National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 3

This exemplar supports assessment against:

Achievement Standard 91573

Apply the geometry of conic sections in solving problems

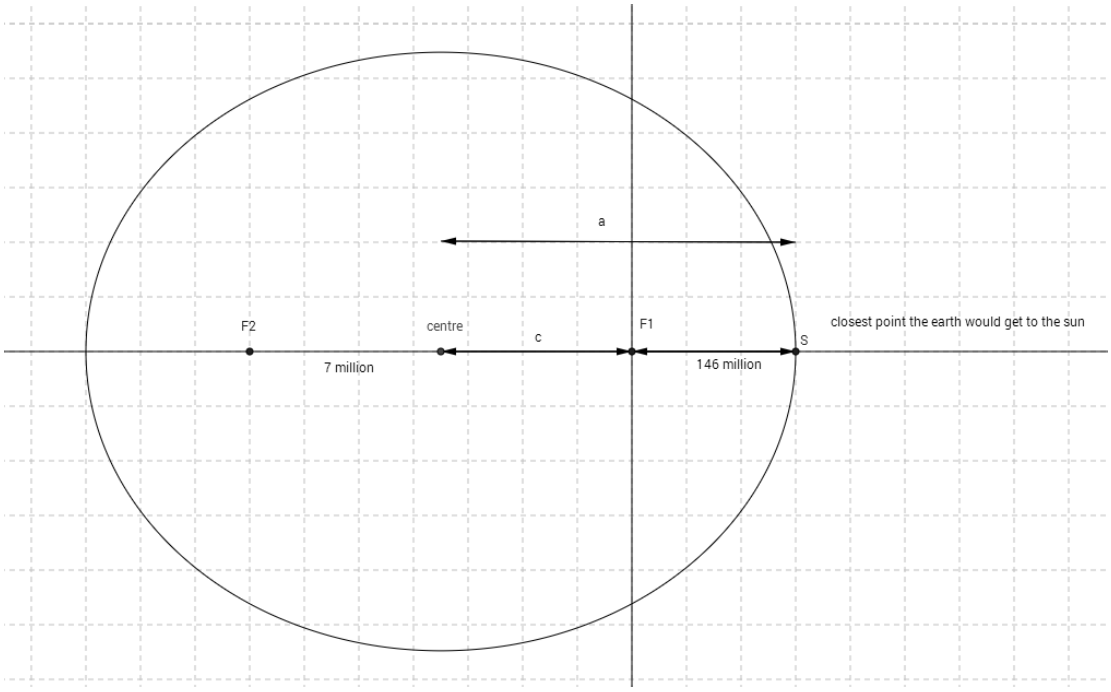
An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to apply the geometry of conic sections, using extended abstract thinking, in solving problems. This involves one or more of: devising a strategy to investigate or solve a problem, identifying relevant concepts in context, developing a chain of logical reasoning or proof, or forming a generalisation, and also using correct mathematical statements or communicating mathematical insight. This evidence is from a student's response to the TKI task 'Orbits'. This student has devised a strategy to determine the distance of the asteroid from the sun when it crosses the orbits of Earth (1), Mars (2) and the comet (3). Correct mathematical statements have been used in the response. For a more secure Excellence, the student could have presented the findings in a written report and interpreted the distances more clearly in context, for example, by relating the value $y = 149.5 \times 10^6$km to the distance of the asteroid from the sun when the asteroid crosses the orbit of Earth.

Earth



F_1 = (0,0) F_2 = $(-7\times10^6,0)$ The foci are at (c,0) and (-c,0)

$2c = 7\times10^6$ $c = 3.5\times10^6\,km$

c = distance from centre to focus, *a* = distance from centre to vertex/x-intercept

$c+146\times10^6 = a = 3.5\times10^6+146\times10^6 = 149.5\times10^6\,km(3sf)$

$b^2 = a^2 - c^2$

$b = \sqrt{(a^2-c^2)} = \sqrt{[(149.5\times10^6)^2-(3.5\times10^6)^2]} = 149\times10^6\,km(3sf)$

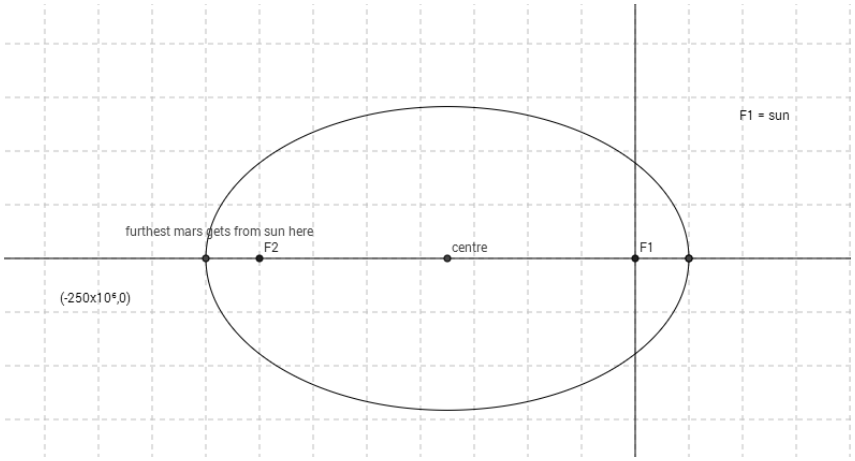
The centre is $3.5\times10^6\,km$ from F_1 so the centre is at $(-3.5\times10^6,0)$

$$\frac{(x+3.5\times10^6)^2}{(149.5\times10^6)^2}+\frac{y^2}{(149\times10^6)^2}=1$$

Intercept = (0,*y*) so y intercept is when *x* = 0 y = 149.5 x 10^6km

1

Mars



2*a* is total horizontal distance, *a* is distance from centre to vertex/x intercept
c is distance from centre to focus

$2a = 460\times10^6$ $\quad a = 230\times10^6$

$250\times10^6 = a + c$ $\quad c = 250\times10^6 - 230\times10^6 = 20\times10^6$

$b = \sqrt{(a^2 - c^2)} = 229\times10^6\, km(3sf)$

Centre must be 20×10^6 km from F_1 which means centre is at (-20×10^6,0)

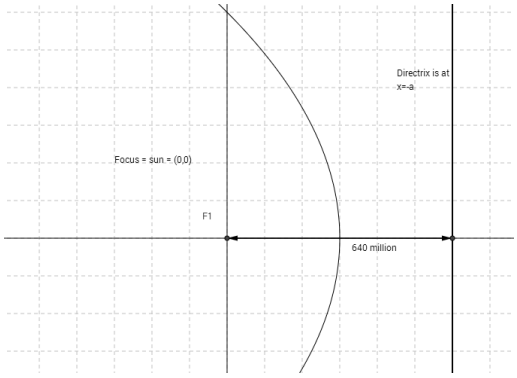
Equation: $\frac{(x+20\times10^6)^2}{(230\times10^6)^2} + \frac{y^2}{(229\times10^6)^2} = 1$

Asteroid crosses path of Mars at point (0,*y*) ie when *x* = 0 and *y* = ?

When x = 0 y = $228\times10^6\, km(3sf)$

2

Comet



$2a = 640\times10^6$ $\quad a = 320\times10^6$

Shift may be 320×10^6 to the right? In which case

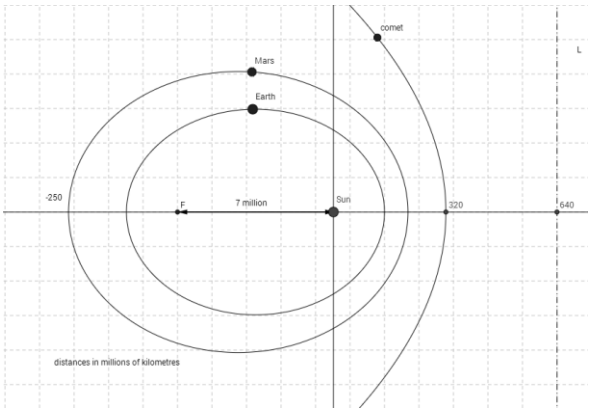
$y^2 = -1280\times10^6(x - 320\times10^6)$

$y^2 = -1280\times10^6(0 - 320\times10^6)$

$y = \sqrt{4.096\times10^{17}} = 640000000km$

3

	Grade Boundary: High Merit
2.	For Merit, the student needs to apply the geometry of conic sections, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, or forming and using a model, and also relating findings to a context or communicating thinking using appropriate mathematical statements. This evidence is from a student's response to the TKI task 'Orbits'. This student has formed a model for the orbit of the comet (1) and for the orbit of Earth (2). The models for the orbits of the comet and Earth have been used to relate the findings to a context by finding the distances from the sun when the asteroid crosses their orbits (3). Appropriate mathematical statements have been used in the response. To reach Excellence, the student could form and use a model for the orbit of Mars.



equation for comet

centre = (320,0) focus = (0,0) focal length = 320

$-y^2 = 4a(x-320)$ $4a = 4\times320 = 1280$

$y^2 = -1280(x-320)$

Position of asteroid when it crosses the orbit of the comet is when x = 0

$y^2 = -1280(x-320) = -1280(0-320) = 409600$

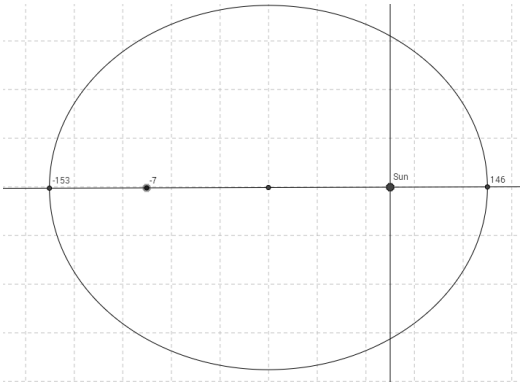
$y = 640$

Asteroid is 640 million km from the sun when it crosses the orbit of the comet.

1

3

equation of earth



foci (0,0) and (-7,0) x-intercepts (146,0) and (-153,0) centre (-3.5,0)

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ a = 149.9 b = ? c = 3.5

$b = \sqrt{(a^2-c^2} = \sqrt{149.5^2-3.5^2} = 149.46$

$\frac{(x+3.5)^2}{149^2}+\frac{y^2}{149.46^2}=1$

2

position of asteroid when it crosses the orbit of the earth

when x = 0

$\frac{(0+3.5)^2}{149^2}+\frac{y^2}{149.46^2}=1$ $5.518\times10^{-4}+\frac{y^2}{149.46^2}=1$

$\frac{y^2}{149.46^2} = 0.999$ $y^2 = 22325.97$

y = 149.41876 = 149 million km

When the asteroid crosses the earth's orbit it is 149 million km from the sun.

3

	Grade Boundary: Low Merit
3.	For Merit, the student needs to apply the geometry of conic sections, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, or forming and using a model, and also relating findings to a context or communicating thinking using appropriate mathematical statements. This evidence is from a student's response to the TKI task 'Orbits'. This student has formed a model for the orbit of the comet (1) and the orbit of Earth (2). The model for the comet has been used to relate the findings to a context by finding the distance the asteroid is from the sun when it crosses this orbit (3). For a more secure Merit, the student could use the model for the orbit of Earth to find the correct distance of the asteroid from the sun when it crosses the orbit.

Equation of comet units in million kilometres

focus (0,0) focal length = 320 million km centre (320,0)

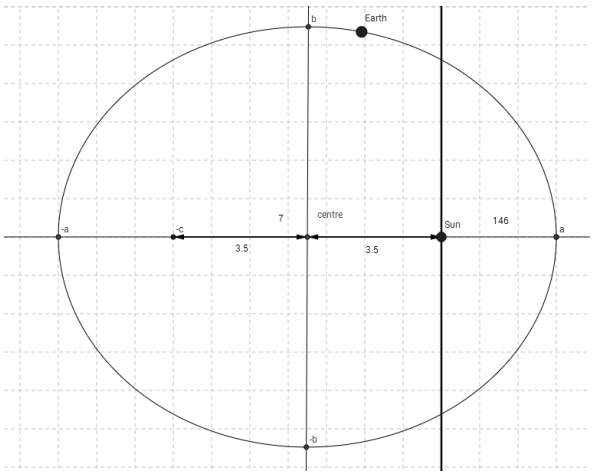
$-y^2 = 4a(x - x_1)$ $-y^2 = 4 \times 320(x - 320)$

$-y^2 = 1280(x - 320)$ $y^2 = -1280(x - 320)$

1

Equation for the earth

Foci (0,0) and (7,0) closest earth gets to sun 146 million km



a = 149.5 b = ? c = 3.5 $b^2 = a^2 - c^2$

$b^2 = 149^2 - 3.5^2 = 22188.75$ b = 148.9589 = 149 million km

$\frac{(x - x_1)^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{(x+3.5)^2}{149.5^2} + \frac{y^2}{149^2} = 1$ $\frac{(x+3.5)^2}{22350.25} + \frac{y^2}{22188.75} = 1$

2

When the asteroid crosses the orbit of the comet it is 640 million kms from the sun

x = 0 y^2 = -1280(0-320) = 409600

y = 640 million km

3

When the asteroid crosses the orbit of the earth *x* = 0

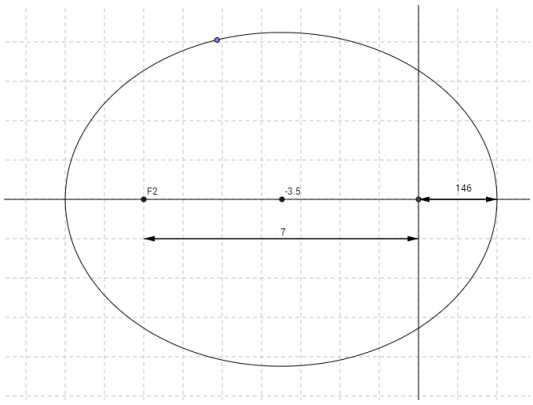
$\frac{(x+3.5)^2}{149.5^2} + \frac{y^2}{149^2} = 1$ $\frac{(0+3.5)^2}{149.5^2} + \frac{y^2}{149^2} = 1$

$5.4809 \times 10^{-4} + \frac{y^2}{149^2} = 1$ $\frac{y^2}{149^2} = -5.4809 \times 10^{-4}$

y^2 = -12.168 *y* =

	Grade Boundary: High Achieved
4.	For Achieved, the student needs to apply the geometry of conic sections in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. This evidence is from a student's response to the TKI task 'Orbits'. This student has selected and used properties of an ellipse and a parabola (1) to form the equation of Earth's orbit and the path of the comet (2). Knowledge of concepts is demonstrated, and this student has communicated using appropriate representations. To reach Merit, the student could consider the path of the asteroid, and use the equations that have been formed to find the distances from the sun to where the asteroid crosses the orbit of Earth and/or the path of the comet.

Earth



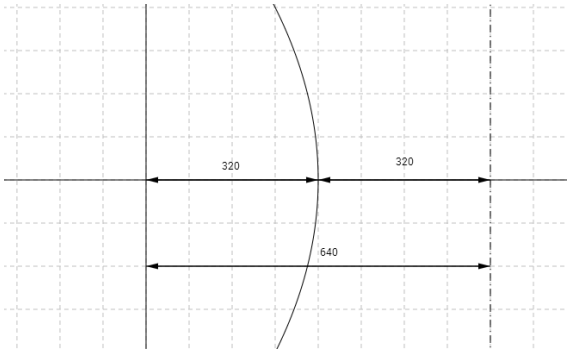
$$\frac{(x+3.5)^2}{149.5^2}+\frac{y^2}{b^2}=1$$

$$b^2=a^2-c^2 \qquad b^2=149.5^2-3.5^2$$
$$=22350.25-12.25=22338$$

$$\frac{(x+3.5)^2}{149.5^2}+\frac{y^2}{22338^2}=1$$

1

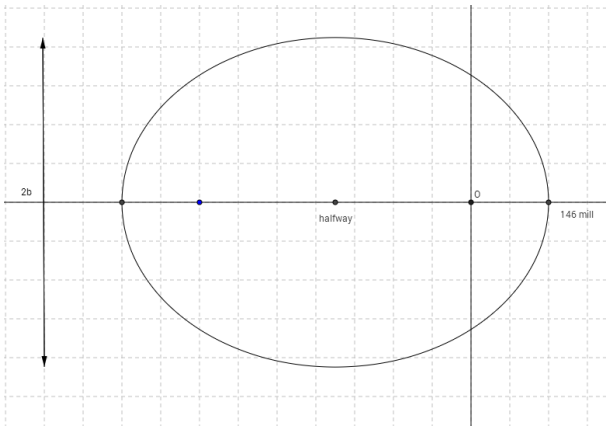
Comet



$$y^2=-4a(x-320)$$
$$a=320$$
$$y^2=-1280(x-320)$$

2

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to apply the geometry of conic sections in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. This evidence is from a student's response to the TKI task 'Orbits'. This student has selected and used the properties of a parabola to find the correct equation of the path of the comet (1). This student has selected and used the properties of an ellipse (2) in starting to find the equations of the ellipse for Earth and Mars, but the subsequent use of properties and equations are incorrect. Some knowledge of concepts is demonstrated, and this student has communicated using appropriate representations. For a more secure Achieved, the student could use the properties of an ellipse to find a correct equation for the orbit of Mars and/or Earth.



Earths orbit

a = 3500000+146000000=149500000

centre of a $7000000 \div 2$ = 3500000

equation of earth orbits

$b = ?$

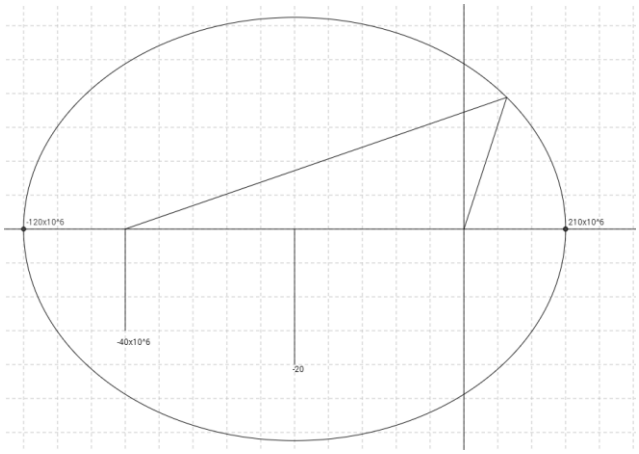
$b^2 = a^2 - c^2$

$= 149500000^2 - 146000000^2$

$b^2 = 1.03425 \times 10^{15}$

$$\frac{(x+3500000)^2}{149500000^2} + \frac{y^2}{1.03425 \times 10^{15}} = 1$$

2



Mars orbit

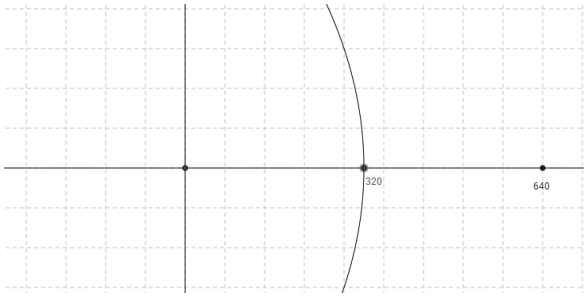
Focus: $(460-250) \times 1 \times 10^6$

$= 210 \times 10^6$ km away from end points

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$b^2 = a^2 - c^2 = 20^2 - 210 \times 10^6 = ?$

$$\frac{(x+20\times10^6)^2}{20^2} + \frac{y^2}{?} = 1$$



comets orbit

$y^2 = -4ax$

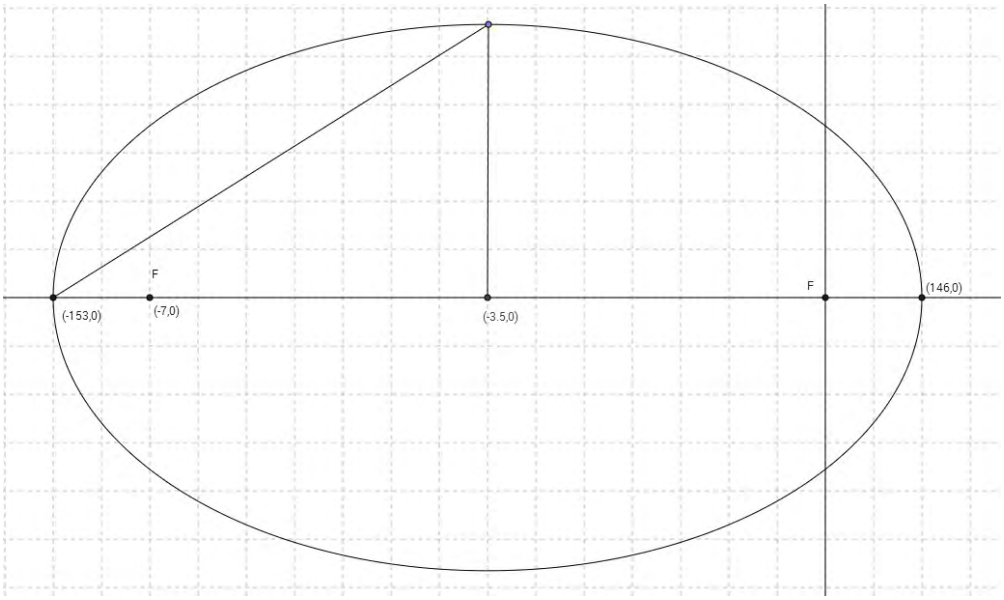
$y^2 = -4 \times 320 \times 10^6 (x - 320 \times 10^6)$

1

	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to apply the geometry of conic sections in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. This evidence is from a student's response to the TKI task 'Orbits'. This student has selected and used the properties of a parabola (1) to find the equation of the path of the comet. To reach Achieved, the student could select and use properties of an ellipse to find the correct equation of one ellipse.

Earth (x and y to million km)

$$\frac{(x-x_1)^2}{a^2}+\frac{(y-y_1)^2}{b^2}=1 \qquad \frac{(x+3.5)^2}{12.25}+\frac{(y-y_1)^2}{b^2}=1$$



Comet (x and y to million km) when (x=0,y=320)

$-y^2=4a(x-320)$

$-320^2=4a(0-320)$

$-102400=4a(-320)$

$-102400=-1280a$

$80=a$

$y^2=-320(x-320)$ 1