Exemplar for internal assessment resource Mathematics and Statistics for Achievement Standard 91574

Constraints $x=$ artichokes $y=$ tomatoes

$$
20 x+10 y \leq 1200 \quad x+y \leq 90 \quad y \geq 30 \quad x \geq 10
$$

Optimisation function
$I=25000 x+10000 y$

Graph of constraints. The unshaded region contains all the feasible points.


Income will be maximised at one of the corners of the region.
Vertex Income = 25000x+10000y
$(10,30) \quad \$ 550000$
$(10,80) \quad \$ 1050000$
$(30,60) \quad \$ 1350000$
$(45,30) \quad \$ 1425000$
To maximise the income in the current year Ted should grow 45 hectares of artichokes and 30 hectares of tomatoes.

For future income, the income for artichokes and tomatoes will be in the ratio 2:1, for example the income could be $\$ 30000$ from artichokes and $\$ 15000$ from tomatoes.
So one possible income is $I=30000 x+15000 y$

| Vertex | Income $=30000 x+15000 y$ |
| :---: | :--- |
| $(10,30)$ | $\$ 750000$ |
| $(10,80)$ | $\$ 1500000$ |
| $(30,60)$ | $\$ 1800000$ |
| $(45,30)$ | $\$ 1800000$ |

In this case growing 30 hectares of artichokes and 60 hectares of tomatoes or 45 hectares of artichokes and 30 hectares of tomatoes will both produce a maximum income of $\$ 180000$

The two points $(30,60)$ and $(45,30)$ lie on the line $20 x+10 y=1200$. This has a gradient of -2.

The future Income function $I=30000 x+15000 y$ also has a gradient of -2 and so it is parallel to the boundary line.

This means that any points on $20 x+10 y \leq 1200$ between $(30,60)$ and $(45,30)$ will all maximise the income, so there are many possible ways of maximising the income.

