National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 3

This exemplar supports assessment against:

Achievement Standard 91574

Apply linear programming methods in solving problems

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to apply linear programming methods, using extended abstract thinking, in solving problems. This involves one or more of: devising a strategy to investigate or solve a problem, identifying relevant concepts in context, developing a chain of logical reasoning, or proof, forming a generalisation and also using correct mathematical statements, or communicating mathematical insight. This evidence is a student's response to the TKI task 'Ted's tomatoes'. The student has determined the number of hectares of artichokes and tomatoes to maximise the current income (1), and identified relevant concepts in context by forming an equation for a possible future income (2). The student has identified the multiple solutions, but made a transfer error when communicating the maximum income (3). They have given an explanation for the multiple solutions (4) and used correct mathematical statements in the response. For a more secure Excellence, the student could generalise the response to the future situation further, for example by investigating a general model for the future income $I = 2kx+ky$.

Student 1: Low Excellence

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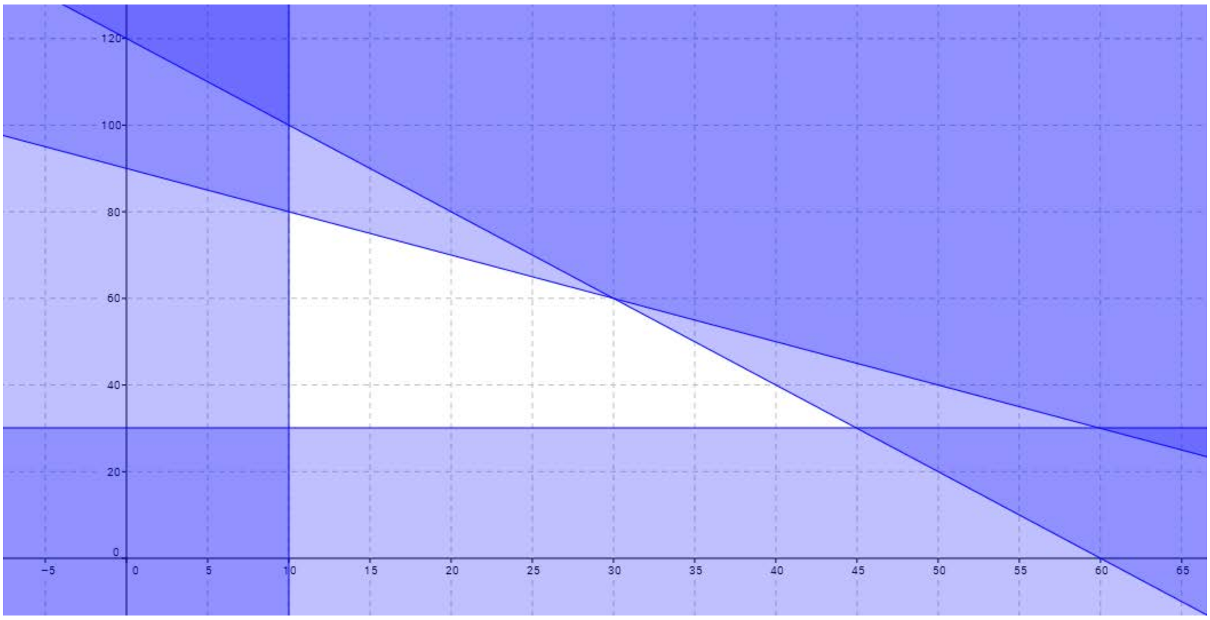
Constraints x = artichokes y = tomatoes

$20x + 10y \le 1200$ $x + y \le 90$ $y \ge 30$ $x \ge 10$

Optimisation function

$I = 25000x + 10000y$

Graph of constraints. The unshaded region contains all the feasible points.



Income will be maximised at one of the corners of the region.

Vertex	Income = 25000x+10000y
(10,30)	$550000
(10,80)	$1050000
(30,60)	$1350000
(45,30)	$1425000

To maximise the income in the current year Ted should grow 45 hectares of artichokes and 30 hectares of tomatoes. ①

For future income, the income for artichokes and tomatoes will be in the ratio 2:1, for example the income could be $30000 from artichokes and $15000 from tomatoes. ②
So one possible income is $I = 30000x + 15000y$

Vertex	Income = 30000x+15000y
(10,30)	$750000
(10,80)	$1500000
(30,60)	$1800000
(45,30)	$1800000

In this case growing 30 hectares of artichokes and 60 hectares of tomatoes or 45 hectares of artichokes and 30 hectares of tomatoes will both produce a maximum income of $180000 ③

The two points (30,60) and (45,30) lie on the line $20x + 10y = 1200$. This has a gradient of -2.

The future Income function $I = 30000x + 15000y$ also has a gradient of -2 and so it is parallel to the boundary line. ④

This means that any points on $20x + 10y \leq 1200$ between (30,60) and (45,30) will all maximise the income, so there are many possible ways of maximising the income.

	Grade Boundary: High Merit
2.	For Merit, the student needs to apply linear programming methods, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This evidence is a student's response to the TKI task 'Ted's tomatoes'. The student has connected different concepts or representations by identifying the feasible region for the inequalities (1) and identifying the number of hectares required for each vegetable to maximise the income (2). A possible income function for future years has been provided (3). The student has identified two solutions for the new income and has selected one as the optimal value for future years (4). They have also related the findings to the context. To reach Excellence, the student could recognise that there are multiple solutions for the situation in future years which are along the line AB, and relate this to the context.

Student 2: High Merit

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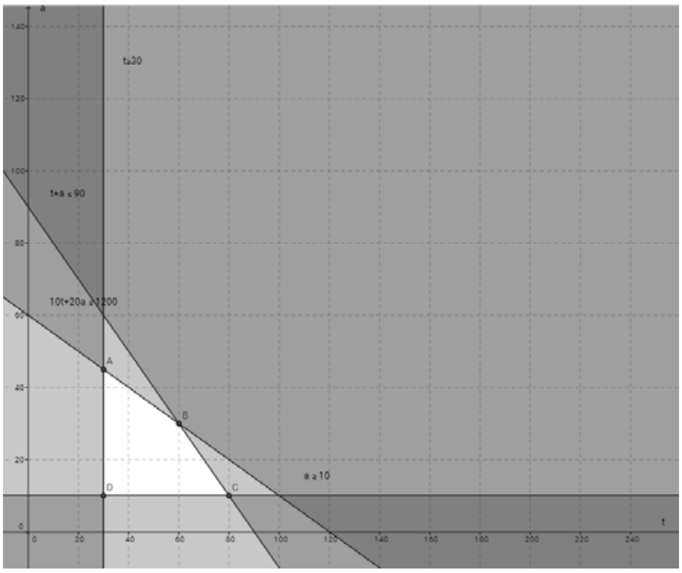
a = artichokes t = tomatoes

equations

$10t + 20a \leq 1200$ $t + a \leq 90$ $t \geq 30$ $a \geq 10$

Income equation: $10{,}000t + 25000a = I$



1

Each set of co-ordinates which are the vertices for the feasible region are put into the profit equation $I = 10{,}000t + 250000a$

Vertices	$10{,}000t + 25000a$	$10{,}000t + 20000a$
A (30,45)	1,425,000	
B (60,30)	1,350,000	
C (80,10)	1,050,000	
D (30,10)	550,000	

In the current year, Ted should plant 30 hectares of tomatoes and 45 hectares of artichokes in order to maximise his income. If he does this, his income will be $1,425,000 according to his expectation regarding how much he will receive per hectare. 2

Future payments of tomatoes: artichokes is predicted at 1:2. As the value was previously $10,000 per hectare of tomatoes and $25,000 per hectare of artichokes, the future value can be estimated at $10,000 for tomatoes and $20,000 for artichokes.
The new income equation will therefore be $I = 10{,}000t + 20000a$ 3

In future years Ted could pant either 30 hectares of tomatoes and 45 hectares of artichokes or 60 hectares of tomatoes and 30 hectares of artichokes, both options providing $1,200,000.

However, seeing as artichokes are very labour-intensive, Ted's best option would be to plant 60 hectares of tomatoes and 30 hectares of artichokes in future years. 4

	Grade Boundary: Low Merit
3.	For Merit, the student needs to apply linear programming methods, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model and also relating findings to a context, or communicating thinking using appropriate mathematical statements. The student has connected different concepts or representations by locating the feasible region for the inequalities (1) and identifying the number of rods and pillars to maximise the profit (2). They have also related the findings to the context. Modified inequalities for the increased hours for drilling and grinding have been given (3). For a more secure Merit, the student would need to investigate the effect of the increased hours on the feasible region and maximum profit.

Student 3: Low Merit

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rods r and pillars p

drilling $0.5r + 1.5p \leq 105$

grinding $r + p \leq 90$

polishing $r + 0.5p \leq 70$

$r \leq 60$

$p \leq 70$

shading out

1

Vertex	Co-ordinate	Profit (300r + 600p)
A	(0,70)	42000
B	(30,60)	45000
C	(50,40)	39000
D	(60,20)	30000
E	(60,0)	18000
F	(0,0)	0

Maximum profit is $45000, 30 rods and 60 pillars 2

Changing drilling and grinding

Drilling plus 5 $0.5r + 1.5p \leq 110$

grinding plus 5 $r + p \leq 95$ 3

polishing the same $r + 0.5p \leq 70$

	Grade Boundary: High Achieved
4.	For Achieved, the student needs to apply linear programming methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms, and communicating using appropriate representations. This evidence is a student’s response to the TKI task ‘Ted’s tomatoes’. The student has formed the linear inequalities for the problem (1) and found the feasible region (2). They have also made some progress towards optimising the solution by determining the income for each vertex of the feasible region (3). To reach Merit, the student could identify the vertex which maximises the income function, in order to make a recommendation regarding the number of hectares for each vegetable.

Student 4: High Achieved

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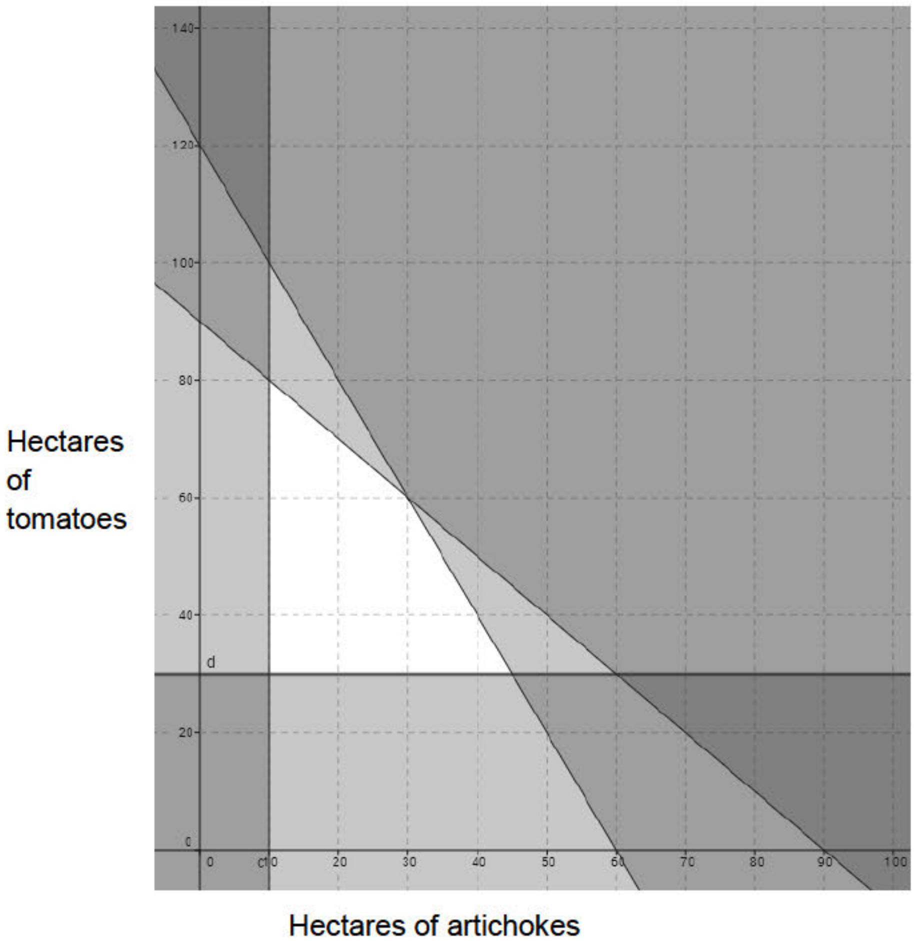
A = x
T = y

$20A + 10T \leq 1200$
$A + T \leq 90$
$T \geq 30$
$A \geq 10$
$25{,}000A + 10{,}000T$

①

(30,60)	$1,350,000
(45,30)	$1,425,000
(10,80)	$1,050,000
(10,30)	$550,000

③



②

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to apply linear programming methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms, and communicating using appropriate representations. This evidence is a student's response to the TKI task 'Ted's tomatoes'. The student has formed the linear inequalities for the problem (1) and found the feasible region (2). For a more secure Achieved, the student could indicate what each variable represents and make some progress towards finding the optimal solution.

Student 5: Low Achieved

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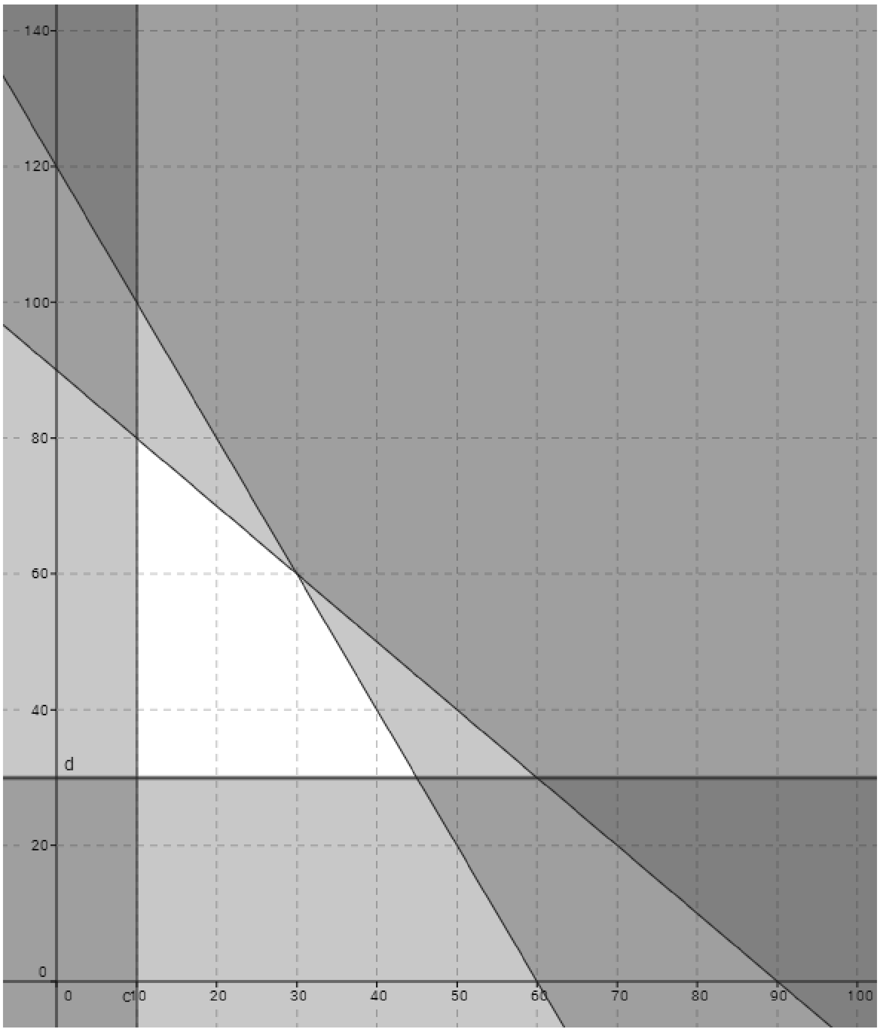
equations

$$20x+10y \le 1200$$
$$x+y \le 90$$
$$y \ge 30$$
$$x \ge 10$$
$$I = 25000x+10000y$$

1



2

Intercepts: (10,30) (10,80) (30,60) (45,30)

Intercepts	$25000x+10000y$
(10,30)	
(10,80)	
(30,60)	
(45,30)	

	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to apply linear programming methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms, and communicating using appropriate representations. This evidence is a student's response to the TKI task 'Ted's tomatoes'. The student has formed some of the linear inequalities (1) and used these to find a feasible region. To reach Achieved, the student could form the equation of the inequality for the hours of labour, and use this to find the correct feasible region for the problem.

Student 6: High Not Achieved

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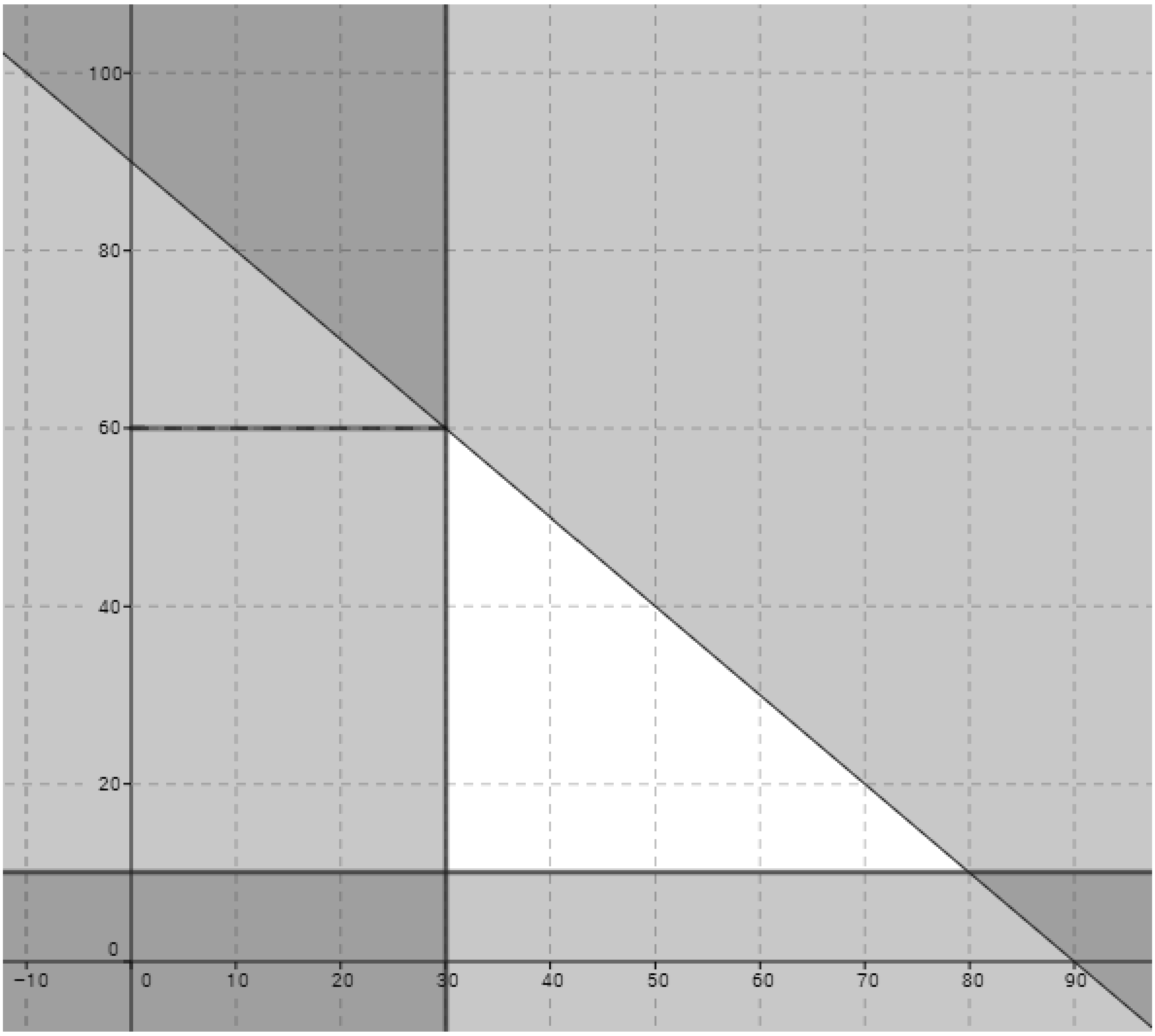
$t + a \leq 90$

$t \geq 30$

$a \geq 10$

1

Shading out



$t \geq 30$ $t + a \leq 90$

vertices	income
(30,10)	
(80,10)	
(60,30)	