Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 2

This exemplar supports assessment against:

Achievement Standard 91269

Apply systems of equations in solving problems

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment from 2014

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to apply systems of equations, using extended abstract thinking, in solving problems.

This involves one or more of: devising a strategy to investigate or solve a problem, identifying relevant concepts in context, developing a chain of logical reasoning or proof, or forming a generalisation, and also using correct mathematical statements, or communicating mathematical insight.

This evidence is a student's response to the TKI task 'Logo Design'.

This student has devised a strategy to investigate a problem by finding the coordinates of the white dot (1) and the black dot (2). The student has also investigated the grey line (3) and use the discriminant to investigate when it is a tangent (4).

For a more secure Excellence, the student could communicate more clearly the reasons for the discriminant being zero. |

$$x^2 - 6x + y^2 = 0$$

$x^2 - 6x + 9 + y^2 = 9$

$(x-3)^2 + y^2 = 9$

centre at (3, 0) & radius 3.
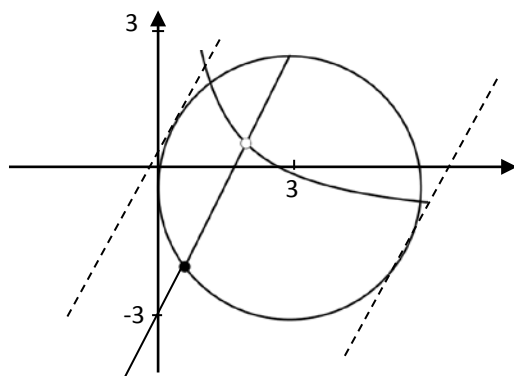
White Dot

The white dot is where $y = 2x - 3$ and

$x(y+1) = 4$ meet.

$x(2x - 3 + 1) = 4$

$x(2x - 2) = 4$

$2x^2 - 2x = 4$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad or \quad x = -1$

(2, 1)　　　　　　　　　　　　①

Black Dot

The black dot is where $(x-3)^2 + y^2 = 9$ and

$y = 2x - 3$ meet.

$(x-3)^2 + (2x-3)^2 = 9$

$x^2 - 6x + 9 + 4x^2 - 12x + 9 = 9$

$5x^2 - 18x + 9 = 0$

$x = 0.6 \ or \ x = 3$

$x = 0.6$

so $y = 2(0.6) - 3$

$y = -1.8$

black dot at (0.6, − 1.8)　　　②

Grey Line

parallel to $y = 2x - 3$

Try $y = 2x - 6$

$x(2x - 5) = 4$

$2x^2 - 5x = 4$

$x = -0.64$ calculator solver

$y = -7.28$

(-0.64, -7.28)　　　　　　　　③

Tangent

$y = 2x + c$

so

$x^2 - 6x + (2x + c)^2 = 0$

$x^2 - 6x + (2x + c)(2x + c) = 0$

$x^2 - 6x + 4x^2 + 4cx + c^2 = 0$

$5x^2 - 6x + 4cx + c^2 = 0$

$(-6 + 4c)^2 - 4 \times 5 \times c^2 = 0$

$(-6 + 4c)(-6 + 4c) - 20c^2 = 0$

$36 - 48c + 16c^2 - 20c^2 = 0$

$36 - 48c - 4c^2 = 0$

$9 - 12c - c^2 = 0$

$c^2 + 12c - 9 = 0$

$c^2 + 12c + 36 = 9 + 36$

$(c + 6)^2 = 45$

$c + 6 = \pm 6.71$　　　　　　　④

$c = -12.71 \quad or \quad 0.71$

tangent 1 $y = 2x + 0.71$

tangent 2 $y = 2x - 12.71$

	Grade Boundary: High Merit
2.	For Merit, the student needs to apply systems of equations, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts and terms, forming and using a model, and also relating findings to a context or communicating thinking using appropriate mathematical statements. This evidence is a student's response to the TKI task 'Logo Design'. This student has connected different concepts or representations by solving simultaneous equations to find the co-ordinates of the white dot (1), the black dot (2) and the dark grey dot (3). Appropriate mathematical statements have been used throughout the task. This student has investigated how the constant of the equation can be changed by guess and check (4). To reach Excellence the student could use simultaneous equations and the discriminant to investigate how the constant of the equation can be changed to solve the final part of the problem.
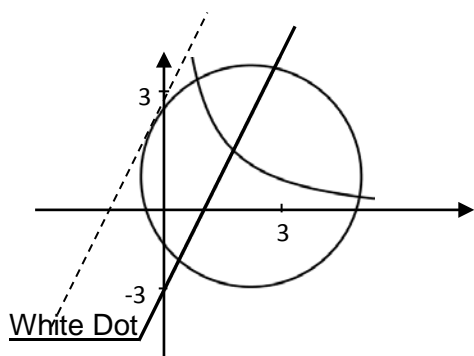
White Dot

Two lines cross so

$$y = 2x - 3$$
$$x(y + 1) = 4$$

$$x(2x - 3 + 1) = 4$$

$$x(2x - 2) = 4$$

$$2x^2 - 2x = 4$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \quad or \quad x = -1$$

White dot at (2, 1)

(1)

Black Dot

$$x^2 - 6x + y^2 = 0 \quad and \quad y = 2x - 3$$
$$x^2 - 6x + (2x - 3)^2 = 0$$
$$x^2 - 6x + 4x^2 - 12x + 9 = 0$$
$$5x^2 - 18x + 9 = 0$$
Using the calculator solver mode

$$x = 0.6 \; or \; x = 3$$

(2)

so for the black dot $x = 0.6$ and $y = -1.8$

Grey Line

parallel to $y = 2x - 3$

$$y = 2x + 6$$

$$x(2x + 7) = 4$$

$$2x^2 + 7x - 4 = 0$$

$$x = 0.5 \text{ calculator}$$

solver

(3)

$$y = 1$$

(0.5, 1)

Tangent

Try $c = 2$

so $y = 2x + 2$

$$x^2 - 6x + (2x + 2)(2x + 2) = 0$$

$$x^2 - 6x + 4x^2 + 4x + 4x + 4 = 0$$

$$5x^2 + 2x + 4 = 0$$

Using the calculator solver mode – no answers

Try $c = 1$

so $y = 2x + 1$

$$x^2 - 6x + (2x + 1)(2x + 1) = 0$$

$$x^2 - 6x + 4x^2 + 2x + 2x + 1 = 0$$

$$5x^2 - 2x + 1 = 0$$

Using the calculator solver mode – no answers

Try $c = 0$

so $y = 2x$

$$x^2 - 6x + 4x^2 = 0$$

$$5x^2 - 6x = 0$$

(4)

Using the calculator solver mode

$$x = 0 \quad or \quad 1.2$$

c must be between 0 and 1.2

so $c = 0.6$

so tangent is $y = 2x + 0.6$

	Grade Boundary: Low Merit
3.	For Merit, the student needs to apply systems of equations, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts and terms, forming and using a model, and also relating findings to a context or communicating thinking using appropriate mathematical statements. This evidence is a student's response to the TKI task 'Logo Design'. This student has connected different concepts and representations by using simultaneous equations to find the co-ordinates of the white dot (1) and the dark grey dot (2). Appropriate mathematical statements have been used throughout the task. This student has made an error in finding the coordinates of the black dot (3). For a more secure Merit, the student could find the correct co-ordinates of the black dot.

White Dot when

$y = 2x - 3$ and $x(y+1) = 4$

$x(2x - 3 + 1) = 4$

$x(2x - 2) = 4$

$2x^2 - 2x = 4$

$x^2 - x - 2 = 0$

$x = 2$

(2, 1) is the white dot   ①

To find Black dot

$y = 2x - 3$

$x^2 - 6x + y^2 = 0$

$x^2 - 6x + (2x - 3)^2 = 4$

$x^2 - 6x + 4x^2 - 9 = 4$

$5x^2 - 6x - 5 = 0$ ③

$x = 2.32$

$y = 1.64$

The Grey line

If the grey line is parallel to the black line but above it then it will cut only the white line.
so $y = 2x - 1$ could be an equation for the grey line.

This will cut $x(y+1) = 4$ when

$x(2x - 1 + 1) = 4$

$2x^2 = 4$

$x = 1.41$

$y = 2 \times 1.41 - 3 = -0.18$

Grey dot is at (1.41, -0.18)   ②

	Grade Boundary: High Achieved
4.	For Achieved, the student needs to apply systems of equations in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms, and communicating using appropriate representations. This evidence is a student's response to the TKI task 'Logo Design'. This student has demonstrated knowledge of concepts and terms by linking the equations to the graphs (1), and interpreted the solution of a system of equations in context by finding the co-ordinates of the white dot (2). This student has made an error in finding the coordinates of the black dot. To reach Merit, the student would need to find the correct coordinates of the black dot and/or the dark grey dot.

White Dot

$$y = 2x - 3 \\ x(y+1) = 4$$

$$x(2x - 3 + 1) = 4$$

$$x(2x - 2) = 4$$

$$2x^2 - 2x = 4$$

$$x^2 - x - 2 = 0$$

using graphics Calculator

$$x = 2$$
$$y = 1$$

so white dot is at (2, 1)

Black Dot

$$x^2 - 6x + y^2 = 0 \\ y = 2x - 3$$

$$x^2 - 6x + (2x - 3)^2 = 0$$

$$x^2 - 6x + 2x^2 - 6x + 9 = 0$$

$$3x^2 - 12x + 9 = 0$$

using graphics Calculator

$$x = 1$$

$$y = 2 \times 1 + 3 = 5$$

Black dot is at (1, 5)

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to apply systems of equations in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms, and communicating using appropriate representations. This evidence is a student's response to the TKI task 'Logo Design'. This student has demonstrated knowledge of concepts and terms by linking the equations to the graphs (1), and interpreted the solution of a system of equations in context by finding the coordinates of the white dot (2). For a more secure Achieved, the student could make some progress towards finding the coordinates of the black and/or grey dots.

The white dot is where the lines

$x(y+1) = 4$ (i)   and $y = 2x - 3$ (ii) cross.

substitute (ii) into (i)

$x(2x - 3 + 1) = 4$

$x(2x - 2) = 4$

$2x^2 - 2x = 4$

using graphics Calculator

$x = 2$

$y = -2 + 3 = 1$

so the white dot is at (2, 1)

①

②

	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to apply systems of equations in solving problems.
	This involves selecting and using methods, demonstrating knowledge of concepts and terms, and communicating using appropriate representations.
	This evidence is a student's response to the TKI task 'Logo Design'.
	This student has connected different representations by linking the equations to the graphs (1).
	To reach Achieved the student could solve the quadratic equation and interpret the solutions in context.

Black dot at the intersection of

$$x^2 - 6x + y^2 = 0$$
$$y = 2x - 3$$

$$x^2 - 6x + (2x-3)^2 = 0$$

①

$$x^2 - 6x + 4x^2 - 12x + 9 = 0$$

$$5x^2 - 18x + 9 = 0$$

$$(5x\quad)(x\quad) = 0$$