National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 2

This exemplar supports assessment against:

Achievement Standard 91264

Use statistical methods to make an inference

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to use statistical methods to make an inference, with statistical insight. This involves integrating statistical and contextual knowledge throughout the statistical enquiry cycle which may involve reflecting on the process, or considering other explanations. The student has integrated statistical and contextual knowledge throughout the statistical enquiry cycle (1), provided some evidence of considering other explanations (2) and reflected on the process (3). For a more secure Excellence, the student could provide further reflection on the process by developing the discussion on taking another sample and making the same claim. For example, the student could explain why the same claim would be expected from different samples.

For the students who completed the 2017 New Zealand Census at School I wonder if the median armspan of year 11, 12 and 13 boys (senior boys) is greater than the median armspan of year 11, 12 and 13 girls (senior girls).

I think that the median armspan of the senior boys will be greater than the median armspan of the senior girls because arm span is approximately equal to your height. By this age people have generally stopped growing and senior boys are generally taller than senior girls so it seems reasonable it would be true for their armspans as well.

①

Someone who would benefit from the result of this investigation would be a school blazer manufacturer. Generally senior students wear blazers more than junior students. The school blazer manufacturer could use the results to decide what length to make the arms on their blazers for males and females and whether the lengths of the arms on the blazers would be different for males and females.

I used a random sample of 50 for each group. Each sample is made up of year 11, 12 and 13 students.

Armspan of year 11, 12, 13 students 2017
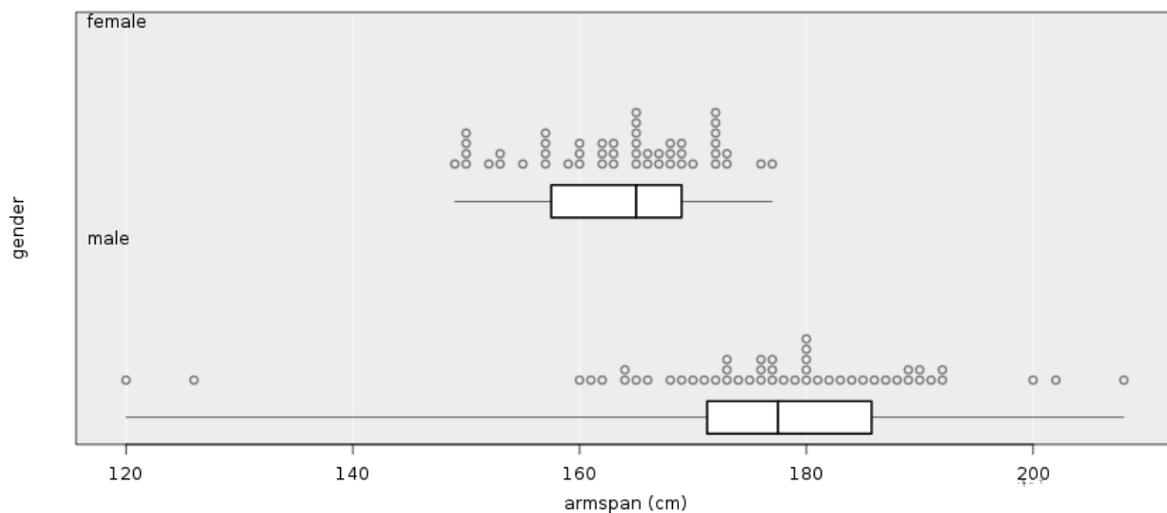
	Min	LQ	Median	UQ	Max	Mean	SD	Num
female	149	157.5	165	169	177	163.4	7.552	50
male	120	171.2	177.5	185.8	208	176.8	15.299	50

From the graph of my samples I notice that the males armspans are shifted further up the scale to the right of the female armspans. The median armspan for the female year 11, 12 and 13 students is 165 cm and for the year 11, 12 and 13 male students the median arm span is 12.5 cm more at 177.5 cm. The box of the middle 50% of the armspans for the males is the right of the box for the females and there is no overlap in the middle 50% of the armspan lengths. This can be seen by the LQ for the year 11, 12 and 13 males being 171.2 cm which is greater than the UQ for the year 11, 12 and 13 female students at 169 cm so more than 75% of the year 11, 12 and 13 males had armspans greater than 25% of the year 11, 12 and 13 females.

①

The IQR for each group is quite similar (11.5 cm for females and 14.6 cm for males) so each of the samples has a similar spread in the middle 50%, but slightly greater for the male group. The overall spread of the male group is larger than the female group. The range for the female armspans is 28 cm compared to the larger range for the males of 88 cm. The

larger overall spread of the males in the sample I took gives the graph a slightly different appearance. On the right hand side there are three points above 200 cm that are removed a bit from the rest of the data but these points seem sensible because the armspan will be approximately equal to height and a senior male student taller than 2 m is reasonable. On the other side of the males graph there are two points below 130 cm. These are smaller than I would expect but could still be reasonable and it is likely these students are quite short. Back in the population I think the data would be more symmetrical than in my samples. There would be more data so I wouldn't expect to see the peaks like there are (172 cm and 150 cm for females) or the gaps in the data and with a body measurement like this I would expect it to more like a normal distribution. I would expect the population data to have a higher median armspan for the year 11, 12 and 13 males but I think the spread for males and females would be similar. I think the males range is only larger for my sample due to the extreme figures (above 200 cm and below 130 cm) and I have no reason to believe there will be more spread for one group than the other.

armspan by gender
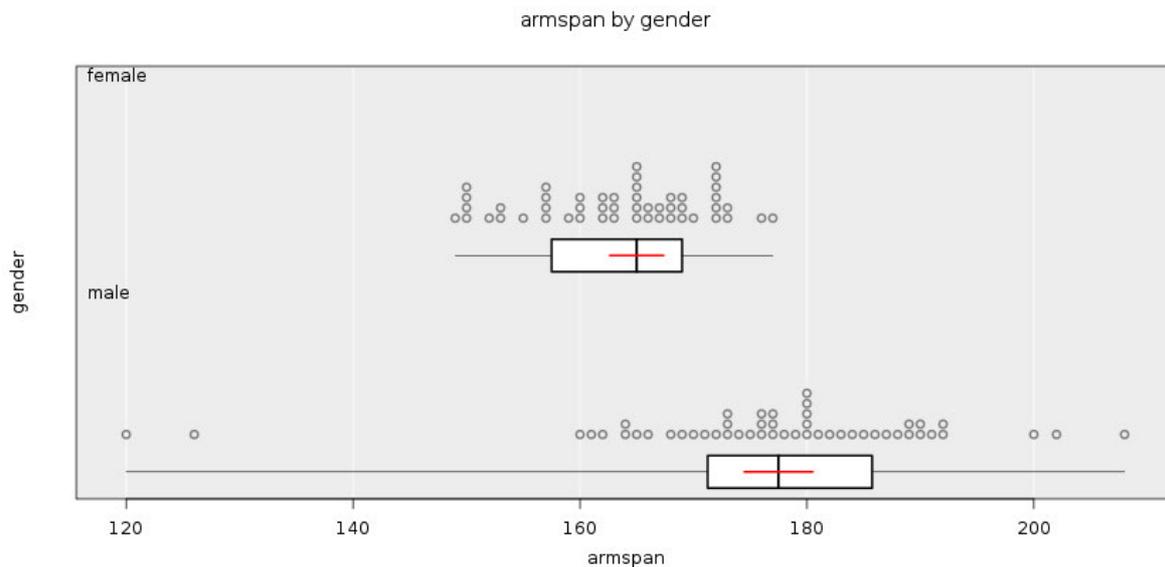
	lower	upper
female	162.5605	167.4395
male	174.4241	180.5759

I have found informal confidence intervals for the median. I can be pretty confident that the population medians will be inside these confidence intervals. I am therefore pretty sure for the students who did the Census at School in 2017 the median armspan for the year 11, 12 and 13 females is between 162.6 cm and 167.4 cm and the median armspan for the year 11, 12 and 13 males is between 174.4 cm and 180.6 cm. Because these intervals don't overlap I can claim that the median armspan of year 11, 12 and 13 males is greater than the median armspan of year 11, 12 and 13 females.

These results are based on my samples. If I was to repeat this sampling process and take another random sample from Census at school 2017, I would get different data in my samples and so get a different median, different quartiles, different minimum and maximum values. The informal confidence intervals might be different but I will still expect to make the same claim.

A school blazer manufacturer could use these results to determine the average length to make the arms on the blazers. They will also know that senior males generally have longer armspans than senior females and therefore they may decide that they may need to make different length arms on the blazers for senior male and females students.

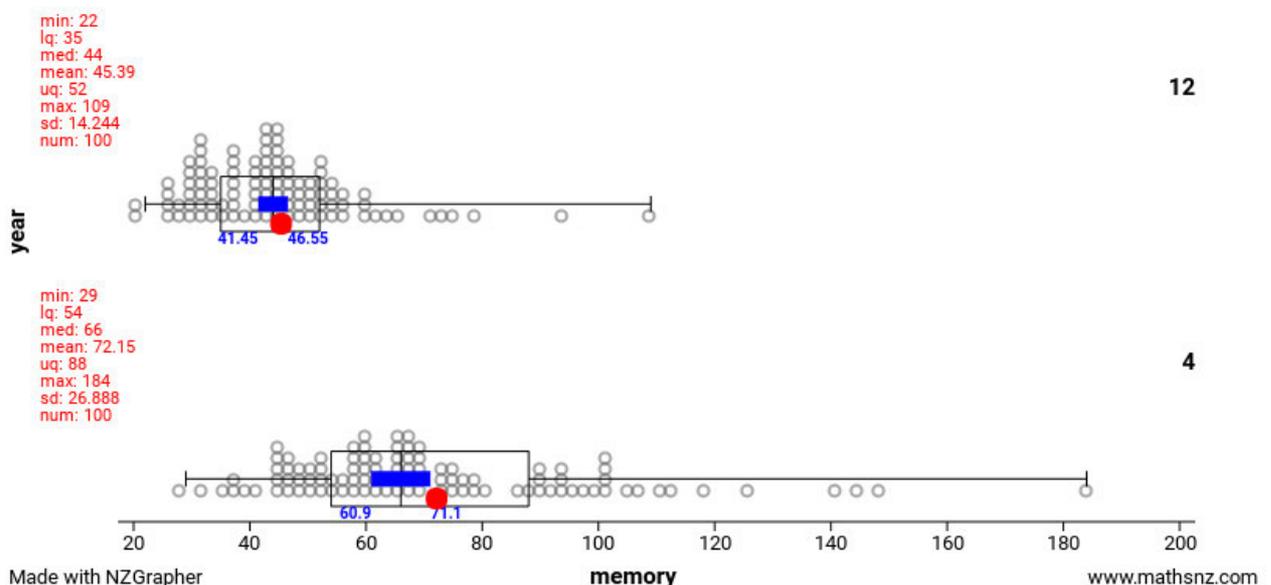|	Grade Boundary: High Merit
2.	For Merit, the student needs to use statistical methods to make an inference, with justification. This involves linking components of the statistical enquiry cycle to the context, and/or to the populations, and referring to evidence such as sample statistics, data values, trends, or features of visual displays in support of statements made. This student's evidence comes from the TKI assessment resource 'The Great Debate'. The student has linked the components of the statistical enquiry cycle to the context and population (1), and provided supporting evidence for the statements made (2). To reach Excellence, the student could demonstrate a clearer understanding about sampling variability, and include some reflection on the process. Additional integration of statistical and contextual knowledge would also be expected, for example linking the conclusion back to the introduction.

Investigative question

Many studies have shown that as we grow older, our brains grow with us and the brain of a teenager will be more developed than the brain of a person at primary school. As our neurons and brain transmitter grow our memories and brain activity increases. I will investigate whether the median time taken to complete a memory game for the year 4s in the New Zealand 2011 Census at School is longer than the median time taken to complete a memory game of the year 12s in the 2011 Census at School. My hypothesis is that the time it takes for year 4s to match all the pairs in the memory game is longer than the time it takes for the year 12's to complete the memory game and find all the pairs. This is because year 12's have a more developed brain than year 4's so should have a better memory.

To get my sample I went to CensusAtSchool data set 2011 to get a random sample of 100 year 12s and 100 year 4s. CensusAtSchool data set 2011 uses a simple random sampling method, this means my sample has no bias because every element has an equal chance of being selected. I chose a sample of 200 - 100 year 12s and 100 year 4s - because I wanted to get a sample big enough so it will be a good representation of all year 4s and 12s that did the census in New Zealand and that the median times in my samples will be close to the median times for all year 4s and 12s in New Zealand 2011 census.

Analysis

Median time in seconds for year 12s and year 4s to complete the memory game

min: 22
lq: 35
med: 44
mean: 45.39
uq: 52
max: 109
sd: 14.244
num: 100

41.45   46.55

min: 29
lq: 54
med: 66
mean: 72.15
uq: 88
max: 184
sd: 26.888
num: 100

60.9   71.1

Made with NZGrapher   memory   www.mathsnz.com

In my sample I can see that there is a large shift of the times for the year 12's middle 50% to the left. This shows that the year 12's tend to complete the memory test faster. The large shift can be seen by the upper quartile of the year 12's (52 sec) being less than the lower quartile of the year 4's (54 sec) so more than 75% of the year 12 times for the game are less than 25% of the year 4 times. This makes sense to me because as we grow our memory functions develop so older students should do the memory game faster.

The year 4's data is visually more spread than the year 12's data. This is true in the middle 50% of the data. The year 4's IQR is exactly double (34 sec) the year 12's IQR (17 sec), which also means a much wider ICI. But to see the overall spread of the data I will use standard deviation. The sd for the year 12's is 14.4s and the sd for the year 4's is 26.8s, almost double. So you can see based on both the standard deviation and IQR, the year 4's data is much more spread.

Most of the year 12s times for the game are tightly packed up but the year 4s times are spread out and there are some large values above 140 sec making it slightly skewed to the right.

The shape of both graphs look unimodal. The year 4's data does look like it could be bimodal because of that spike at around 100 sec and the gap just after 80 sec but the gap is most likely noise and if I was to get a larger sample, that gap and spike would probably flatten out.

Conclusion

I can use my sample to estimate the population medians. Based on the confidence intervals, I can be pretty sure that the median time to find all the pairs in the memory game for year 12's in the 2011 CensusAtSchool is between 41.5 sec and 46.6 sec and the median time for year 4's in 2011 CensusAtSchool is between 60.9 sec and 71.1 sec. The confidence intervals do not overlap so therefore I can confidently infer that the median time to complete the memory game for year 4's in the 2011 CensusAtSchool is longer than the median time to complete the game for year 12's in the 2011 CensusAtSchool. This backs up what my analysis shows. There was a large shift between the two medians, 44 seconds for the year 12's median time and 66 seconds for the year 4's median. This is reasonably big difference.
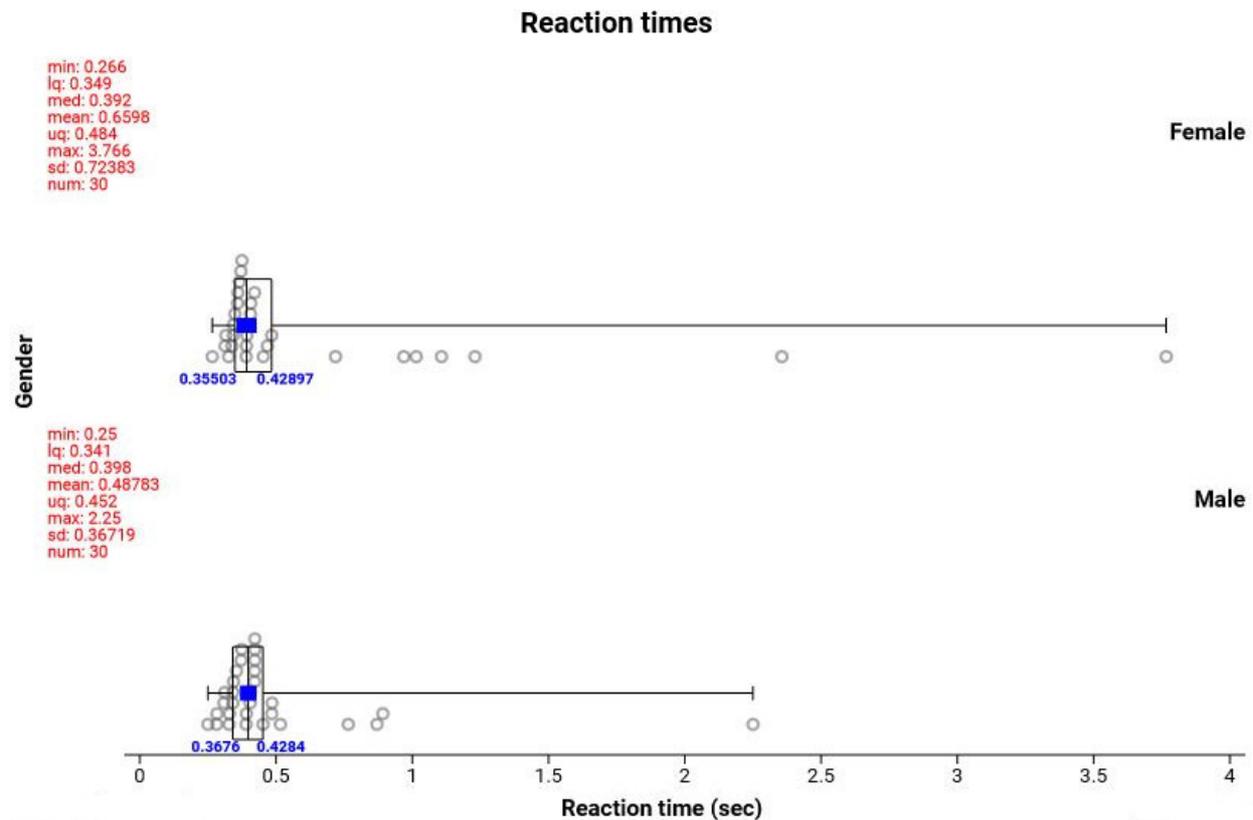
If I had a smaller sample size, I wouldn't be able to make as good estimates for the population's median and my ICI's would be wider, but I would still be reasonably sure that the population median would be within the ICI's. At some point the sample size would get to small and the ICI's would overlap meaning I would no longer be able to make an inference for the population. However, If I were to take another sample I would most certainly get different statistics (median, upper lower quarters etc, but I would still expect to get the same result most of the time.

	Grade Boundary: Low Merit
3.	For Merit, the student needs to use statistical methods to make an inference, with justification.

This involves linking components of the statistical enquiry cycle to the context, and/or to the populations, and referring to evidence such as sample statistics, data values, trends, or features of visual displays in support of statements made.

This student's evidence comes from the TKI assessment resource 'The Great Debate'.

The student has provided supporting evidence for statements made (1), and provided some evidence of linking the components of the statistical enquiry cycle to the context and to the population.

For a more secure Merit, the student could provide stronger evidence of linking the components of the statistical enquiry cycle to the context, and to the population. For example, the discussion of the sampling distributions could be linked more clearly to the context by ensuring that the comments refer to reaction times. |

I wonder if the median reaction time of the year 11 – 13 males that completed the 2011 CensusAt School New Zealand is less than the median reaction time of the year 11 – 13 females that completed the 2011 CensusAt School New Zealand.

The reaction time data is the time it takes for the student to notice that the green button has turned red. The quicker a person is at noticing then the shorter reaction time they will have. I think the males will react in a shorter time than the females. The time I took to notice the green light had changed colour to red was 0.336 seconds.

I am going to get random samples of 30 males and 30 males to answer my question.

Reaction times

min: 0.266
lq: 0.349
med: 0.392
mean: 0.6598
uq: 0.484
max: 3.766
sd: 0.72383
num: 30

Female

0.35503   0.42897

min: 0.25
lq: 0.341
med: 0.398
mean: 0.48783
uq: 0.452
max: 2.25
sd: 0.36719
num: 30

Male

0.3676   0.4284

Reaction time (sec)

For the samples I got the reaction times for both groups are quite similar which surprised me because I thought the males would have shorter times and so there would be a shift to the left of their times. The median for the females is 0.392 seconds and for males 0.398 seconds. The female times are skewed to the right. There is slightly more spread in the middle 50% of the times for the females. The iqr for the female times is 0.135 sec and for males it is 0.111 sec.

If I did the sampling again I would probably get different data and so would get different times for the median and iqr etc. The graphs would therefore be different.

The median reaction time for the year 11 – 13 females in the 2011 CensusAt School New Zealand is likely to be between 0.355 sec and 0.429 sec and the median reaction time for the year 11 – 13 males in the 2011 CensusAt School New Zealand is likely to be between 0.368 sec and 0.428 sec. The confidence intervals for the median reaction times overlap so I am not able to say that the median reaction time for the year 11 – 13 males in the 2011 CensusAt School New Zealand is less than the median reaction time for the year 11 – 13 males in the 2011 CensusAt School New Zealand.
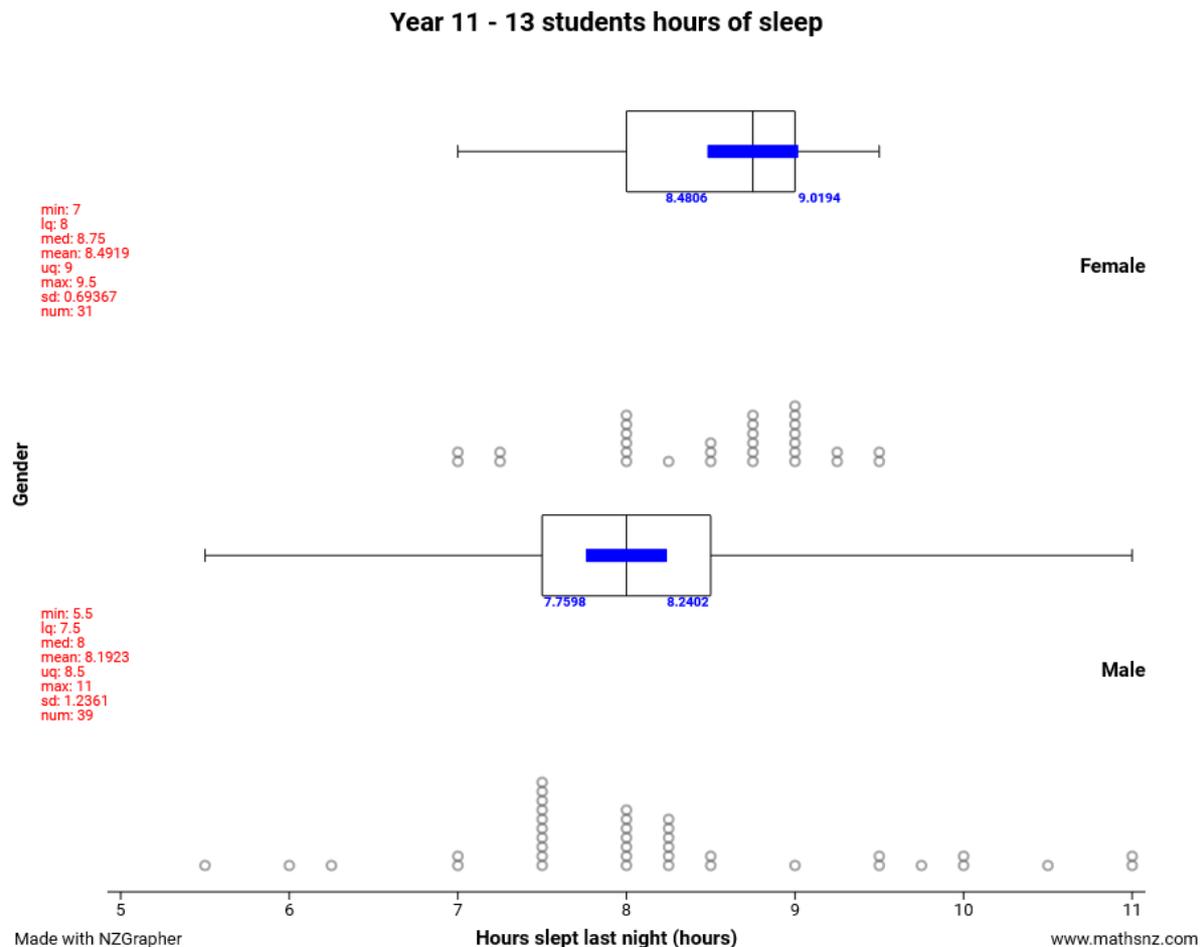
	Grade Boundary: High Achieved
4.	For Achieved, the student needs to use statistical methods to make an inference. This involves showing evidence of using each component of the statistical enquiry cycle to make an inference. This student's evidence comes from the TKI assessment resource 'The Great Debate'. The student has posed an appropriate investigative comparison question (1), selected random samples (2), selected and used appropriate displays and measures (3), discussed sample distributions (4), discussed sampling variability, including the variability of estimates (5), made an inference (6), and communicated findings in a conclusion (7). To reach Merit, the student could justify the call with specific evidence from the displays, and include the actual population with the interval estimates for the population medians.

Question:

Is the median hours of sleep of Year 11 - 13 females who did the Census at School NZ 2011 longer than the median hours of sleep of Year 11 - 13 males who did the Census at School NZ 2011?

①

I am going to use NZgrapher to get a random sample for analysis. My variable is the hours of sleep the previous night. The census is usually completed at school but I don't know which day so don't know what night it was but it shouldn't be Friday or Saturday if it is collected at school.

②

Year 11 - 13 students hours of sleep

8.4806   9.0194

min: 7
lq: 8
med: 8.75
mean: 8.4919
uq: 9
max: 9.5
sd: 0.69367
num: 31

Female

③

Gender

7.7598   8.2402

min: 5.5
lq: 7.5
med: 8
mean: 8.1923
uq: 8.5
max: 11
sd: 1.2361
num: 39

Male

5   6   7   8   9   10   11

Made with NZGrapher   **Hours slept last night (hours)**   www.mathsnz.com

I notice in my sample that the females median hours slept (8.75 hr) is longer than the males median hours slept (8 h). That's a difference of 0.75 h.
The graph for the females hours is more squashed up than the male hours which is quite spread out but most of the male hours of sleep are between 7.5 hours and 8.5 hours. This can also be seen in the range of the male hours of sleep (5.5 h) is more than the range of the female hours of sleep.

④

The interquartile range (middle 50%) for the males in my sample is 1 hour and the interquartile range (middle 50%) for the females in my sample is also 1 hour and the boxes in the graph are the same length. This means there is the same amount of variation in the middle 50% hours of sleep for both males and females in my sample.
The middle 50% of females hours of sleep also overlaps the male hours of sleep.
One of the males in my sample only had 5.5 hours of sleep for the previous night and then there are two more males with 6 hours then 6.25 hours of sleep. This isn't much sleep but it is only what they thought they got the night before so they may have had a late night or they

may have an early morning job or sports practice. At the other end of the male graph there are some large values but these are also reasonable and as it is only one night (hours of sleep the previous night) it doesn't mean they are sleeping 11 hours every night. A different sample of males and females will have different hours of sleep to what I got so the graphs are going to look different. The median and middle 50% might be in a different place as well.

⑤

Conclusion:
The confidence intervals on the graph show where the population median should be. In the population I expect that the median hours of sleep of year 11 – 13 females to fall between 8.4806 h and 9.0194 h. For the population of year 11 - 13 males I would expect the median hours of sleep to be between 7.7598 h and 8.2402 h (the males confidence interval).

⑦

Answer Question: My question was Is the median hours of sleep of Year 11 - 13 females who did the Census at School NZ 2011 longer than the median hours of sleep of Year 11 - 13 males who did the Census at School NZ 2011? Looking at the informal confidence intervals I can make the call that the median hours of sleep the previous night of year 11 - 13 females in the Census at School NZ 2011 is longer than the median hours of sleep the previous night of year 11- 13 males in the Census at School NZ 2011.

⑥

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to use statistical methods to make an inference.

This involves showing evidence of using each component of the statistical enquiry cycle to make an inference.

This student's evidence comes from the TKI assessment resource 'The Great Debate'.

The student has posed an appropriate investigative comparison question (1), selected random samples (2), selected and used appropriate displays and measures (3), discussed sample distributions (4), discussed sampling variability, including the variability of estimates (5), made an inference by giving an interval estimate for the median time standing on the left leg (6), and communicated findings in a conclusion (7).

For a more secure Achieved, the student could use the informal confidence intervals to make a comparative statement about the median times for the two population groups standing on the left leg.

The student could also provide stronger evidence of discussing the sample distributions in context by referring to the times standing on the left leg, and include the variable with both interval estimates. |

I wonder if male's can stand on their left leg longer than females in the dataset from the 2017 Census at school database. I think that male's can stand longer because they tend to have a more muscular build, meaning their legs are stronger. I will use the median time (seconds) to make an inference to see how long male's and female's can stand on their left leg in the dataset from the 2017 Census at school database.

①

I will use simple random sampling to select 30 male and 30 female.

②

Standing on left leg comparison
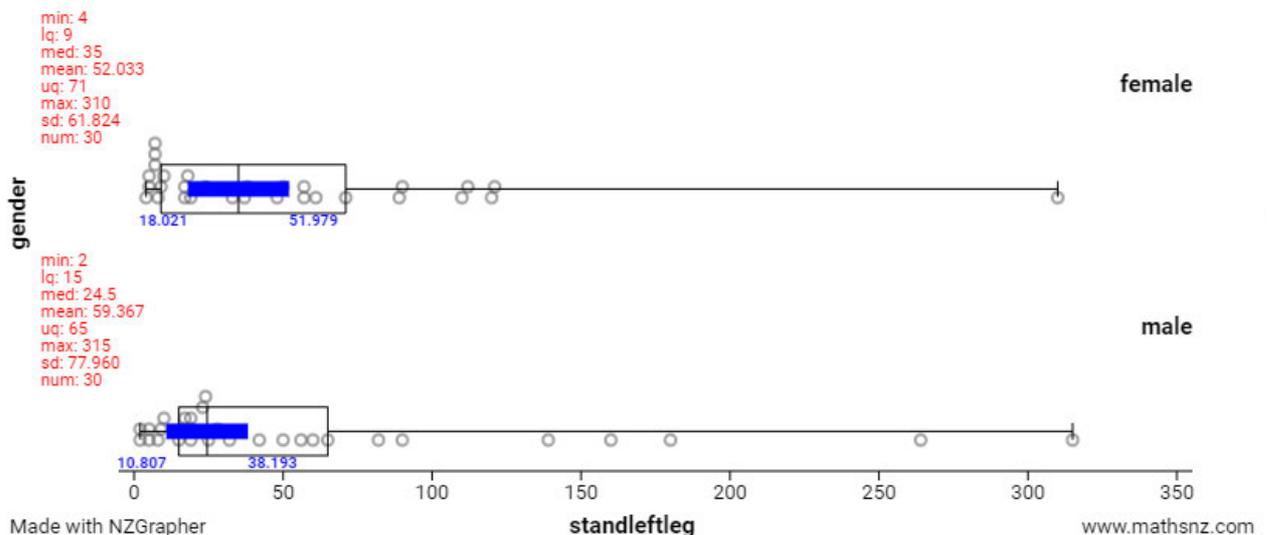
From the graph above I notice that the sample median for female's is 35 seconds which is larger than the males of 24.5 seconds. This means females generally tend to stand longer on their left leg rather than male's in the 2017 Census at school database.

I notice that the middle 50% of data (IQR) for females is (71-9) 62 seconds which is larger than the middle 50 % of data (IQR) for males which is (65-15) 50 seconds. This means there is more variability within the sample of females.

④

I also notice that the shape of the females data is skew and the shape of the males data is symmetrical with outliers.

If I did another sample my statistics would be different. This means the median and confidence intervals would also be different. However the population median would most likely still fall within the confidence intervals. Although if I took a larger sample the interval would have less variability and the confidence interval would be smaller.

⑤

From my statistics gathered I am reasonably confident that from the dataset from the 2017 Census at school database the median for females standing on their left leg lies between 18.021 seconds and 51.979 seconds and I am reasonably confident that from the dataset from the 2017 Census at school database between 10.807 seconds and 38.193 seconds is the time for males.

⑥

In conclusion from the results gathered the confidence intervals overlap therefore I cannot say that male can stand on their left leg longer than females in the dataset from the 2017 Census at school database.

⑦

	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to use statistical methods to make an inference.

This involves showing evidence of using each component of the statistical enquiry cycle to make an inference.

This student's evidence comes from the TKI assessment resource 'SURFing'.

The student has posed an investigative comparison question (1), selected random samples (2), selected and used appropriate displays and measures (3), discussed sample distributions (4), discussed sampling variability, including the variability of estimates (5), and communicated findings in a conclusion (6).

To reach Achieved, the student would need to make an inference by answering the investigative question about the median weekly incomes in the population. An understanding of the comments about weekly incomes applied to 2001 should also be evident. |
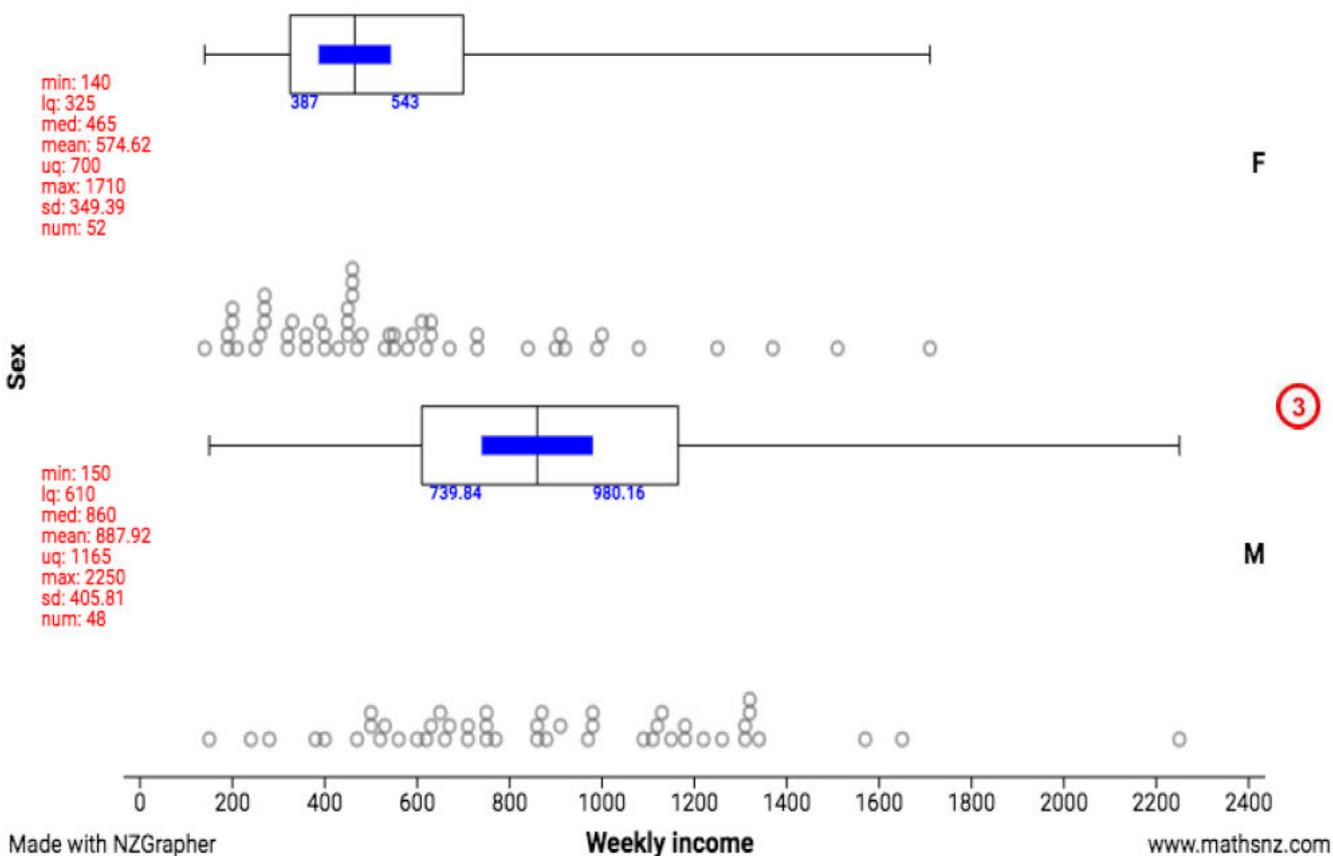
Question:

In the statistics nz survey (SURF) is the median weekly income for a female in New Zealand less than the median weekly income for a male in New Zealand?   ①

Prediction:

I predict that the median weekly income amount a male gets will be more than the median weekly income a female gets because males generally work longer hours.

Sampling:

I have chosen a sample size of 100 because it is a good amount to work with, it isn't too large or too small and I am able to see accurate results. I have chosen to go with a simple random sample because it's an easy way to collect data without being bias.   ②

min: 140
lq: 325
med: 465
mean: 574.62
uq: 700
max: 1710
sd: 349.39
num: 52

387   543

F

③

min: 150
lq: 610
med: 860
mean: 887.92
uq: 1165
max: 2250
sd: 405.81
num: 48

739.84   980.16

M

Sex

0   200   400   600   800   1000   1200   1400   1600   1800   2000   2200   2400

Weekly income

Made with NZGrapher   www.mathsnz.com

Centres:

I notice that the median amount a male gets per week ($860) is more than the median amount a female gets per week ($465). There is a difference of $395. The males mean ($887.92) is also greater than the females mean ($574.62) This means for this sample there is quite a big difference between the weekly income amount of a male in NZ and the weekly income amount of a female in NZ.

Shapes:

The shape of the distribution of data for the weekly income amount that a male gets tends towards a more uniform distribution whereas the shape of the distribution of data for the weekly income amount a female gets tends towards more of a right skew.

Spread:

I notice that the IQR of the males ($555) is more than the IQR of the females ($375). The males ($2100) also have a bigger range than the females ($1570). The data for the males sits just right of the females data with the boxes slightly overlapping.

The smaller IQR for the females means that there are more values clustered around the median whereas since the males IQR is larger it means that the values are more spread out.

Back in the population the range would suggest that the males tend to get a bigger weekly income amount than the females do due to the fact that the ranges are so different in size.

Overlap:

50% of the males data is more than 75% of the females data. This means that quite a few males get a bigger weekly income amount than the females. Back in the population we would expect that quite a few males would get a bigger weekly income amount as the difference is quite significant.

Unusual features:

In the males data there is an outlier that sits around the $2200-$2400 weekly income amount mark. This could be due to this person having a very difficult job that pays extremely well like a surgeon, a CEO of a company or owning a building company.

Confidence intervals:

The confidence intervals don't overlap, $387 to $543 for females and $739.84 to $980.16 for males. In fact there is quite a difference between them so we can be reasonably confident that males get a bigger weekly income amount than the females.

Sampling variability:

If we took another sample we would almost certainly get different results as the sample data would be different and the confidence intervals wouldn't be the same. However we would expect the median to lie within the confidence intervals generated by the sample.

Conclusion:

In conclusion I found that in this sample my prediction was correct and the median weekly income amount a male gets tends to be more than the median weekly income amount a female gets. This could be due to a number of different things and because it's such a controversial topic it is hard to know for certain who earns more but for this specific sample of data it does show that males get a bigger weekly income amount. If I were to sample differently this could affect the outcome because if I were to take a smaller sample the data would be closer together and I am fairly confident that the confidence intervals would end up overlapping more.