National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 2

This exemplar supports assessment against:

Achievement Standard 91268

Investigate a situation involving elements of chance using a simulation

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to investigate a situation involving elements of chance using a simulation, with statistical insight. This involves integrating statistical and contextual knowledge throughout the simulation process and may involve reflecting about the process, or considering other variables. This student's evidence is a response to the TKI assessment resource 'Fruity Freezes'. The student has designed a simulation for a given situation by identifying the tool to be used (1), defining a trial and deciding the number of trials (2) and determining data recording methods (3). They have also carried out the simulation and recorded the outcomes (4), selected and used appropriate displays and measures (5) and communicated findings in a conclusion (6). The student has reflected on the process by linking the assumptions to the design of the simulation (7). For a more secure Excellence, the student could consider other variables. For example, by considering a simulation for a situation where two friends swap symbols so that they can collect a set more quickly. The student could also develop the comment about the simulation results being different if the experiment was repeated, by providing an explanation for the results being estimates.

I will use my calculator to find a maximum of 10 random numbers from 1 to 10. The formula I will use is 10RAN#+1. I will ignore decimals. (1)

I will allocate letters/symbols to numbers. Each number represents 10%.

Apple	0.4 (40%)	4 numbers allocated:	1-4
Pineapple	0.3 (30%)	3 numbers allocated:	5-7
Grape	0.2 (20%)	2 numbers allocated:	8-9
Strawberry	0.1 (10%)	1 number allocated:	10

A successful trial is when all four letters/symbols are found.
Eg 3,6,9,10 – apple, pineapple, grape strawberry.
An unsuccessful trial is when all four letters/symbols are not found.
Eg 1,3,2,4,7,6,5,9,9,8 – there is no strawberry here. (3)
10 random numbers is the maximum to be found as the trial goes on for ten days, one ice block bought per day. (2)
The trial will be repeated 30 times.

Results of the simulation

Trial number	Random numbers	Win/lose	Number of ice blocks bought
1	9 6 6 4 6 8 3 9 7 9	×	10
2	4 6 8 4 7 4 10	✓	7
3	3 8 6 1 1 2 4 9 6 8	×	10
4	7 6 3 5 3 5 3 9 10	✓	9
5	1 7 4 6 1 7 6 2 2 8	×	10
6	4 5 9 7 1 3 9 5 9 10	✓	10
7	1 7 8 1 1 2 7 10	✓	8
8	7 2 1 10 2 1 1 4 10 6	×	10
9	5 9 5 5 4 4 6 8 9 2	×	10
10	5 1 3 5 8 5 5 1 2 1	×	10
11	1 1 7 2 5 5 1 3 9 1	×	10
12	8 1 3 6 4 4 3 7 10	✓	9
13	9 9 5 10 4	✓	5
14	3 2 6 7 1 10 9	✓	7
15	2 2 4 7 3 2 6 7 9 1	×	10
16	6 1 1 2 9 2 1 8 5 4	×	10
17	3 4 8 4 5 1 5 1 4 5	×	10
18	4 10 6 10 9	✓	5
19	7 8 10 6 10 1	✓	6
20	1 2 1 2 9 4 6 6 10	✓	9
21	6 9 4 1 5 6 9 5 7 10	✓	10
22	6 9 7 6 6 7 9 2 4 3	×	10
23	4 1 2 6 10 2 9	✓	7
24	4 4 3 9 3 2 6 8 6 7	×	10
25	1 2 8 6 6 3 9 10	✓	8
26	2 2 6 3 1 10 10 7 1 6	×	10
27	4 4 1 6 10 6 5 4 4 1	×	10
28	5 1 6 5 2 8 4 5 9 8	×	10
29	2 7 6 7 6 3 3 3 8 1	×	10
30	2 7 9 7 10	✓	5

(4)

Probability of winning = $\frac{\#\ of\ wins}{\#of\ trials} = \frac{14}{30} = 0.47$ (5)

Average number of ice blocks to win = $\frac{\#oficeblocksbought}{\#oftrials} = \frac{265}{30} = 8.83$

The estimated probability of winning a movie ticket is 0.47
The estimated average number of ice blocks needed to win a movie ticket is 8.83.

Any results found in this experiment should be treated with caution as they are estimates thus they are likely to be different if the experiment was repeated again. (6)
I would expect different results if I repeated the experiment.

In this simulation there are several assumptions which are made which must be taken into consideration.

- It is assumed that the proportions of symbols is kept the same throughout the competition by the suppliers. This means that the probabilities (0.4, 0.3, 0.2 and 0.1) are kept constant.
- It is assumed that the ice blocks are distributed evenly across all the outlets as are the different symbols (i.e. no clustering of certain symbols in a particular outlet, as this will alter the probability of Grace getting all 4 sticks to win a movie ticket.)
- It is assumed that the 4 symbols are all released at the same time (none are held back affecting the probabilities of collecting a symbol).
- It is assumed the price of the ice blocks is not changed during the competition which may affect the number of ice blocks Grace purchases. (7)
- It is also assumed that each symbol is independent from another. For example collecting a certain symbol will have no influence on the next symbol/ ice block purchased.

	Grade Boundary: High Merit
2.	For Merit, the student needs to investigate a situation involving elements of chance using a simulation, with justification. This involves linking components of the simulation process to the context, explaining relevant considerations made in the design of the simulation, and supporting findings with statements which refer to evidence gained from the simulation. This student's evidence is a response to the TKI assessment resource 'Fruity Freezes'. The student has designed a simulation for a given situation by identifying the tool to be used (1), defining a trial and deciding the number of trials (2) and determining data recording methods (3). They have also carried out the simulation and recorded the outcomes (4), selected and used appropriate displays and measures (5) and communicated findings in a conclusion (6). The student has explained relevant considerations made in the design of the simulation (7) and supported findings with statements which refer to evidence from the simulation (8). To reach Excellence, the student could clearly link the assumptions to the design of the simulation and provide a fuller explanation for the probability and mean being estimates.

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Student 2: High Merit

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Tool: I am going to use random number generator on my graphics calculator to generate numbers between 1 and 10.
1 – 4 will represent an apple stamp
5 – 7 will represent a pineapple stamp
8 – 9 will represent a grape stamp
10 will represent a strawberry stamp

1

Trial: For each trial I will generate 10 random numbers to symbolise the 10 ice blocks Grace is able to buy. However, if Grace gets one of each stamp before she has brought 10 ice blocks the trial will finish there. I will do 20 trials. A successful trial will be if Grace gets one of each stamp (at least one of 1, 2, 3 or 4, at least one of 5, 6 or 7 at least one of 8 or 9 and 10)
eg. 9 7 6 4 1 2 3 8 9 1 = not successful
10 3 2 4 1 2 5 6 8 7 = successful

2

3

Trial	Random numbers	success	No of days
1	3, 8, 9, 6, 1, 8, 8, 3, 1, 9	×	10
2	10, 7, 5, 5, 10, 9, 10, 2	✓	8
3	10, 3, 8, 6	✓	4
4	3, 6, 9, 6, 8, 2, 3, 7, 5, 10	✓	10
5	8, 10, 9, 6, 7, 5, 9, 6, 6, 5	×	10
6	9, 2, 3, 7, 3, 2, 9, 3, 4, 2	×	10
7	2, 7, 7, 7, 8, 5, 9, 3, 5, 7	×	10
8	5, 8, 4, 1, 3, 9, 1, 4, 8, 10	✓	10
9	6, 2, 7, 4, 1, 6, 10, 2, 4, 8	✓	10
10	5, 1, 8, 5, 6, 4, 8, 10	✓	8
11	8, 9, 10, 2, 2, 7	✓	6
12	6, 10, 6, 8, 9, 4	✓	6
13	4, 1, 9, 8, 5, 6, 6, 5, 8, 4	×	10
14	5, 5, 8, 4, 4, 5, 1, 6, 1, 5	×	10
15	2, 9, 5, 10	✓	4
16	2, 8, 8, 10, 1, 7,	✓	6
17	4, 2, 10, 4, 2, 6, 4, 8	✓	8
18	4, 10, 3, 8, 6	✓	5
19	1, 8, 10, 6	✓	4
20	4, 9, 6, 4, 7, 5, 6, 7, 3, 8	×	10

4

The results I have gathered are only an estimate and any probabilities taken from my results are only estimates. If I were to do my trials again or if someone else were to do my simulation, the results may differ.

8

$$\frac{\text{Total number of days}}{\text{No of trials}} = \frac{159}{20} = 7.95$$

$$\frac{\text{No of successful days}}{\text{No of trials}} = \frac{13}{20}$$

5

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From my simulation I estimated the mean number of days Grace buys an ice block to win a movie ticket for to be 7.95 and the chances of Grace winning a movie ticket to be 0.65. However, this is only an estimate because if I did it again I might get different results. 6

The assumptions I have made are that the price of the ice blocks remains the same. If the price increases or decreases the number of ice blocks Grace can buy will change. I am also assuming that the stamped ice block sticks are available in these proportions in each shop because otherwise this will change the probabilities and affect Graces chance of winning a movie ticket. That Grace will not swap ice block stamps as this will increase her chances of winning and that Grace will continue to have enough money to buy 10 ice blocks otherwise that will change my probabilities. 7

An alternative approach could be to do the simulation with two people so that the chance of winning a prize increases. You could also keep each trial going so it finishes when a movie ticket is won instead of finishing after 10 ice blocks.

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	Grade Boundary: Low Merit
3.	For Merit, the student needs to investigate a situation involving elements of chance using a simulation, with justification. This involves linking components of the simulation process to the context, explaining relevant considerations made in the design of the simulation, and supporting findings with statements which refer to evidence gained from the simulation. This student's evidence is a response to the TKI assessment resource 'Fruity Freezes'. The student has designed a simulation for a given situation by identifying the tool to be used (1), defining a trial and deciding the number of trials (2) and determining data recording methods (3). They have also carried out the simulation and recorded the outcomes (4), selected and used appropriate displays and measures (5) and communicated findings in a conclusion (6). The student has explained relevant considerations made in the design of the simulation (7) and supported findings with statements which refer to evidence from the simulation (8). For a more secure Merit, the student would need to expand on the assumptions made in designing the simulation.

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Student 3: Low Merit

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I will use the random number function on my calculator to generate random numbers between 1 and 10. Each number represents the type of symbol stamped on an ice-block (i.e. apple, pineapple, grape or strawberry). Digits 1 to 4 represent an apple stamp, 5 to 7 represent a pineapple stamp, 8 and 9 represent a grape stamp and digit 10 represents a strawberry stamp. Each trial consists of generating random numbers between 1 & 10 until at least one of each symbol is obtained (i.e. until at least one of 1, 2, 3 or 4, one of 5, 6 or 7, one of 8 or 9 and a digit10 is obtained) or until a maximum of 10 random numbers is obtained as Grace can only buy an ice-block per day for a maximum of 10 days. A successful outcome of a trial would be to obtain at least one of each symbol, for example 2, 9, 9, 8, 5, 10 would be the equivalent of Grace getting an apple, grape, grape, grape, pineapple and strawberry symbol in that order, one per day. This would be a successful trial as Grace got at least one of each symbol and she would win a free movie ticket. I will do 20 trials. I will record the results on a table and record which trials are successful and which are not, and also the number of days Grace has to buy ice blocks for each trial, in order to calculate the mean number of days Grace buys ice-blocks for.

1 2 3

		outcome	
Trial number	Random numbers	Successful or not	No of days
1	9, 6, 6, 8, 7, 7, 4, 6, 10	✓	9
2	3, 10, 1, 9, 9, 9, 7	✓	7
3	8, 10, 5, 2	✓	4
4	1, 2, 10, 1, 6, 10, 5, 4, 8	✓	9
5	6, 2, 1, 8, 5, 1, 6, 2, 9, 8	×	10
6	3, 4, 9, 10, 8, 9, 10, 6	✓	8
7	5, 9, 2, 10	✓	4
8	8, 5, 10, 3	✓	4
9	8, 2, 9, 2, 4, 2, 5, 5, 7, 7	×	10
10	10, 4, 5, 6, 5, 8	✓	6
11	2, 5, 1, 3, 7, 1, 6, 9, 9, 9	×	10
12	6, 7, 7, 5, 6, 6, 8, 7, 1, 3	×	10
13	8, 9, 3, 8, 7, 2, 6, 8, 2, 10	✓	10
14	6, 5, 4, 6, 10, 2, 7, 9	✓	8
15	4, 3, 2, 8, 7, 7, 1, 2, 3, 4	×	10
16	4, 8, 1, 3, 3, 7, 4, 8, 2, 4	×	10
17	6, 4, 4, 4, 4, 3, 7, 6, 3, 5	×	10
18	7, 6, 2, 2, 4, 1, 9, 6, 8, 4	×	10
19	10, 7, 6, 6, 10, 3, 8	✓	7
20	4, 9, 6, 10	✓	4

4

total successful = 12

total no of days = 160 $\frac{160}{20}$ = 8 days $\frac{12}{20} = \frac{3}{5}$

5

The probability she wins a movie ticket is $\frac{3}{5}$ and the mean number of days to win the movie ticket is 8 days.

8

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According to the results of my simulation, the estimated mean number of days Grace buys ice block is 8 days. However, this value is likely to change if the simulation is repeated because repeating the simulation is likely to give different random numbers. Thus 8 is only an estimate of the mean number of days Grace buys ice-blocks. 6

Assumptions I have made include the following:
The fraction of symbols for the fruits is the same in every shop so that I can use the same random numbers for the fruits every time. 7

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	Grade Boundary: High Achieved
4.	For Achieved, the student needs to investigate a situation involving elements of chance using a simulation. This involves showing evidence of using each component of the simulation process. This student's evidence is a response to the TKI assessment resource 'Fruity Freezes'. The student has designed a simulation for a given situation by identifying the tool to be used (1), defining a trial and deciding the number of trials (2) and determining data recording methods (3). They have also carried out the simulation and recorded the outcomes (4), selected and used appropriate displays and measures (5) and communicated findings in a conclusion (6). To reach Merit, the student would need to discuss assumptions made in designing the simulation.

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Student 4: High Achieved

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Tool: I will generate random numbers using the random number generator on my calculator between 1 – 10 to represent the four different fruit symbols. (1)

1, 2, 3, 4 will represent the apple symbol P(0.4)
5, 6, 7 will represent the pineapple symbol P(0.3)
8, 9 will represent the grape symbol P(0.2)
10 will represent the strawberry symbol P(0.1)

Trial: I will continue to generate random numbers until I have at least one of each of the four different symbols so a free movie ticket can be won. However, if I do not have at least one of each of the four different symbols when I reach 10 numbers I will stop as this represents Grace only having enough money to buy ice blocks for a maximum of ten days. (2)
A successful trial will be if I obtain at least one of the four different symbols within 10 numbers e.g. 8, 10, 6, 6, 3. An unsuccessful trial will be after 10 numbers I still do not have at least one of each of the symbols e.g. 3, 4, 2, 4, 8, 4, 6, 9, 3, 1 so therefore a free movie ticket cannot be won.
I will repeat this 20 times to represent the 20 trials. (3)

Trial	Random numbers	Phone won/success	Number of days
1	10, 9, 6, 2	✓	4
2	1, 2, 9, 9, 2, 8, 5, 8, 6, 5	×	10
3	10, 8, 9, 1, 9, 9, 1, 2, 3, 3	×	10
4	10, 4, 6, 6, 5, 10, 9	✓	7
5	3, 4, 5, 5, 4, 6, 8, 4, 6, 6	×	10
6	8, 10, 7, 2	✓	4
7	9, 4, 3, 6, 10	✓	5
8	8, 5, 6, 6, 2, 5, 6, 8, 6, 5	×	10
9	7, 3, 2, 9, 5, 1, 1, 1, 7, 6	×	10
10	8, 5, 9, 2, 8, 5, 1, 2, 4, 4	×	10
11	2, 1, 5, 4, 4, 2, 7, 7, 5, 3	×	10
12	2, 7, 8, 5, 6, 2, 9, 6, 10	✓	9
13	2, 5, 3, 8, 7, 10	✓	6
14	3, 3, 5, 8, 8, 4, 6, 9, 2, 3	×	10
15	3, 8, 6, 5, 3, 1, 5, 5, 4, 7	×	10
16	2, 10, 5, 6, 1, 1, 8	✓	7
17	9, 8, 1, 9, 1, 9, 10, 8, 1, 10	×	10
18	5, 3, 5, 9, 1, 7, 10	✓	7
19	7, 8, 3, 6, 10	✓	5
20	1, 10, 4, 10, 1, 6, 6, 2, 1, 6	×	10

(4)

My results and calculations are only based on my simulation. Results should be treated with caution as another simulation could give a different result.

Mean number of days Grace buys ice blocks for = $\frac{\text{total number of days}}{\text{total number of trials}} = \frac{164}{20} = 8.2$ days (5)

Probability of winning a movie ticket = $\frac{\text{Number of successes}}{\text{Number of trials}} = \frac{9}{20} = 0.45$

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Based on my simulation I estimated that the mean number of days Grace will buy ice blocks for is 8.2 days and her probability of winning a free movie ticket is 0.45. However this result may vary as the numbers generated are random therefore if this trial was repeated the numbers generated could be different therefore they may well generate a different mean instead of 8.2 days.

6

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	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to investigate a situation involving elements of chance using a simulation. This involves showing evidence of using each component of the simulation process. This student's evidence is a response to the TKI assessment resource 'Fruity Freezes'. The student has designed a simulation for a given situation by identifying the tool to be used (1), defining a trial and deciding the number of trials (2) and determining data recording methods (3). They have also carried out the simulation and recorded the outcomes (4), selected and used appropriate displays and measures (5) and communicated findings in a conclusion (6). For a more secure Achieved, the student could strengthen the findings by more clearly communicating the probability of winning a movie ticket. The student could also calculate the mean number of days Grace will buy ice blocks.

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Student 5: Low Achieved

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I will use the random number function on my calculator to generate numbers from 1 – 10. Each number represents a different symbol.
Numbers 1 – 4 represent an apple, numbers 5 – 7 represent a pineapple, numbers 8 – 9 represent a grape and number 10 represents a strawberry. (1)

One trial consists of generating random numbers until all four are obtained or until a maximum of 10 random numbers are generated. A successful trial is when all four symbols are obtained within the 10 number maximum i.e. (3, 8, 1, 5, 9, 10) is a successful outcome. (2)

I will repeat this process 20 times. I will also record the number of days it took for each trial. This I will use to find the mean number of days she will buy ice blocks for.

1 – 4 = apple 5 – 7 = pineapple 8 – 9 = grape 10 = strawberry

Trial	Results	No of days	All symbols
1	2, 7, 3, 1, 5, 6, 2, 1, 3, 3	10	×
2	3, 8, 9, 6, 1, 8, 8, 3, 10	9	✓
3	1, 10, 9, 6	4	✓
4	3, 6, 9, 19	4	✓
5	5, 9, 5, 4, 2, 8, 6, 5, 5, 2	10	×
6	1, 4, 3, 8, 2, 10, 4, 7	8	✓
7	2, 7, 7, 7, 8, 5, 10	7	✓
8	5, 8, 4, 1, 3, 9, 1, 4, 8, 4	10	×
9	9, 2, 7, 4, 10	5	✓
10	5, 1, 8, 5, 6, 10	6	✓
11	5, 5, 8, 4, 4, 5, 1, 6, 1, 5	10	×
12	4, 4, 3, 2, 6, 5, 4, 10, 7, 9	10	✓
13	4, 1, 9, 8, 5, 6, 6, 5, 10	9	✓
14	4, 6, 9, 10	4	✓
15	2, 4, 9, 2, 1, 1, 4, 8, 8, 6	10	×
16	2, 8, 8, 1, 7, 1, 7, 2, 8, 8	10	×
17	9, 2, 3, 7, 3, 2, 9, 3, 4, 2	10	×
18	4, 1, 3, 8, 10, 3, 2, 2, 3, 6	10	✓
19	1, 4, 4, 6, 9, 7, 6, 6, 6, 10	10	✓
20	9, 8, 4, 9, 6, 4, 7, 1, 9, 1	10	×
Total =		166	12

(3) (4) (5)

Grace chances of winning a free movie ticket are 12 times in 20. (6)

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	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to investigate a situation involving elements of chance using a simulation. This involves showing evidence of using each component of the simulation process. This student's evidence is a response to the TKI assessment resource 'Fruity Freezes'. The student has defined a trial and decided the number of trials (1), determined data recording methods (2), carried out the simulation and recorded the outcomes (3) and selected and used appropriate displays and measures (4). To reach Achieved, the student would need to identify the tool used in the simulation and clearly communicate the findings in a conclusion.

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Student 6: High Not Achieved

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Tool:
Grace has only enough money to buy them for a maximum of ten days.

Apple: 40% (1, 2, 3, 4)
Pineapple: 30% (5, 6, 7) 4 symbols ①
Grape: 20% (8, 9)
Strawberry: 10% (10)

I will do the simulation until I get at least one of each or reach 10 times.

Trial:

1	2, 2, 1, 9, 6, 10 ✓ 6 times	✓	
2	7, 10, 6, 2, 4, 6, 5, 8 ✓ 8 times	✓	
3	9, 3, 4, 2, 7, 3, 2, 6, 8, 6	×	10 times
4	4, 5, 7, 2, 4, 7, 3, 9, 6, 4	×	〃
5	5, 6, 6, 5, 1, 1, 7, 7, 7, 2	×	〃
6	3, 2, 1, 3, 9, 8, 10, 1, 1, 2	×	〃
7	7, 3, 5, 8, 9, 6, 9, 8, 5, 8	×	〃
8	3, 5, 1, 3, 6, 9, 8, 1, 10	✓	9 times
9	6, 5, 6, 1, 8, 9, 10	✓	7 times
10	1, 4, 10, 9, 9, 9, 8, 9, 4, 10, 2	×	10 times
11	10, 8, 9, 6, 1	✓	5 times
12	1, 10, 4, 9, 9, 6	✓	6 times
13	8, 7, 2, 7, 2, 10	✓	6 times
14	3, 5, 3, 8, 9, 5, 10	✓	7 times
15	4, 9, 9, 6, 4, 6, 2, 8, 8, 7	×	10 times
16	8, 9, 7, 4, 8, 7, 5, 3, 6, 10	✓	10 times
17	9, 10, 6, 4, 9	✓	5 times
18	9, 6, 3, 6, 3, 10	✓	6 times
19	9, 2, 7, 7, 10	✓	5 times
20	1, 5, 10, 4	✓	4 times

② ③

$\frac{154}{20} = 7.7$ ④

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