National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 2

This exemplar supports assessment against:

Achievement Standard 91259

Apply trigonometric relationships in solving problems

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

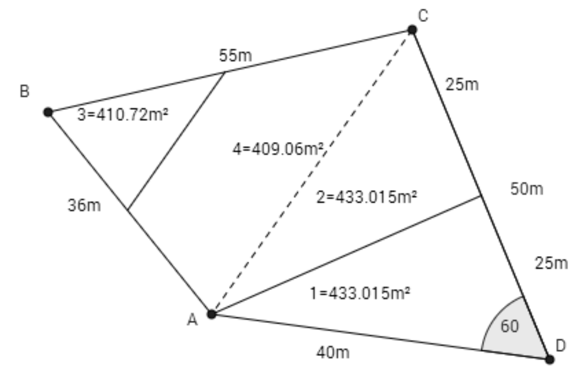
New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to apply trigonometric relationships, using extended abstract thinking, in solving problems. This involves one or more of: devising a strategy to investigate or solve a problem, identifying relevant concepts in context, developing a chain of logical reasoning, or proof, forming a generalisation, and also using correct mathematical statements, or communicating mathematical insight. This student's evidence is a response to the TKI task 'School Spare Land Subdivision'. The student has devised a strategy to investigate the situation of subdividing the land for the sale. The student has shown that the total area can be subdivided into four sections of at least 400 m^2 (1). The student has also shown how four sections can be created, not all of which are triangles that satisfy the requirement of the sale (2). Correct mathematical statements have been used throughout the response. For a more secure Excellence, the student could improve the communication, for example by clearly explaining how subsections 3 and 4 are created from ΔABC, and also by finding and stating clearly the dimensions of the four subsections.

Student 1: Low Excellence

NZQA Intended for teacher use only



Length of pipeline =

$$a^2 = 40^2 + 50^2 - 2 \times 40 \times 50 \times \cos 60$$
$$a^2 = 2100$$
$$a = 45.83m$$

area $\triangle$ACD $= \frac{1}{2} bc \sin A = \frac{1}{2} \times 40 \times 50 \times \sin 60 = 866.03m^2 (3sf)$

$$\angle ABC = \frac{36^2 + 55^2 - 45.83^2}{2 \times 36 \times 55} = 0.56$$
$$\cos^{-1} 0.56 = 55.9^o = \angle ABC$$

Area $\triangle$ABC $= \frac{1}{2} bc \sin A = \frac{1}{2} \times 36 \times 55 \times \sin 55.9 = 819.78m^2$

Total area is 819.78+866.03 = 1685.81m^2

$1685.81 \div 4 = 421.4$ so it can be divided into 4 sections of at least 400m^2. 1

$\triangle$ACD half the base of CD to get two triangles with half the area of ACD.

Subsection 1= $\frac{1}{2} \times 25 \times 40 \times \sin 60$ which is 433.015m^2. This means

Subsection 2 is also 433.015m^2 because 866.03-433.015=433.015

Subsection 3 = $\frac{1}{2} \times 31 \times 32 \times \sin 55.9$ which is 410.72m^2 which means

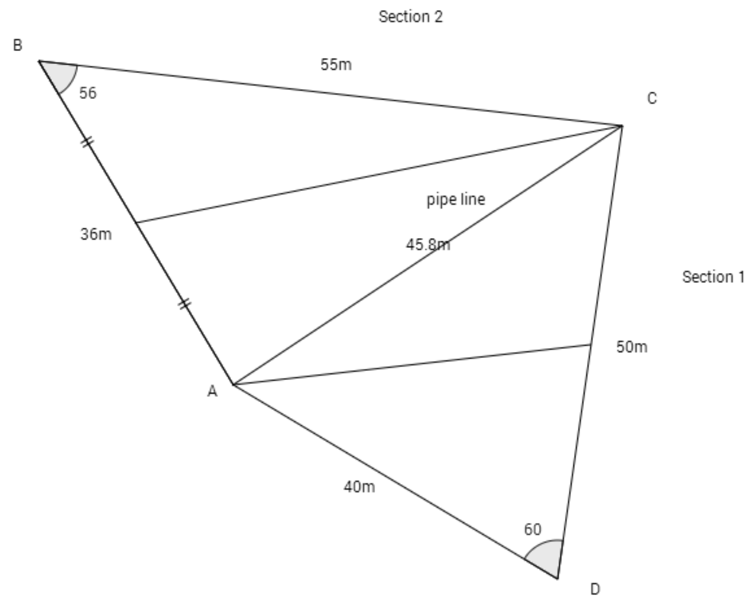
Subsection 4 is 819.78-410.72 = 409.06m^2.

There is 4 subsections with at least 400m^2 in each one and they are not all triangles. 2

	Grade Boundary: High Merit
2.	For Merit, the student needs to apply trigonometric relationships, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model, and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI task 'School Spare Land Subdivision'. The student has selected and carried out a logical sequence of steps to calculate the areas of the two triangles on either side of the pipeline (1), and to show that each triangle can be subdivided into two sections with an area of more than 400 m^2 (2). Appropriate mathematical statements have been used throughout the response. To reach Excellence, the student would need to provide a subdivision into four sections, not all of which are triangles.

Student 2: High Merit

NZQA Intended for teacher use only



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 50^2 + 40^2 - 2\times 50\times 40\times\cos 60$$
$$a^2 = 2100$$
$$a = 45.8m(3sf)$$

Area Section 1

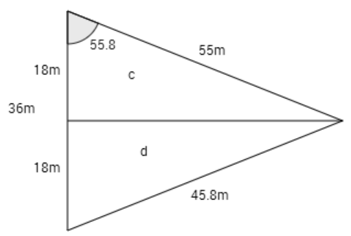
$$= \frac{1}{2}bc\sin A = \frac{1}{2}\times 40\times 50\times\sin 60 = 866m^2(3sf)$$ 1

$$\angle B = \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{55^2 + 36^2 - 45.8^2}{2\times 55\times 36}$$
$$A = 55.8(3sf)$$

Area Section 2

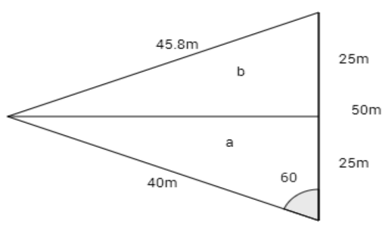
$$= \frac{1}{2}bc\sin A = \frac{1}{2}\times 36\times 55\times\sin 55.8 = 818.8m^2(1dp)$$ 1

Section 2



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 18^2 + 55^2 - 2\times 18\times 55\times\cos 55.8$$
$$a^2 = 7736$$
$$a = 47.3m$$
$$areac = \frac{1}{2}ab\sin C = \frac{1}{2}\times 18\times 55\times\sin 55.8 = 409.4m^2(1dp)$$ 2
$$aread = 818.8 - 409.4 = 409.4m^2$$

Section 1



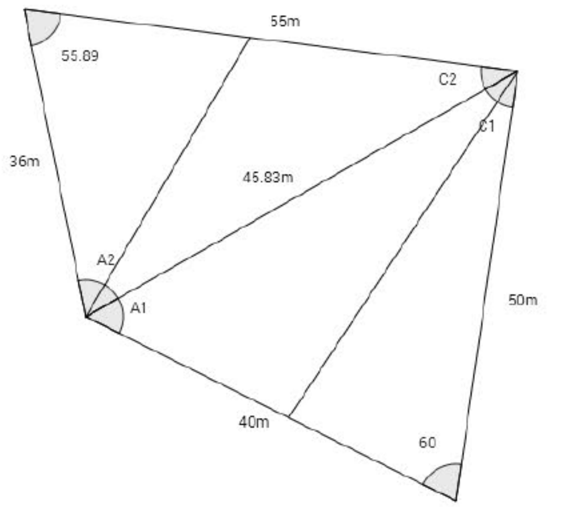
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 25^2 + 40^2 - 2\times 25\times 40\times\cos 60$$
$$a^2 = 1225$$
$$a = 35m$$
$$areaa = \frac{1}{2}ab\sin C = \frac{1}{2}\times 25\times 40\times\sin 60 = 433m^2$$
$$areab = 866 - 433 = 433m^2$$ 2

If you split section 1 between CD and join up with A and section 2 between AB and join up with C you can create 4 sections all over 400m^2. 2

	Grade Boundary: Low Merit
3.	For Merit, the student needs to apply trigonometric relationships, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model, and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI task 'School Spare Land Subdivision'. The student has selected and carried out a logical sequence of steps to connect the areas of the triangles to four sections of at least 400 m^2 (1). Appropriate mathematical statements have been used. For a more secure Merit, the student could start to investigate possible dimensions for the four triangular subdivisions on the diagram to meet the requirements that each of them is more than 400 m^2.

Student 3: Low Merit

NZQA Intended for teacher use only



$AC^2 = 50^2 + 40^2 - 2 \times 50 \times 40 \times \cos 60$

$AC^2 = 2100$

$\sqrt{ans}$

$AC = 45.83m$

Area of triangle ACD = $\frac{1}{2} \times 40 \times 50 \times \sin 60 = 866.03m^2$

Half area of triangle ACD = one section $\frac{1}{2} \times 866.03 = 433.01m^2$ 1

$\frac{\sin A1}{50} = \frac{\sin 60}{45.83}$ (x50) sin A1=0.94 ($\sin^{-1}$) A1=70.88°

C1 = 180 – 70.88 – 60 = 49.12° (angle sum in triangle is 180)

$\cos B = \frac{55^2 + 36^2 - 45.83^2}{2 \times 55 \times 36}$ cosB = 0.56 ($\cos^{-1}$) B=55.89

Area of triangle BCA = $\frac{1}{2} \times 55 \times 36 \times \sin 55.89 = 819.68m^2$

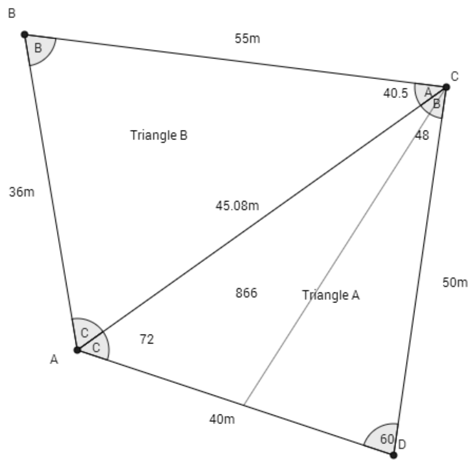
Half area of triangle BCA = one section $\frac{1}{2} \times 819.68 = 409.84m^2$ 1

Therefore all sections are at least 400m^2.
In triangle ACD the sections are 433.01m^2 each.
In triangle BCA the sections are 409.84m^{2}each.

	Grade Boundary: High Achieved
4.	For Achieved, the student needs to apply trigonometric relationships in solving problems. This involves selecting and using methods, demonstrating knowledge of trigonometric concepts and terms and communicating using appropriate representations. This student's evidence is a response to the TKI task 'School Spare Land Subdivision'. The student has selected and used the cosine rule to find the length of the pipeline (1), the sine rule to find an angle in a triangle (2), and the formula for the area of a triangle to find the areas of Triangle A and Triangle B (3). The student has communicated their working using appropriate representations. To reach Merit, the student could relate the two areas of the triangles to the requirement for sections of at least 400 m^2. The additional line on the diagram (4) shows the start of subdividing the land.

Student 4: High Achieved

NZQA Intended for teacher use only



4

$AC^2 = 40^2 + 50^2 - 2 \times 40 \times 50 \times \cos 60$

$AC^2 = 2100$ Length of pipeline = 45.8m 1

$AC = 45.8m$

area ACD = $\frac{1}{2} \times 40 \times 50 \times \sin 60 = 866m^2$ 3

$\angle CAD = \frac{\sin C}{50} = \frac{\sin 60}{45.8}$

$\sin C = 0.95$ 2

$C = 72^o$

$\Delta B = \cos A = \frac{45.8^2 + 55^2 - 36^2}{2 \times 45.8 \times 55}$

$A = 40.5^o$

$areaBCA = \frac{1}{2} \times 45.8 \times 55 \times \sin 40.5$ 3

$= 818m^2$

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to apply trigonometric relationships in solving problems. This involves selecting and using methods, demonstrating knowledge of trigonometric concepts and terms and communicating using appropriate representations. This student's evidence is a response to the TKI task 'School Spare Land Subdivision'. The student has selected and used the cosine rule to find the length of the pipeline (1), and the formula for the area of a triangle to find the area of section 1 (2). The student has communicated using appropriate representations. For a more secure Achieved, the student could make progress towards finding the area of the second triangle.

Student 5: Low Achieved

NZQA Intended for teacher use only

pipeline = a

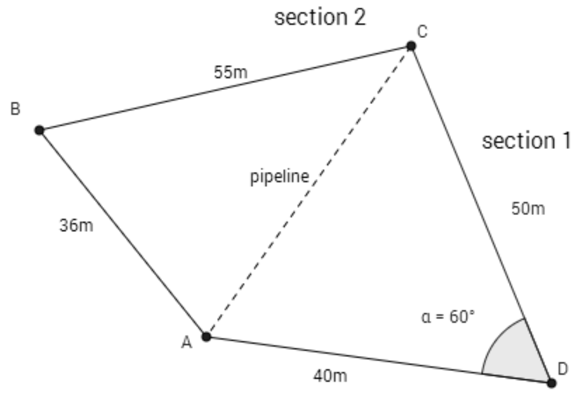
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 40^2 + 50^2 - 2\times 40\times 50\times \cos 60$$
$$a^2 = 2100$$
$$a = 45.8m$$

1

section 1

$$= \frac{1}{2}bc\sin A = \frac{1}{2}\times 40\times 50\times \sin 60$$
$$= 866m^2$$

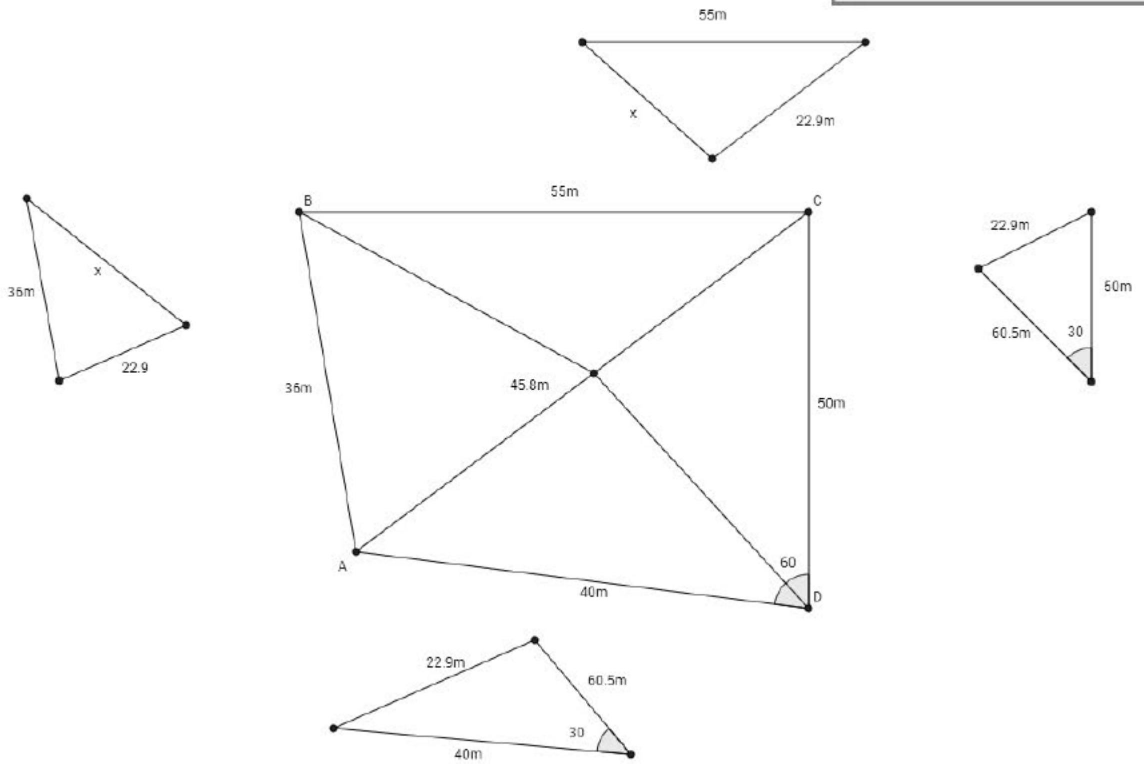
2



	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to apply trigonometric relationships in solving problems. This involves selecting and using methods, demonstrating knowledge of trigonometric concepts and terms and communicating using appropriate representations. This student's evidence is a response to the TKI task 'School Spare Land Subdivision'. The student has selected and used the cosine rule to find the length of the pipeline (1). The student has incorrectly thought that halving the angle will result in the length of the opposite side being halved and the subsequent calculations are wrong (2). This student has attempted to use the sine rule but has misinterpreted the answer as a length (3). To reach Achieved, the student would need to select and use one more method correctly whilst making progress towards solving the problem, for example by finding the area of triangle ADC.

Student 6: High Not Achieved

NZQA Intended for teacher use only



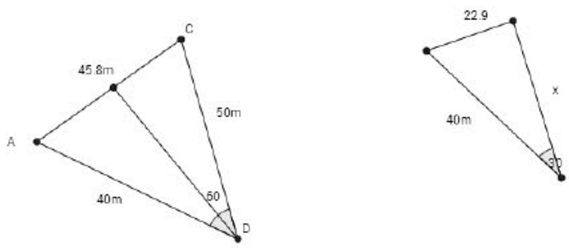
1. By calculating the length of the pipeline running through the land

$x^2 = 40^2 + 50^2 - 2 \times 40 \times 50 \times \cos 60$

$x^2 = 2100(\sqrt{\ })$ 1

$x = 45.8m$

2. Land can be divided into 4 sections each of more than 400m^2



60 divided by 2 = 30
45.8 divided by 2 = 22.9

2

$x = \frac{\sin 30}{22.9} \times 40 = 0.87(\sin^{-1}) = 60.5m$ 3

$Area = \frac{1}{2} \times 22.9 \times 40 \times \cos 60.5$

$Area =$