National Certificate of Educational Achievement
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Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 3

This exemplar supports assessment against:

Achievement Standard 91575

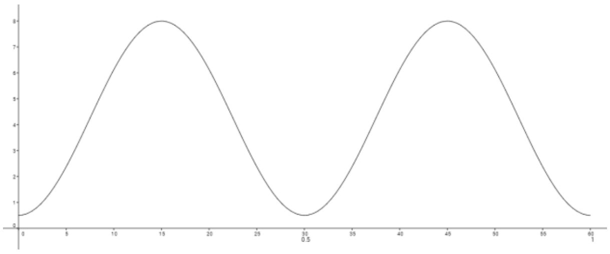
Apply trigonometric methods in solving problems

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to apply trigonometric methods, using extended abstract thinking, in solving problems. This involves one or more of: devising a strategy to investigate or solve a problem, identifying relevant concepts in context, developing a logical chain of reasoning or proof, or forming a generalisation, and also using correct mathematical statements or communicating mathematical insight. This evidence is a student's response to the TKI task 'Maths End Ferris Wheels'. This student has devised a strategy to investigate a problem by finding a model for the Kiddy-wheel (1) and a model for the Flying-high wheel (2), although the latter model is incorrect and it does not simplify the problem. The student has used these to solve the problem (3). Correct mathematical statements have been used throughout the response. For a more secure Excellence, the student could have found the correct equation for the Flying-high wheel. The student could also consider and explain that one of the intervals is not a sensible solution, and have chosen to discard it.



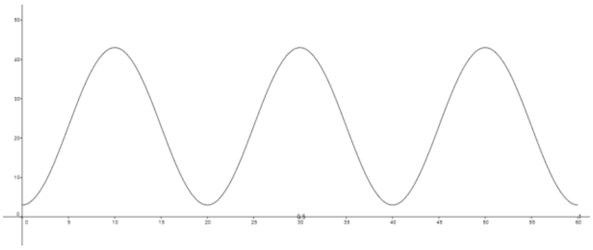
Jade – Kiddy Wheel

Height 0.5m to 8m
2 revolutions per minute
8 – 0.5 = 7.5
$7.5 \div 2 = 3.75$

$y = A\sin B(t - C) + D$ A = 3.75 D = 3.75 + 0.5 = 4.25 $B = \frac{2\pi}{30} = \frac{\pi}{15}$

So by a process of elimination

Kiddy Wheel is $h(t) = 3.75\sin\frac{\pi}{15}(t - 7.5) + 4.25$ 1

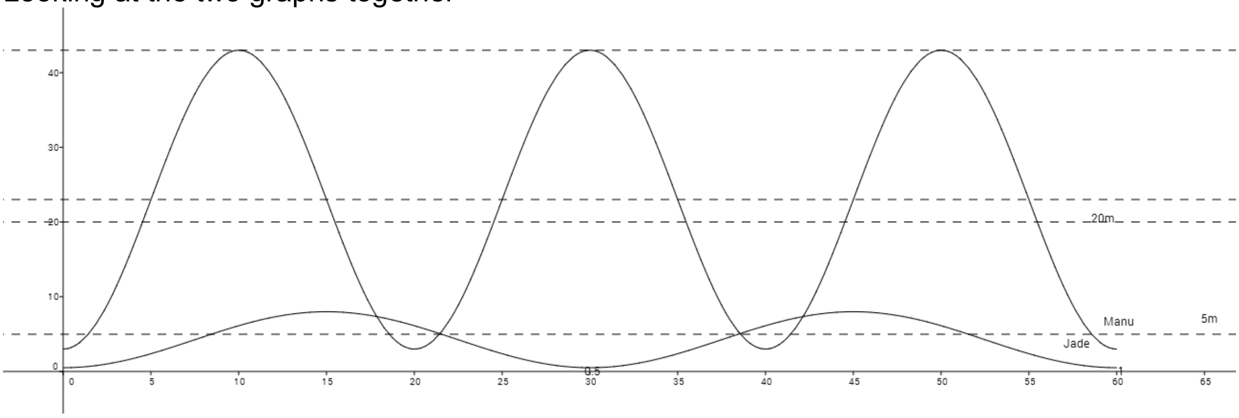


Manu – Flying - High Ferris
Height 3m to 43m
3 revolutions per minute
43 – 3 = 40
$40 \div 2 = 20$

A = 20 D = 23 $B = \frac{2\pi}{20} = \frac{\pi}{10}$ C = 5

So $h(t) = 20\sin\frac{\pi}{10}(t - 5) + 23$ 2

Looking at the two graphs together



Where can Jade see Manu

Jade ≥ 5 $3.75\sin\frac{\pi}{15}(t - 7.5) + 4.25 \geq 5$

t is between 8.461 & 21.5385 and 38.461 & 51.5385 GC

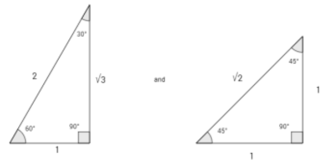
Manu – going up, ≥ 5 and ≤ 20

$5 \leq 20\sin\frac{\pi}{10}(t - 5) + 23 \leq 20$

t is between 1.44 & 4.52 and 21.44 & 24.52 and 41.44 & 44.52 GC
so the intersection of these solutions is the time when Jade can see Manu in the first 60 seconds 3
t is between 21.44 to 21.54 sec and 41.44 to 44.52 sec
this will happen every 60 seconds for the duration of the ride.

	Grade Boundary: High Merit
2.	For Merit, the student needs to apply trigonometric methods, using relational thinking in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, or forming and using a model, and also relating findings to a context or communicating thinking using appropriate mathematical statements. This evidence is a student's response to the TKI task 'Exact Values'. This student has demonstrated an understanding of concepts by finding the value of the reciprocal trigonometric functions for angles found using the compound angle formulae and double angle formulae (1). The values for tan120° and cot120° are incorrect (2). In finding general solutions, the student has found all the angles that have a sine and cosine of $1/\sqrt{2}$ and a tangent of 1, but has not clearly communicated what the angles represent (3). The statements for the reciprocal ratios are incorrect (4). To reach Excellence, the student could correct the errors, and the generalisations need to be extended to exact values other than those for the angles in the special triangles.

Special angles



I can determine that

	sin	cos	tan
30^0, $\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45^0, $\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60^0, $\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Using these triangles and

$\sin = \frac{O}{H}$ and $\cos = \frac{A}{H}$ and $\tan = \frac{O}{A}$

because $\cot\theta = \frac{1}{\tan\theta}$

cot 30^0 = $\sqrt{3}$, cot 45^0 = 1, cot 60^0 = $\frac{1}{\sqrt{3}}$ 1

because $\sec\theta = \frac{1}{\cos\theta}$

sec 30^0 = $\frac{2}{\sqrt{3}}$, sec 45^0 = $\sqrt{2}$, sec 60^0 = 2

because cosec$\theta = \frac{1}{\sin\theta}$

cosec 30^0= 2, cosec 45^0 = $\sqrt{2}$, cosec 60^0 = $\frac{2}{\sqrt{3}}$

Compound angles

$15°, \frac{\pi}{12}$

$$\sin(60-45) = \sin 60\cos 45 - \cos 60 \sin 45 = (\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}) - (\frac{1}{2}\times\frac{1}{\sqrt{2}}) = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$ 1

$\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$ therefore cosec$15° = \frac{2\sqrt{2}}{\sqrt{3}-1}$

$$\cos(60-45) = \cos 60\cos 45 + \sin 60\sin 45 = (\frac{1}{2}\times\frac{1}{\sqrt{2}}) + (\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}) = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$\cos 15° = \frac{1+\sqrt{3}}{2\sqrt{2}}$ therefore sec$15° = \frac{2\sqrt{2}}{1+\sqrt{3}}$

$$\tan(60-45) = \frac{\sqrt{3}-1}{1+(\sqrt{3}\times 1)} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ therefore cot$15° = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

Double angles

$$120°, \frac{2\pi}{3}$$

$$\sin 2A = 2\sin A\cos A$$

$$\sin 2(60°) = 2\sin 60\cos 60 = 2(\frac{\sqrt{3}}{2}\times\frac{1}{2}) = 2(\frac{\sqrt{3}}{2\sqrt{2}}) = \frac{2\sqrt{3}}{4\sqrt{2}}$$

$\sin 120° = \frac{2\sqrt{3}}{4\sqrt{2}}$ and $\mathrm{cosec}\,120° = \frac{4\sqrt{2}}{2\sqrt{3}}$ 1

$$\cos 2A = 2\cos^2 A - 1$$

$$A = 60°$$

$$\cos 2(60°) = 2(\frac{1}{2})^2 - 1 = \frac{1}{2} - 1 = -0.5$$

cos 120^0 = -0.5 and sec 120^0 = $\frac{1}{-0.5}$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$A = 60$$

$$\tan 2(60°) = \tan 120° = \frac{2\frac{\sqrt{3}}{1}}{1-(\frac{\sqrt{3}}{1})^2} = \frac{\frac{2\sqrt{3}}{2}}{1-3} = \frac{\frac{2\sqrt{3}}{2}}{-2}$$

cot 120^0 = $\frac{-2}{2\frac{\sqrt{3}}{2}}$ 2

General solutions

$\sin x = \frac{1}{\sqrt{2}}$	$x = \sin^{-1}\frac{1}{\sqrt{2}}$	x=45^0	x=n180^0 + $(-1)^n 45^0$
$\cos x = \frac{1}{\sqrt{2}}$	$x = \cos^{-1}\frac{1}{\sqrt{2}}$	x=45^0	x=2(180^0)n +/- 45^0
$\tan x = 1$	$x = \tan^{-1} 1$	x=45^0	x=n180^0+45^0

3

Therefore

cosec $x = n\frac{1}{180} + (-1)^n\frac{1}{45}$

$$\sec x = n(\frac{1}{360}) \pm \frac{1}{45°}$$

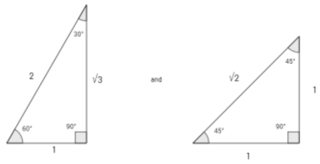
$$\cot x = n\frac{1}{180} + \frac{1}{45}$$

4

	Grade Boundary: Low Merit
3.	For Merit, the student needs to apply trigonometric methods, using relational thinking in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, or forming and using a model, and also relating findings to a context or communicating thinking using appropriate mathematical statements. This evidence is a student's response to the TKI task 'Exact Values'. This student has connected different concepts in finding the value of the reciprocal trigonometric functions for the angles found using the compound angle formulae (1). For a more secure Merit, the student could use the unit circle or graphs to investigate other angles.

Student 3: Low Merit

Intended for teacher use only



deg	rad
30^0	$\frac{\pi}{6}$
90^0	$\pi/2$
45^0	$\frac{\pi}{4}$
60^0	$\frac{\pi}{3}$

$\sin\theta = \frac{O}{H}$	$\sin 30 = \frac{1}{2}$	$\sin 45 = \frac{1}{\sqrt{2}}$	$\sin 60 = \frac{\sqrt{3}}{2}$
$\cos\theta = \frac{A}{H}$	$\cos 30 = \frac{\sqrt{3}}{2}$	$\cos 45 = \frac{1}{\sqrt{2}}$	$\cos 60 = \frac{1}{2}$
$\tan\theta = \frac{O}{A}$	$\tan 30 = \frac{1}{\sqrt{3}}$	$\tan 45 = 1$	$\tan 60 = \sqrt{3}$
$\operatorname{cosec}\theta = \frac{H}{O}$	$\operatorname{cosec} 30 = \frac{2}{1} = 2$	$\operatorname{cosec} 45 = \frac{\sqrt{2}}{1} = \sqrt{2}$	$\operatorname{cosec} 60 = \frac{2}{\sqrt{3}}$
$\sec\theta = \frac{H}{A}$	$\sec 30 = \frac{2}{\sqrt{3}}$	$\sec 45 = \frac{\sqrt{2}}{1} = \sqrt{2}$	$\sec 60 = \frac{2}{1} = 2$
$\cot\theta = \frac{A}{O}$	$\cot 30 = \frac{\sqrt{3}}{1} = \sqrt{3}$	$\cot 45 = 1$	$\cot 60 = \frac{1}{\sqrt{3}}$

$$\sin 75 = \sin(45+30) = \sin 45\cos 30 + \cos 45\sin 30 = \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\times\frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\sin 105 = \sin(45+60) = \sin 45\cos 60 + \cos 45\sin 60 = \frac{1}{\sqrt{2}}\times\frac{1}{2} + \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$$\sin 90 = \sin(30+60) = \sin 30\cos 60 + \cos 30\sin 60 = \frac{1}{2}\times\frac{1}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$$

$$\cos 75 = \cos(45+30) = \cos 45\cos 30 - \sin 45\sin 30 = \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\times\frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\cos 105 = \cos(45+60) = \cos 45\cos 60 - \sin 45\sin 60 = \frac{1}{\sqrt{2}}\times\frac{1}{2} - \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} = \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

$$\cos 90 = \cos(30+60) = \cos 30\cos 60 - \sin 30\sin 60 = \frac{\sqrt{3}}{2}\times\frac{1}{2} - \frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

$$\tan 75 = \tan(45+30) = \frac{\tan 45 + \tan 30}{1 - \tan 45\tan 30} = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$$

$$\tan 105 = \tan(45+60) = \frac{\tan 45 + \tan 60}{1 - \tan 45\tan 60} = \frac{1+\sqrt{3}}{1-\sqrt{3}}$$

$$\tan 90 = \tan(30+60) = \frac{\tan 30 + \tan 60}{1 - \tan 30 \tan 60} = \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{1-1} = \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{0} = \text{unidentified}$$

$$\sin 15 = \sin(45-30) = \sin 45 \cos 30 - \cos 45 \sin 30 = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\cos 15 = \cos(45-30) = \cos 45 \cos 30 + \sin 45 \sin 30 = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\tan 15 = \tan(45-30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

1

cosec 75 = $\frac{2\sqrt{2}}{\sqrt{3}+1}$ $\sec 75 = \frac{2\sqrt{2}}{\sqrt{3}-1}$ $\cot 75 = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$

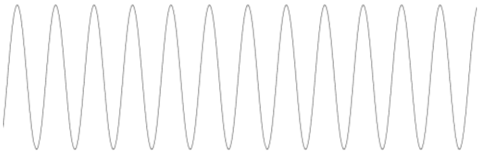
cosec 105 = $\frac{2\sqrt{2}}{1+\sqrt{3}}$ $\sec 105 = \frac{2\sqrt{2}}{1-\sqrt{3}}$ $\cot 105 = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

cosec 90 = 1 $\sec 90 = 0$ cot 90 = unidentified

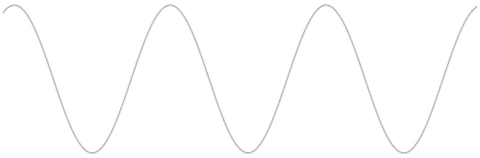
cosec 15 = $\frac{2\sqrt{2}}{\sqrt{3}-1}$ $\sec 15 = \frac{2\sqrt{2}}{1+\sqrt{3}}$ $\cot 15 = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$

	Grade Boundary: High Achieved
4.	For Achieved, the student needs to apply trigonometric methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. This evidence is a student's response to the TKI task 'Maths End Ferris Wheels'. This student has selected and used properties of trigonometric functions by identifying the correct equation of the Kiddy-wheel (1) and finding the correct equation of the Flying-high wheel (2). This student has solved trigonometric equations to find the correct intervals for both Ferris wheels (3). To reach Merit, the student could link the solutions back to the problem to find an interval when Jade can see Manu and show a contextual understanding of the problem.

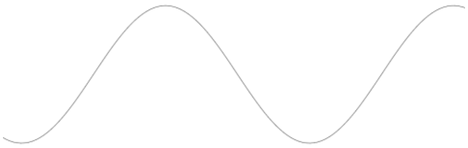
Jade



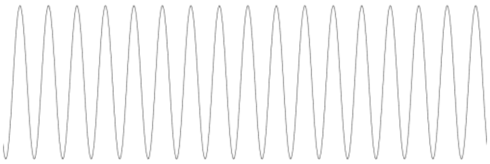
Drew on GC
$h(t) = 3.75\pi t + 4.5$
60 rotations in 60 seconds - unbelievable



$h(t) = 3.75\sin\frac{\pi}{15}(t - 7.5) + 4.25$
Drew on GC
Looks good



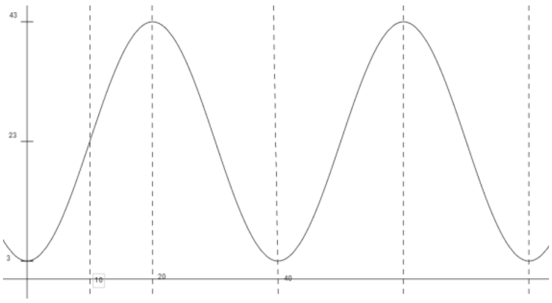
$h(t) = 4\frac{\pi}{15}(t - 7.5) + 4.25$
Drew on GC
-period not correct



$h(t) = 4\cos\frac{\pi}{30}(t - 15) + 4.25$
Drew on GC
60 rotations - unbelievable

So $\Rightarrow h(t) = 3.75\sin\frac{\pi}{15}(t - 7.5) + 4.25$ is the correct equation. 1

Manu:



$\frac{2\pi}{40}$

$h(t) = 20\sin\frac{\pi}{20}(t - 10) + 23$ 2

Jade:

$3.75\sin\frac{\pi}{15}(t - 7.5) + 4.25 \geq 5$

$8.461 \leq t \leq 21.5385$

$38.461 \leq t \leq 51.5385$

Manu:

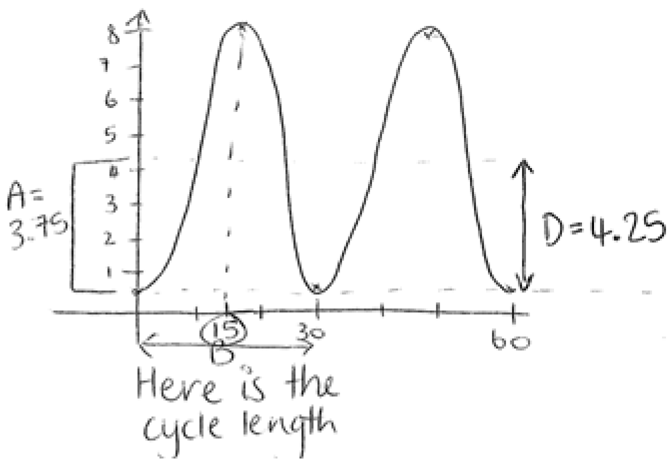
$5 \leq 20\sin\frac{\pi}{20}(t - 10) + 23 \leq 20$ going up

$2.87 \leq t \leq 9.04$

$42.87 \leq t \leq 49.04$

$82.87 \leq t \leq 89.04$ 3

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to apply trigonometric methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. This evidence is a student's response to the TKI task 'Maths End Ferris Wheels'. This student has selected and used properties of trigonometric functions in finding the correct equation of the Kiddy-wheel (1) and solved a trigonometric equation to find an interval when Jade is above 5 m (2). As above. For a more secure Achieved, the student could complete the equation for the Flying-high wheel and find an interval for Manu.



Kiddy wheel: $h = 3.75\sin\frac{\pi}{15}(t-7.5)+4.25$

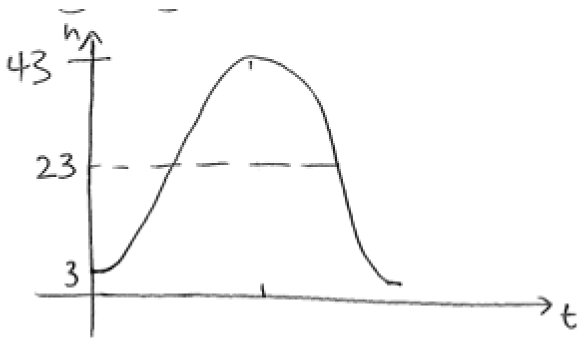
A B C?? D

1

$(43-3)\div 2$

Flying High $h = 20\sin B(t-C)+23$ moved up 23 (3+23)

B: 3 revolutions in 2 minutes?



2

Jade > 5m t between 8.46 and 21.54 (GC)
Manu < 20m and going up…?????

	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to apply trigonometric methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. This evidence is a student's response to the TKI task 'Exact Values'. This student has selected and used reciprocal trigonometric functions to find exact values for some of the reciprocal functions for the angles in the special triangles (1). The use of the double angle formula to find sin90° is correct, but this is a known angle (2). To reach Achieved, the student could determine the exact value for sin135°.

Student 6: High Not Achieved

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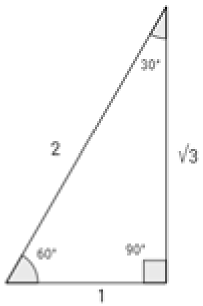
$\sin 30° = \frac{1}{2}$ $\qquad$ $\text{cosec}\,30° = 2$

sin(2x30)=2sin30xcos30



$\cos 60° = \frac{1}{2}$ $\qquad$ $\sec 60° = 2$

1

$\sin 45° = \frac{1}{\sqrt{2}}$ $\qquad$ $\text{cosec}\,45° = \sqrt{2}$

$\cos 45° = \frac{1}{\sqrt{2}}$ $\qquad$ $\sec 45° = \sqrt{2}$

$\sin(45 \times 2) = 2 \times \sin 45 \times \cos 45 = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 2 \times \frac{1}{2}$

2

$\sin 90° = 1$

$\cos 90° = 0$

$\sin(90 + 45) = \sin 90 \times \cos 45 + 0 \times \sin 45$

$\sin 135° =$