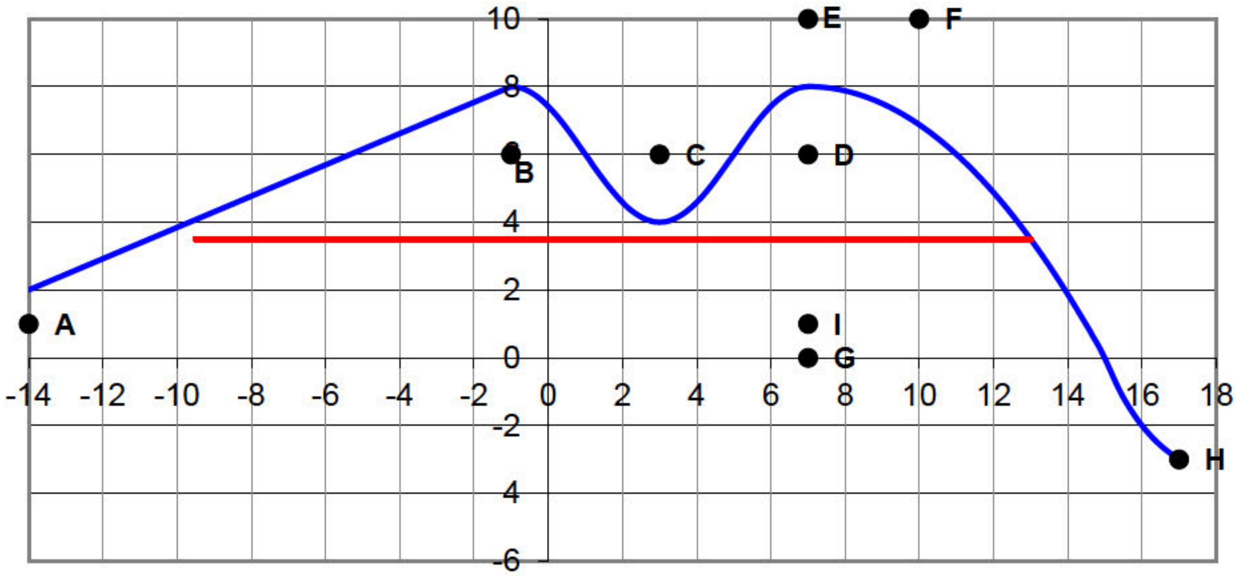
$0=-\frac{kw^2}{4}+12$

$k=\frac{48}{w^2}$

	Grade Boundary: High Merit
2.	For Merit, the student needs to apply graphical methods, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model, and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI task 'Motorcycle School'. The student has connected different concepts and representations to form the models for four sections of the course (1) and provide a possible path for the instructor (2). To reach Excellence, the student could provide a detailed domain for the path of the instructor by considering appropriate points of intersection. The student could also discuss the margin of safety on both sides of the path, the appropriateness of the model for the course, and provide further communication of the strategy used to determine the models.

Student 2: High Merit

NZQA Intended for teacher use only



1 square is 1 metre

Section 1, straight line from (-14,2) to (-1,8) is $y = \frac{6}{13}x + 8.46$ (1)

Section 2 will be a sin graph. Amplitude is 2 (up and down 2 about y = 6) and the period is 8.

$y = 2\sin\frac{2\pi}{8}(x-5) + 6$ (1)

Section 3 is a parabola. Vertex is at (7,8). It goes through (15,0). $y = \frac{-1}{8}(x-15)(x+1)$ (1)

Section 4 is exponential y = $A.2^{x-B} + C$. The curve is moved over 15. y = $-2^{x-15} + 1$ (1)

The instructor is the red line, y = 3.5 between x = -9.5 and x = 13 (2)

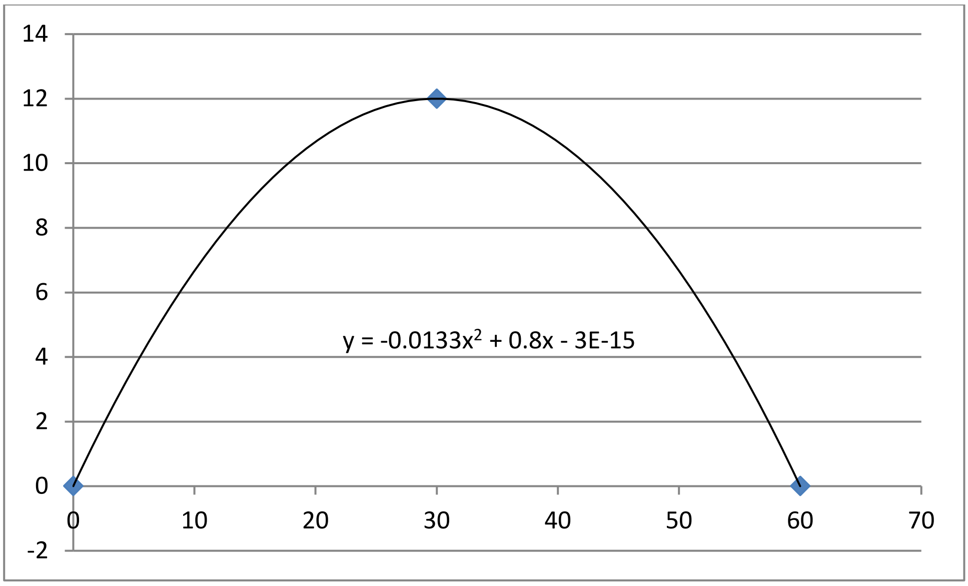
	Grade Boundary: Low Merit
3.	For Merit, the student needs to apply graphical methods, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model, and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student’s evidence is a response to the TKI task ‘Bridges’. The student has formed and used models in determining appropriate functions for the first half of the bridge (1) and the whole bridge (2). The findings have been related to the context. For a more secure Merit, the student could strengthen the communication by explaining their thinking more clearly using appropriate mathematical statements. The cubic model is not symmetric (3), and the student could also consider a piecewise model consisting of two cubic functions for the whole bridge.

Student 3: Low Merit

NZQA Intended for teacher use only

The function that models the bridge needs to go through the points (0,0) and (30,12). Because the top of the bridge is at the point (30,12) it will be symmetrical about this point and the function will also go through (60,0).

I put these three points into Excel and fitted a quadratic model to them.

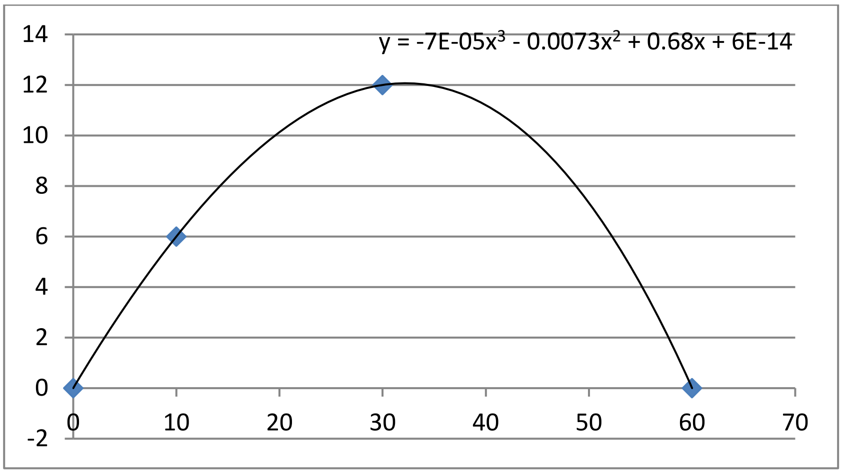


The equation of the function is $y = -0.0133x^2 + 0.8x$ as the constant is practically zero and I know it goes through the origin so it must be zero..

I only need the part of the function from x=0 to x= 30 so I will restrict the domain to these values.

This quadratic goes through these points, has a vertex at (30,12), is pointing down and has x=30 as an axis of symmetry.

I am also going to fit a cubic. I need at least four points to do this in Excel so I am going to estimate that the bridge also passes through (10,6). This looks a good idea from the graph above.



1

This has equation $y = -0.00007x^3 - 0.0073x^2 + 0.68x$ The constant at the end is practically zero again and it needs to go through the origin so it must be zero. We need to restrict the values of x to between 0 and 30 again as we only want the first half of the bridge.

Cubics don't really have many features apart from the points where it crosses the axes which are (0,0) and (60,0). They are S shaped and don't have reflective symmetry. I would guess, but can't be sure, that this cubic has rotational symmetry about the origin. Cubics always have rotational symmetry but the centre changes from curve to curve.

I can't fit any of the other types of functions you suggest on Excel so I am not going to try a third one.

To complete the bridge we need functions that go from x = 30 to x = 60. They need to be the reflections of the first functions in x = 30 so the bridge is symmetrical about the top point.

The quadratic is easy because it is already symmetrical about the top so we can use the (2)
same function $y = -0.0133x^2 + 0.8x$ with x from 0 to 60.

The cubic looks nearly symmetrical about x = 30 too so we can use the same function (3)

$y = -0.00007x^3 - 0.0073x^2 + 0.68x$ with x = 0 to x = 60.

	Grade Boundary: High Achieved
4.	For Achieved, the student needs to apply graphical methods in solving problems. This involves selecting and using methods, demonstrating knowledge of the properties of functions and graphs and communicating using appropriate representations. This student's evidence is a response to the TKI task 'Bridges'. The student has selected and used the graph of a logarithmic function, its features and equation (1), the properties of the logarithmic function by considering the asymptote and domain (2) and the graph, features, equation and properties of the quadratic model (3). The student has also demonstrated knowledge of the properties of functions and graphs and communicated using appropriate representations. To reach Merit, the student would need to develop equations to model the complete bridge. The student could also develop the discussion on the steepness of the logarithmic model at the beginning by referring to specific points.

Student 4: High Achieved
NZQA Intended for teacher use only

Part 1.

I am going to take the bottom left hand point of the bridge as the origin (0,0).
The graphs need to pass through (0,0), (30,12) and (60,0)

I am going to fit a ln function and quadratic function

1. The ln function passes through (0,0) and (30,12). The basic ln function passes through (1,0) so I need to translate the basic function 1 to the right and then make sure it goes through (30,12) (1)

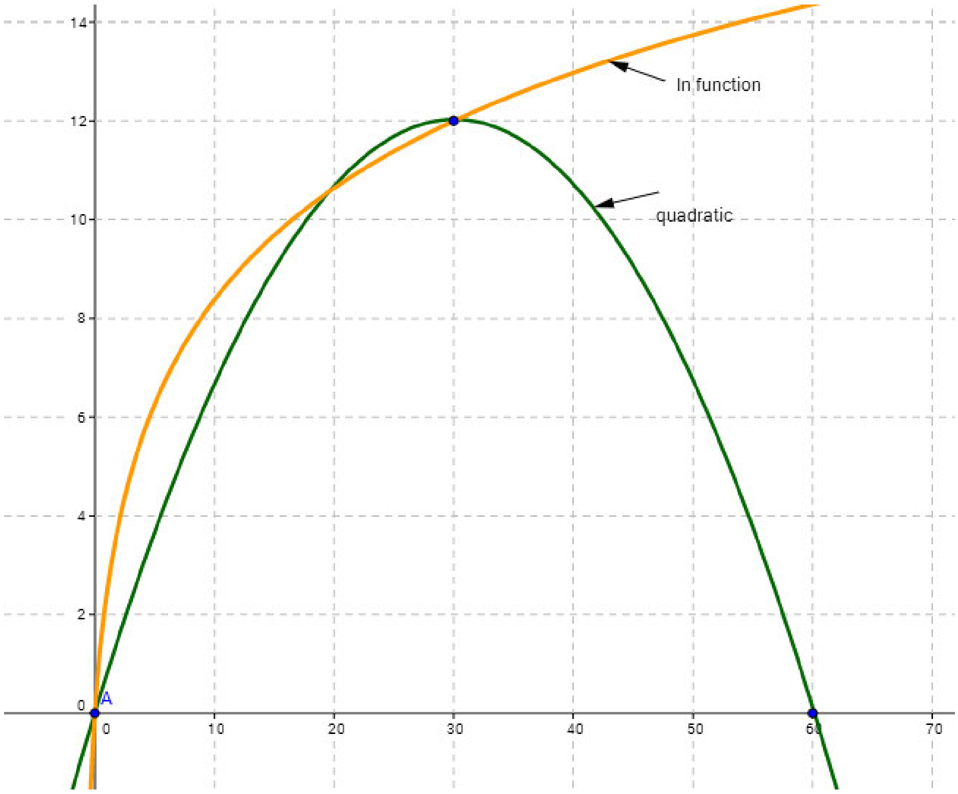
$y = k\ln(x+1)$ $12 = k\ln 31$ k = 3.494 $y = 3.494\ln(x+1)$
As we only want the function from 0 to 30 the domain must be limited to $0 \le x \le 30$.

The function goes through the correct points, but goes up too steeply at the beginning and goes on up after the top of the bridge. It has a vertical asymptote when x = -1. (2)

2. The quadratic passes through (0,0) has a vertex at (30,12) and passes through (60,0)
I have put these three points into my graphics calculator and fitted a quadratic model

$$y = -0.01333x^2 + 0.8x$$

The whole function has these features: (3)
It passes through (0,0), (30,12) and (60,0), it has a vertex at (30,12), it is a sad quadratic, and it has x=30 as an axis of symmetry.
This also needs a domain of $0 \le x \le 30$ for the first part of the bridge.
Here are my graphs, plotted only for the first part of the bridge.

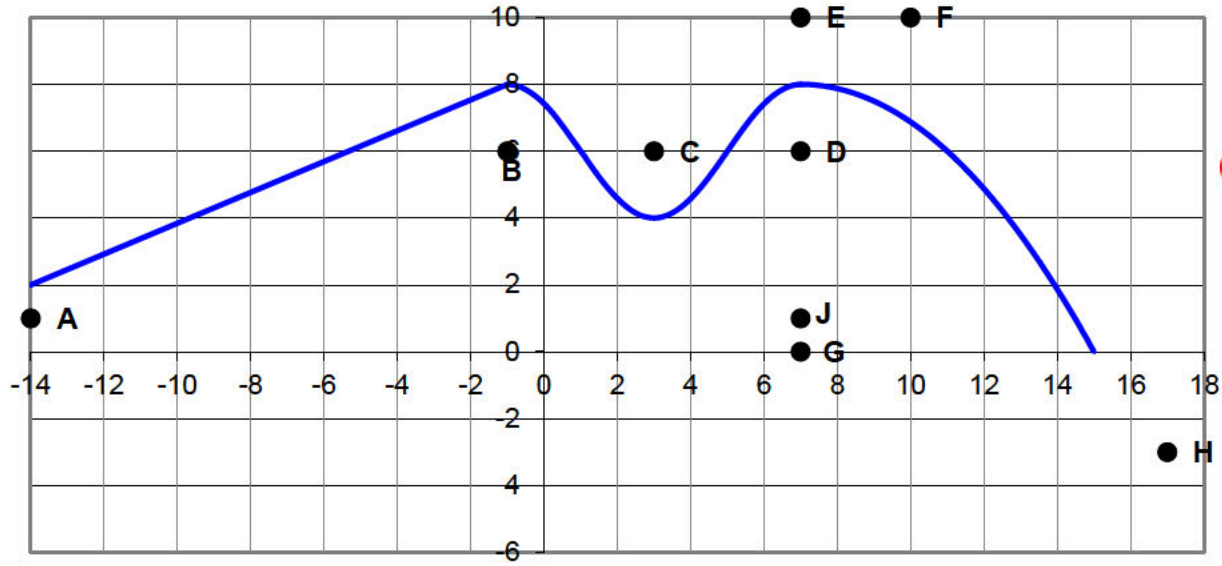


ln function
quadratic
A

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to apply graphical methods in solving problems. This involves selecting and using methods, demonstrating knowledge of the properties of functions and graphs and communicating using appropriate representations. This student's evidence is a response to the TKI task 'Motorcycle School'. The student has selected and used graphs for the first three sections of the course (1), features of the trigonometric function in order to determine two of the constants in the model (2), and a feature of the parabola (3). The student has also demonstrated knowledge of properties of functions and graphs, and communicated using appropriate representations. For a more secure Achieved, the student could determine the correct equations for Sections 2 and 3 and discuss the features and properties of these graphs.

Student 5: Low Achieved

NZQA Intended for teacher use only



1

Section 1 $y - y' = m(x - x')$

$8 - 2 = m(-1 - {}^{-}14)$

$6 = m \times 13$

$6/13 = m$

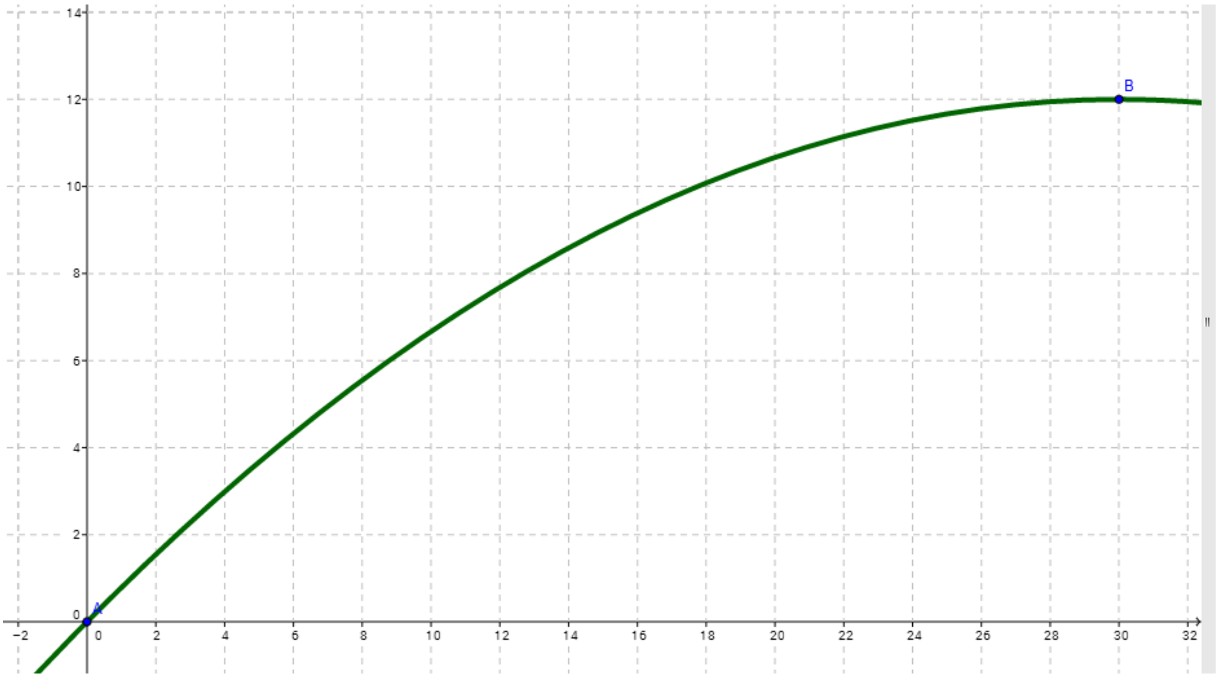
Section 2 $y = 2\sin(x - 5) + 6$ 2

Section 3 parabola with vertex (7,8) $y = (x - 7)^2 + 8$ 3

	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to apply graphical methods in solving problems. This involves selecting and using methods, demonstrating knowledge of the properties of functions and graphs and communicating using appropriate representations. This student's evidence is a response to the TKI task 'Bridges'. The student has selected and used the properties of the quadratic function by considering the domain, vertex and symmetry of the quadratic (1). To reach Achieved, the student would need to determine the equation of the quadratic, and consider using other graphs to model the half bridge.

Student 6: High Not Achieved

NZQA Intended for teacher use only



I drew a quadratic on geogebra which went through (0,0) and (30,12)
Here is a screenshot of my function.
We only need the bit of the graph from x = 0 to 30 for the first half of the bridge.
The quadratic works because it has a vertex of 12 when x =30, which is the top of the bridge.
If I continued it it would go through (60,0) because it is symmetrical about the vertex. 1