National Certificate of Educational Achievement
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Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 2

This exemplar supports assessment against:

Achievement Standard 91258

Apply sequences and series in solving problems

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to apply sequences and series, using extended abstract thinking, in solving problems. This involves one or more of: devising a strategy to investigate a situation, identifying relevant concepts in context, developing a chain of logical reasoning, or proof, or forming a generalisation, and also using correct mathematical statements, or communicating mathematical insight. This student's evidence is in response to the TKI task 'Property Development'. The student has devised a strategy to investigate and find the total weekly rent for the buildings with 15 floors in both areas (1) (2). The student has found a solution for the number of floors required in each building in order for the income difference to be less than $1000 (3). For a more secure Excellence, the student needs to improve the communication by clearly indicating what it is they are calculating and the decisions that they made when selecting and using formulae. The student should also use correct mathematical statements throughout the response.

Student 1: Low Excellence

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Business Area Highrise (BAH) Geometric

-window $120 per window 120x24 = $2880 r = 1.05
-inside $102 per window 102x8 = $816

Industrial Area (IAH) Artihmetic

-window $103 per window 103x28 = $2884 3x28 = 84 d = 84
-inside $65 per window 65x16 = $1040

BAH $\frac{2880(1-1.05^n)}{(1-1.05)}+816n$ n =15 = $74386.26 (2 decimal places) 1

IAH $\frac{n}{2}(2\times 2884+(n-1)\times 84)+(1040\times n)$ n = 15 = $67680 2

Found the sums of each building weekly rent if they both had 15 floors. They are quite close in number but BAH is geometric so its size will get bigger quicker than IAH which is arithmetic.

BAH n = 20 = 111549.95 (2 decimal places)
n = 22 = 128847 (6sf)
n = 23 = 138087.8 (1 decimal place)

IAH n = 30 = 154260
n = 25 = 123300
n = 26 = 129324

IAH = 26 = $129324
BAH = 22 = $128847 3

Because IAH has a much bigger floor level amount range and through guess and check I have discovered it can be a much higher number.
So I took the biggest floor amount (BAH) and got the sum and found could IAH not match it by less than 1000. So I dropped BAH by one floor and found I could match it.

I suggest IAH should have 26 floors and BAH should have 22 floors. It is the most money that can be made weekly with them less than 1000 apart.

	Grade Boundary: High Merit
2.	For Merit, the student needs to apply sequences and series, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model, and also relating findings to a context or communicating thinking using appropriate mathematical statements. This student's evidence is in response to the TKI task 'Property Development'. The student has formed and used a model to find the total weekly rent for an industrial area building (1) and a business area building (2). The student has used appropriate mathematical statements. To reach Excellence, the student could improve the communication. This could be done by explaining why floors 17 and 19 were chosen, and by confirming that the difference is less than $1000.

Student 2: High Merit

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Business area high rise

At least 15 floors high below 23 floors high
$24 \times 120 = \$2880$ = ground floor outside window
$8 \times 102 = \$816$ = inside office on any floor

$t_n = (2880 \times 1.05^{n-1}) + 816 =$ total floor cost for any floor

$s_n = \frac{2880(1-1.05^n)}{(1-1.05)} + 816 \times n =$ total weekly rent for business building. (2)

$s_{15} = \$74386.26$ $s_{17} = \$88292.26$

Developer should build business building 17 floors high.

Industrial area high rise

At least 15 floors high below 32 floors high

28x103 = 2884 = ground floor outside window
16x65 = 1040 = inside offices on any floor
$84 added each floor on outside windows

a = 3924 d = 84
$t_n = 3924 + (n-1)84 =$ total floor cost for any floor

$s_n = \frac{n}{2}(2 \times 3924 + (n-1) \times 84) =$ total cost of building per week (1)

$s_{15} = \$67680$ $s_{19} = \$88920$

Developer should build the industrial building 19 floors high

	Grade Boundary: Low Merit
3.	For Merit, the student needs to apply sequences and series, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model, and also relating findings to a context or communicating thinking using appropriate mathematical statements. This student's evidence is in response to the TKI task 'Property Development'. The student has formed and used models to investigate the total weekly rent for the business area (1) and industrial area buildings (2). For a more secure Merit, the student needs to use eight inside offices rather than six, and for the solution, the difference in rents needs to be less than $1000. The student should also use appropriate mathematical statements throughout the response.

Student 3: Low Merit

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1) Business area high rise building:
Floor 10:

$t_{10} = 120 \times 1.05^9 = 186.16 =$ outside office

186.12 x 24 = 4467.84 102 x 8 = 816 total $5283.84

Industrial area high rise building

Floor 10:

$t_{10} = 103 + 9 \times 3 = 130 \Rightarrow$ outside office

130 x 28 = 3640 65 x 16 = 1040 total $4680

2)
Building 1

$s_{15} = \frac{120(1-1.05^{15})}{-0.05} = 2589.43 \times 24 = \$62146.32$
$61246.32 + 15 \times 102 \times 6 = \$71326.32$

$s_{23} = \frac{120(1-1.05^{23})}{-0.05} = 4971.66 \times 24 = \$119319.84$
$119319.84 + 23 \times 102 \times 6 = \$133395.84$

The total rent for the building 1 is between $71326.32 and $133395.84 (1)

$s_{22} = \frac{120(1-1.05^{22})}{-0.05} = 4620.63 \times 24 = \$110895.12$
$110895.12 + 22 \times 102 \times 6 = \$124359.12$

Building 2

$s_{15} = 7.5(206 + 14 \times 3) = 1860 \times 28 = \$52080$
$52080 + 15 \times 65 \times 16 = 67680$
$s_{32} = 16(206 + 31 \times 3) = 4784 \times 28 = \$133952$
$133952 + 32 \times 65 \times 16 = \$167232$

The total weekly rent for building 2 is between $67680 and $167232 (2)

$167232 - 133395 = 33837$
$s_{25} = 12.5(206 + 24 \times 3) = 3475 \times 28 = \$97300$
$97300 + 25 \times 65 \times 16 = 123300$
$s_{27} = 13.5(206 + 26 \times 3) = 3834 \times 28 = \$107352$
$107352 + 27 \times 65 \times 16 = 135432$
$s_{26} = 13(206 + 25 \times 3) = 3653 \times 28 = \$102284$
$102284 + 26 \times 65 \times 16 = \$129324$

The developed should build 22 floors for building 1 and 25 floors for building 2.

	Grade Boundary: High Achieved
4.	For Achieved, the student needs to apply sequences and series in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. This student's evidence is in response to the TKI task 'Property Development'. The student has selected and used a partial sum of an arithmetic sequence in correctly finding the total weekly rent for a 15 floor building in the industrial area (1), and a partial sum of a geometric sequence in finding the total weekly rent for a 15 floor building in the business area (2). To reach Merit, the student needs to start to investigate different building sizes. The student would also need to more clearly communicate what they are calculating.

Student 4: High Achieved

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Weekly rent for industrial – 15 floors
Inside offices 16 x 65 = $1040 per floor = $15600 inside offices for entire building

Outside offices first floor = 28 x 103 = $2884

Entire building (outside offices)

$$s_n = \frac{n}{2}(2a + (n-1)d) \times 28$$
$$= 7.5(206 + 14 \times 3) \times 28$$
= $52080 a week outside offices

Entire building a week = $67680 (15 floors) (1)

Weekly rent for a city centre building 15 floors

Inside offices 8 x 102 = $816 per floor
816 x 15 = $12240 entire building (inside)
Outside offices first floor = 24 x 120 = 2880 per week

All outside offices in the building

$$s_n = \left(\frac{a(1-r^n)}{(1-r)}\right) \times 24 = \left(\frac{120(1-1.05^{15})}{(1-1.05)}\right) \times 24$$ =$62146.32

Entire building per week 15 floors = $74386 (2)

Two 15 floor buildings

Industrial = $67680 a week
City centre = $74386 per week

Add one floor to industrial
$t_n = a + (n-1)d = 103 + 15 \times 3 = 148$
67680+148 x 28 +15600 = $87424 per week for 16 floor building rent (indus)

Weekly floor rent for any floor in industrial

$(103 + (floor - 1) \times 3) \times 28 + 1040$
Weekly floor rent for any floor in city centre

$(120 \times 1.05^{floor-1}) \times 24 + 816$

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to apply sequences and series in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. This student's evidence is in response to the TKI task 'Property Development'. The student has selected and used the general term of a geometric sequence in correctly finding the total rent of the 15th and 23rd floor of the business area high rise building (1), and a partial sum of a geometric sequence in finding the total rent for a 15 floor building in the business area (2). For a more secure Achieved, the student, when finding the total floor cost for the industrial area, could use the correct common difference in calculations that involve finding the cost of the outside offices.

Student 5: Low Achieved

BA High rise

Ground floor = (102 x 8) + (120 x 24) = $3696 (120 x 24) = a

Floor 15 = $t_n = ar^{n-1} = 2880 \times 1.05^{14}$ = $5702.20

Total = $6518.2

1

Floor 23 = $t_n = ar^{n-1} = 2880 \times 1.05^{22}$ =$8424.75

Total = $9240.75

Total of 15 = $s_n = \frac{a(r-1^n)}{(r-1)}$ = $62146.26+(102 x 8) x 15 = $74386.26

2

IA High rise

Ground floor = (65 x 16) + (103 x 28) = $3924 (103 x 28) = a

Floor 15 = $t_n = a + (n-1)d = 2884(15-1)3$=$2926

Total = $3966

Floor 32 = $t_n = a + (n-1)d = 2884(32-1)3$=$2977

Total = $4017

Total of 15 = $s_n = \frac{n}{2}(2a + (n-1)d) = \frac{15}{2}(5768 + (14 \times 3))$ = $43575 + ((65x16)15) = $59175

Total of 20 = $s_n = \frac{n}{2}(2a + (n-1)d) = \frac{20}{2}(5768 + (19 \times 3)) + ((65 \times 16)15) =$ $73850

	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to apply sequences and series in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. This student's evidence is in response to the TKI task 'Property Development'. The student has selected and used a partial sum of an arithmetic sequence in correctly finding the total costs of renting one outside office for 15 floors and for 32 floors in the industrial area high rise building (1). To reach Achieved, the student needs to select and use one further method, for example by finding the sum of all the offices on the floor, or by correctly using the 5% extra for a business area office. The student should also give their answers in context, for example, by including the $ sign when finding cost.

Student 6: High Not Achieved

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Business area high rise

If this building will be 15 floors and each office on a higher floor is 5% similar or more 6$ so

$t_{15} = 120 + (15-1) \times 6 = 204$ outside + inside = 2430 + (102 x 15) = 3960

$$s_{15} = \frac{15}{2}(2 \times 120 + (15-1) \times 6) = 2430$$

If the building will be 23 floors high

$t_{23} = 120 + (23-1) \times 6 = 252$

$$s_{23} = \frac{23}{2}(2 \times 120 + (23-1) \times 6) = 4278$$

Outside + inside = 4278 + (102 x 15) = 5808

Industrial area high rise

If the building will be 15 floors high:

$t_{15} = 103 + (15-1) \times 3 = 145$

$$s_{15} = \frac{15}{2}(2 \times 103 + (15-1) \times 3) = 1860$$

Total: 1860+(65 x 15) = 2835

If the building will be 32 floors high

$t_{32} = 103 + (32-1) \times 3 = 196$

$$s_{32} = \frac{32}{2}(2 \times 103 + (32-1) \times 3) = 4784$$

Total: 4784 + (65 x 32) = 6864

1