National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 2

This exemplar supports assessment against:

Achievement Standard 91260

Apply network methods in solving problems

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to apply network methods, using extended abstract thinking, in solving problems. This involves one or more of: devising a strategy to investigate a situation, identifying relevant concepts in context, developing a chain of logical reasoning, or proof, forming a generalisation, and also using correct mathematical statements, or communicating mathematical insight. This student's evidence is a response to the TKI task 'Waikato Cycleway'. The student has identified relevant concepts in context by finding the correct shortest path for the Taupo Club (1), the correct minimum spanning tree for the Hamilton Club (2), the correct maximum spanning tree for the Tirau Club (3), addressing the Tokoroa Club's question correctly (4) and suggesting an appropriate compromise network (5). The student has communicated using mathematical insight in constructing and justifying an appropriate compromise network (6). For a more secure Excellence, the student could strengthen the communication of the strategy used to create a network that satisfies the priorities of the three clubs, for example by communicating fully why the three additional edges were chosen.

Student 1: Low Excellence

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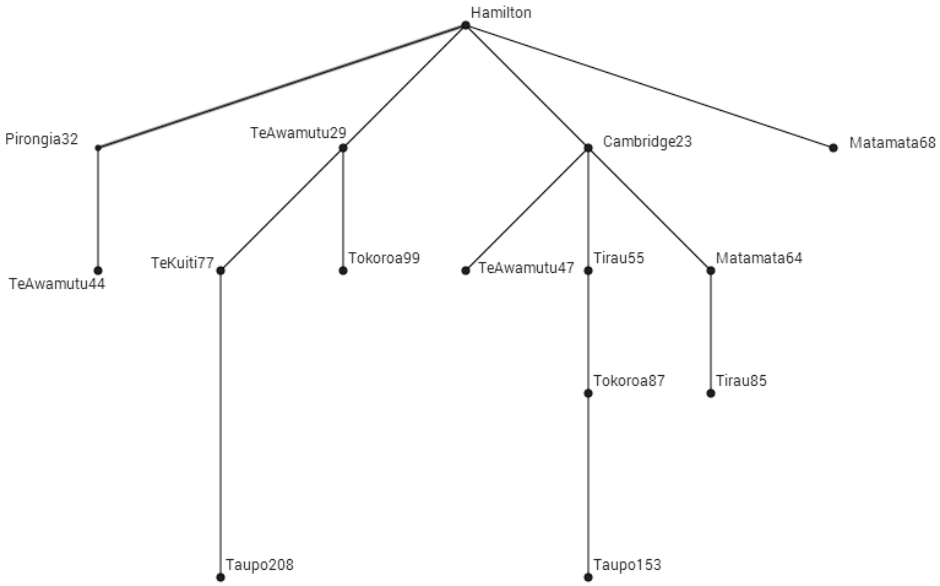
For the Taupo Club I need to find the shortest route from Hamilton to Taupo.

I used the tree method do this, starting in Hamilton. I draw lines to each of the towns I can go to at each step and record the total distance from Hamilton. If I have two ways or more to get a town I choose the shortest one and don't go any further with the diagram except from the shortest distance.



The tree shows me the shortest distance from Hamilton to Taupo is 153km, travelling from Hamilton to Cambridge, to Tirau, to Tokoroa and then to Taupo. (1)

For the Hamilton Club I need the spanning tree with the minimum cost. To find this I used the edge deletion method, removing the highest edge at each turn provided I don't disconnect a town from the network. As there are 9 towns in the network I know I will need 8 edges, so I must delete 8 of the edges on the network as there are 16 altogether.

I deleted the edges that cost:
4560, 3930, 1920, 1830, 1120, 1020, 800, 725 in that order.

This leaves me with this minimum spanning tree which has the minimum cost of $6 230 000. (2)
The tree has these edges:

Hamilton to Cambridge	Cambridge to Matamata	Matamata to Tirau
Cambridge to Te Awamutu	Te Awamutu to Pirongia	Te Awamutu to Te Kuiti
Te Awamutu to Tokoroa	Tokoroa to Taupo	

For the Tirau Club I need the spanning tree with the maximum scenic value. To find this I used the edge deletion method, removing the lowest edge at each turn provided I don't disconnect a town from the network. I have to delete 8 edges again.

I deleted the edges with scenic values:
2, 3, 3, 4, 4, 5, 5 then I have choice of two edges worth 6, either Hamilton to Matamata or Cambridge to Te Awamutu. I am going to delete the second one.
The leaves me with a maximum scenic value of 67 (3)

The edges in the spanning tree are:

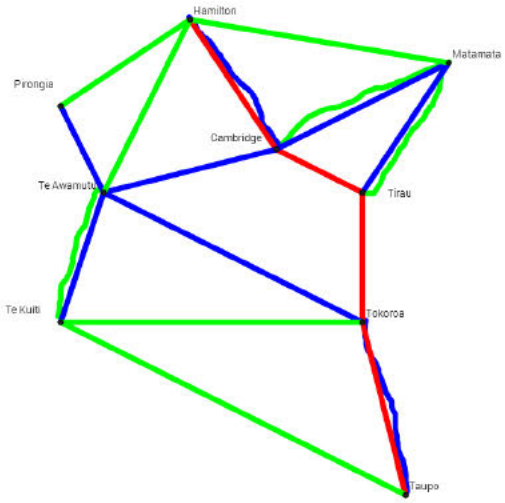
Hamilton to Pirongia	Hamilton to Te Awamutu	Te Awamutu to Cambridge
Cambirdge to Matamata	Matamata to Tirau	Te Awamutu to Te Kuiti
Te Kuiti to Tokoroa	Te Kuiti to Taupo	

For the Tokoroa club I need to see if the network is traversable.

The order of the nodes are;

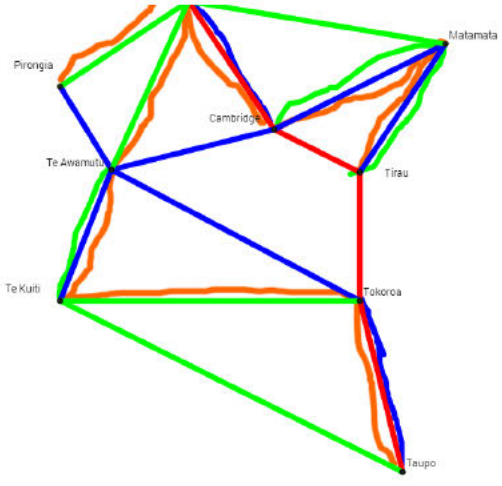
Places	Ham	Mat	Pir	Cam	TeA	Tir	TeK	Tok	Tau
Order	4	3	2	4	6	4	3	4	2

Because Matamata and TeKuiti are odd and the rest even the network is not traversable starting and finishing at Tokoroa. But it could be traversed and finished at the 2 odd nodes, so starting at Matamata and finishing at TeKuiti or vice versa. (4)



This diagram shows the shortest route from Hamilton to Taupo (in Red), the minimum cost tree (in Blue) and the maximum scenic value tree (in Green). Because these three don't overlap there is no one route that will satisfy all three clubs

To satisfy as best I can all three clubs I am going to first select the edges which are in more than one colour as they are common to two clubs. This gives me 5 of the edges in the compromise network (HamCam, CamMat, MatTir, TokTau and TeATeK) and I need another three. I am going to choose edges which have the highest scenic value to complete my network because one of the main reasons that tourists visit New Zealand and the Waikato is for the scenery. (6)



I have no choice but to add in HamTeA and HamPir but I have a choice of adding TeKTok or TeKTau. These have scenic ratings of 9 and 4 so I will use the first one to maximise the scenic value.

My chosen compromise is in orange on the diagram. (5)

	Grade Boundary: High Merit
2.	For Merit, the student needs to apply network methods, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model, and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI task 'Waikato Cycleway'. The student has demonstrated an understanding of concepts by finding the correct shortest path for the Taupo Club (1), by finding and justifying the correct minimum spanning tree for the Hamilton Club (2), the correct maximum spanning tree for the Tirau Club (3) and addressing the question for the Tokoroa Club (4). The student has communicated using appropriate mathematical statements. To reach Excellence, the student could fully justify the shortest route for the Taupo club and further develop the recommendation for a compromise network (5).

Student 2: High Merit

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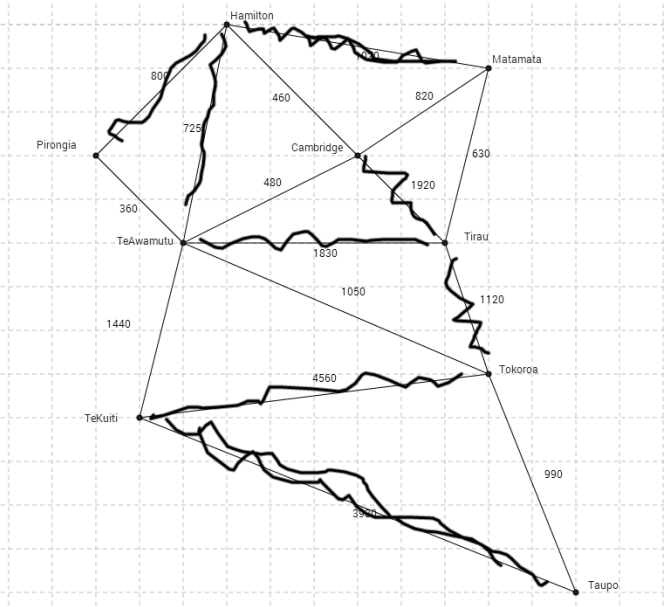
For the Taupo Club I need to find the shortest route from Hamilton to Taupo.

The route Hamilton – TeAwamutu – Tokoroa – Taupo = 29 + 70 + 66 = 165km
The route Hamilton – Cambridge – Tirau – Tokoroa – Taupo = 23 + 32 + 32 + 66 = 153km
The route Hamilton – Cambirdge – Te Awamutu – Tokoroa – Taupo = 23 + 24 + 70 + 66 = 183km

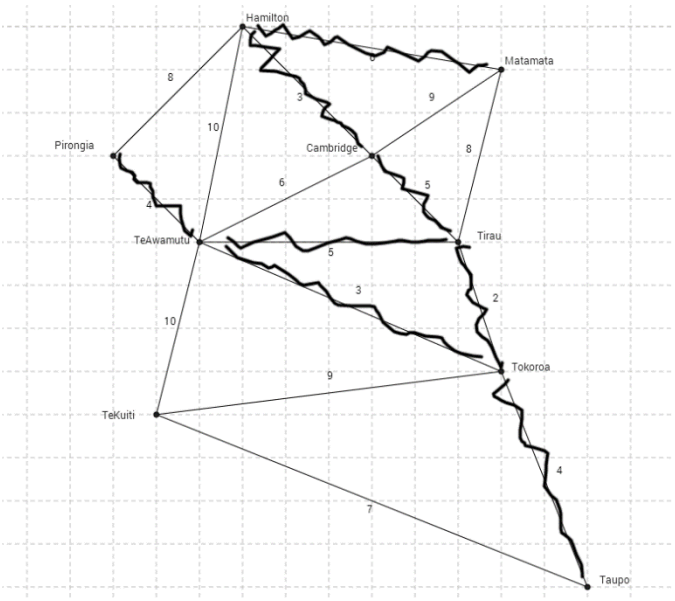
So this route is the shortest Hamilton – Cambridge – Tirau – Tokoroa – Taupo = 153km (1)



For the Hamilton Club I need the minimum spanning tree based on cost. To find this I remove the most expensive edges one by one until I only have a tree, which has no loops, with all towns connected.

I removed the edges that cost 4560, 3930, 1920, 1830, 1120, 1020, 800 and 725 in that order.

The minimum cost is the total of the remaining edges which is $6 230 000. The tree is the edges not crossed off on the diagram (2)



For the Tirau Club I need the spanning tree with the maximum scenic value. I removed the lowest scenic value roads in this order to get the tree in the diagram, with all the towns connected.

2, 3, 3, 4, 4, 5, 5 and 6 (I had a choice for the 6)

The maximum scenic value is 67. (3)

For the Tokoroa club I need to see if the network is traversable. A network can only have even nodes and it is only traversable if there are 0 or 2 odd nodes. I can work out the order of each node by counting how many roads go into them

Hamilton 4 Matatamata 3 Pirongia 2 Cambridge 4 TeAwamutu 6 Tirau 4 TeKuiti 3 Tokoroa 4 Taupo 2

Because Matamata and TeKuiti are odd and the rest even the network is not traversable starting and finishing at the same place. 4

There is no network that can satisfy all three clubs because the networks for the Hamilton, Taupo and Tirau club are all different. Either of the trees for Hamilton and Tirau will still allow people to go from Hamilton to Taupo, although it will be a longer route. I don't think that is important because cyclists like long trips and the exercise will do them good.

I think the best thing to do is to choose one of the other two networks I have found. Which one depends on what the priorities of the Government are. If they want to attract tourists to the region that it would be best to select the maximum scenic route, but that is going to cost more as it is not the minimum cost network. I think this is still the best idea as the investment will be returned by the number of people attracted into the region. If the cost is too much they could consider removing the Tekuiti to Taupo road and replacing it with the Tokoroa to Taupo road. 5

	Grade Boundary: Low Merit
3.	For Merit, the student needs to apply network methods, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model, and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI task 'Waikato Cycleway'. The student has demonstrated an understanding of concepts by finding the correct shortest path for the Taupo Club (1), the correct minimum spanning tree for the Hamilton Club (2), the correct maximum spanning tree for the Tirau Club (3) and addressing, with justification, the question for the Tokoroa Club (4). The student has communicated using appropriate mathematical statements. For a more secure Merit, the student could further explain and justify their solutions for the Taupo, Hamilton and Tirau clubs, for example by showing that they have considered all possible routes from Hamilton to Taupo for the shortest path, and giving the order in which the edges are added into the minimum spanning tree for the Hamilton and Tirau clubs.

Student 3: Low Merit

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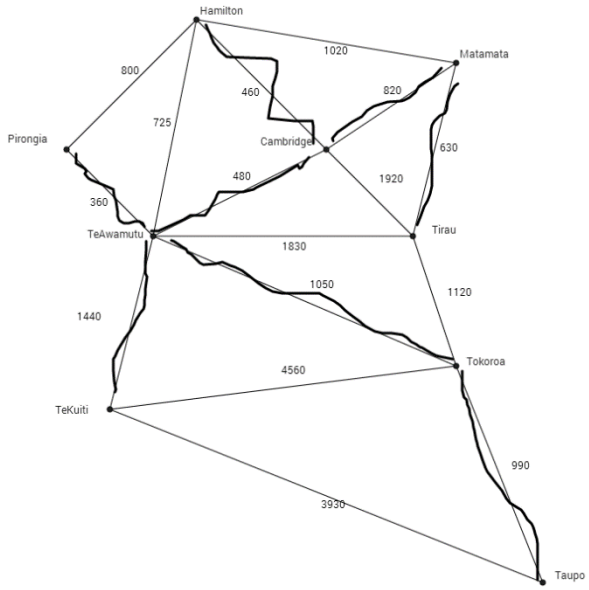
For the Taupo Club I need to find the shortest route from Hamilton to Taupo.

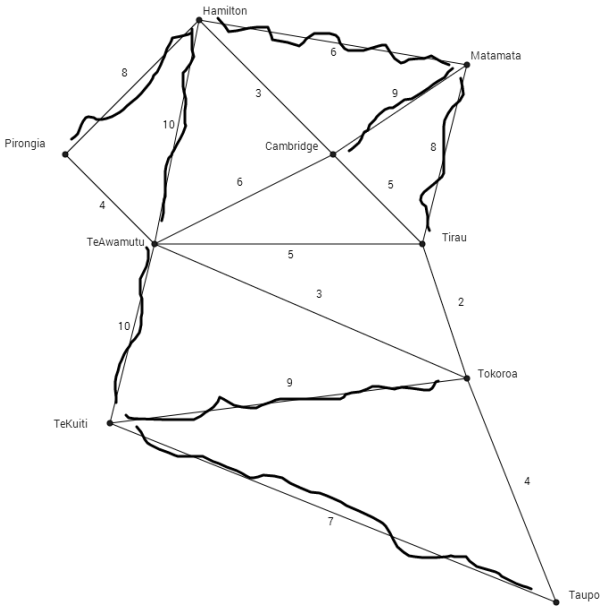
The route Hamilton – TeAwamutu – Tokoroa – Taupo = 29 + 70 + 66 = 165km
The route Hamilton – Cambridge – Tirau – Tokoroa – Taupo = 23 + 32 + 32 + 66 = 153km

So this route is the shortest Hamilton – Cambridge – Tirau – Tokoroa – Taupo = 153km (1)



For the Hamilton Club I need the minimum spanning tree based on cost. To find this I add the cheapest roads one by one until I only have a tree with all towns connected.

The minimum cost is the total of the roads I have put in which is $6 230 000. The tree is the roads marked with wiggly lines on the diagram. (2)



For the Tirau Club I need the spanning tree with the maximum scenic value. I add the highest scenic value roads in order to get the tree in the diagram.

The roads I chose are marked with wiggly lines on the diagram.

The maximum scenic value is 67. (3)

For the Tokoroa club I need to see if the network is traversable. I can work out the order of each node by counting how many roads go into them.
Hamilton 4 Matatamata 3 Pirongia 2 Cambridge 4 TeAwamutu 6 Tirau 4 TeKuiti 3 Tokoroa 4 Taupo 2

Because Matamata and TeKuiti are odd and the rest even the network is not traversable starting and finishing at the same place. 4

There is no network that can satisfy all three clubs but I think the best thing to do is to choose the minimum cost one as that saves money and we don't have much of that. It should be spent on trains instead.

	Grade Boundary: High Achieved
4.	For Achieved, the student needs to apply network methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. This student's evidence is a response to the TKI task 'Waikato Cycleway'. The student has selected and used methods for a shortest path (1), a minimum spanning tree (2) and traversability (3), and communicated using appropriate representations for the cycle paths for the four clubs. To reach Merit, the student would need to justify their solutions to the cycle paths for the four clubs, for example by explaining how the minimum spanning tree was built up, and providing more information as to how the shortest path was found by trial and error.

Student 4: High Achieved

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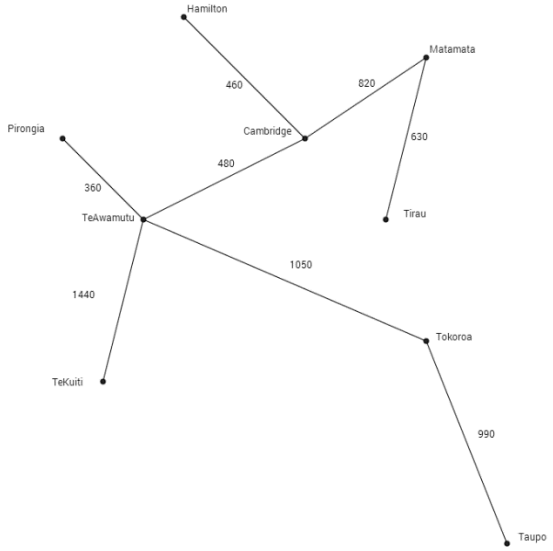
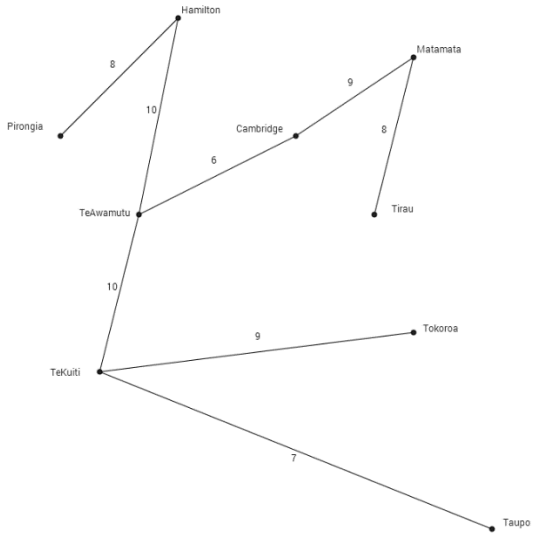
The shortest route is Hamilton to Cambridge to Tirau to Tokoroa to Taupo, which is 153km long. I found this out by trial and error (1)

The minimum cost is $6 230 000. The roads I used are shown on the diagram.



The maximum scenic value is 67. The roads I used are on this diagram (2)



For the Tokoroa club I need to see if the network is traversable.

Because Matamata and TeKuiti are odd and the rest even the network is not traversable starting and finishing at the same place. But you could start at Matamata and finish at TeKuiti, the two odd nodes. (3)

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to apply network methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. This student's evidence is a response to the TKI task 'Waikato Cycleway'. The student has selected and used methods for a minimum spanning tree (1) and traversability (2), and communicated the solutions using appropriate representations. For a more secure Achieved, the student could find the correct shortest path between Hamilton and Taupo and provide the correct maximum scenic value.

Student 5: Low Achieved

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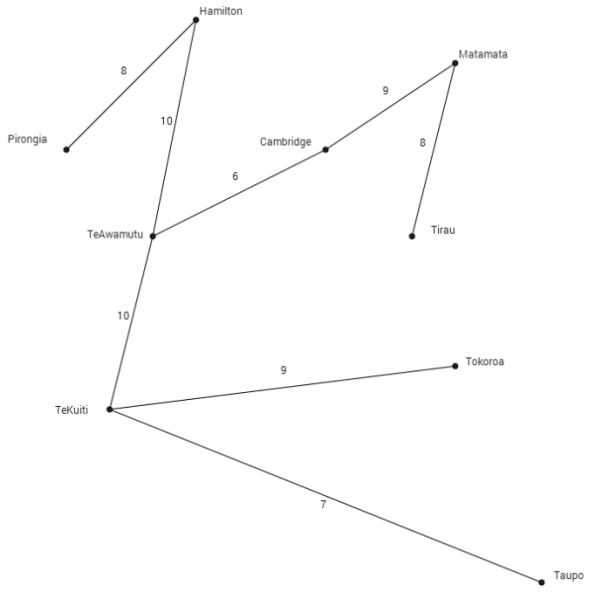
Taupo Club:
The shortest route is Hamilton – TeAwamutu – Tokoroa – Taupo = 29 + 70 + 66 = 165km

Hamilton Club: The minimum cost is $6 230 000. (1)

This uses the roads
Hamilton to Cambridge (460)
Cambridge to Matamata (820)
Matamata to Tirau (630)
Cambridge to Te Awamutu (480)
TeAwamutu to Pirongia (360)
TeAwamutu to Te Kuiti (1440)
Te Awamutu to Tokoroa (1050)
Tokoroa to Taupo (990)

Tirau Club: the maximum scenic value is 64. The roads I used are on this diagram



Tokoroa club

Because Matamata and TeKuiti are odd and the rest even the network is not traversable starting and finishing at the same place. But it could be traversed and finished at the 2 odd nodes, so starting at Matamata and finishing at TeKuiti or the other way round. (2)

	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to apply network methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. This student's evidence is a response to the TKI task 'Waikato Cycleway'. The student has selected and used traversability to address the question for the Tokoroa Club (1). To reach Achieved, the student needs to select and use one further method correctly, for example by finding the shortest path for the Taupo club or the correct minimum spanning tree for the Hamilton club.

Student 6: High Not Achieved

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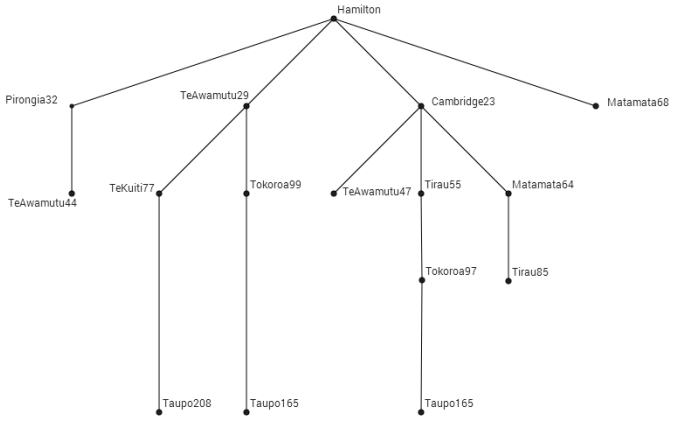
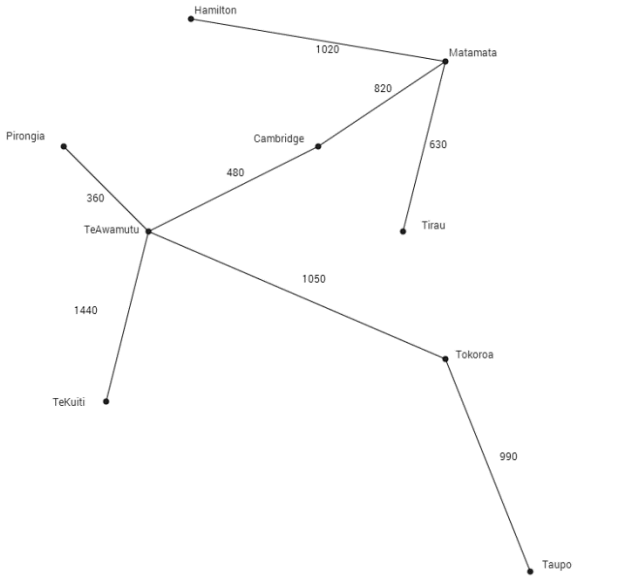
Taupo Club:



There are two shortest routes of 165km shown on this diagram, so the Hamilton people have a choice of the shortest route.



This is the minimum cost network.
The total cost is $6 790 000.

Traversing.

Places	H	M	P	C	TA	TI	TE	TO	TA
Order	4	3	2	4	6	4	3	4	2

Because Matamata and TeKuiti are odd and the rest even the network is not traversable starting and finishing at the same place. But it could be traversed and finished at the 2 odd nodes, so starting at Matamata and finishing at TeKuiti or the other way round. (1)