National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 3

This exemplar supports assessment against:

Achievement Standard 91580

Investigate time series data

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to investigate time series data, with statistical insight.

This involves integrating statistical and contextual knowledge throughout the statistical enquiry cycle, and may include reflecting about the process, considering other relevant variables, evaluating the adequacy of any models, or showing a deeper understanding of models.

This evidence is from a student's response to the TKI task 'Food for thought'.

The student has researched the context, used this to develop a clear purpose for the investigation (1), and has selected and used appropriate displays throughout their response.

There is evidence of statistical and contextual knowledge from appropriate research being integrated in the discussion of features through the consideration of population and takeaway sales as relevant variables (2).

The appropriateness of the model used to make forecasts has been discussed with an evaluation of the adequacy of the model and discussion of the reliability of the calculated seasonal effects (3).

This extract is from a student response which also included the student comparing actual values with predicted values and prediction intervals to confirm the robustness of the fitted model. The student also communicated findings in a conclusion to an appropriate level for the award of Excellence.

For a more secure Excellence the student would need to develop their comments about the model. For example, explaining the unusual residuals by linking their comments more closely to contextual knowledge gained from research. Interpreting the prediction intervals would also show a deeper understanding of the adequacy of the model. |

It has been widely publicised that both NZ and the rest of the world have recently experienced a global financial crisis towards the late 2000's. This has meant that most of society then had to closely consider how much money they were unnecessarily spending, where could they cut back and what was essential or not. According to a report from the Ministry for the Environment dated the end of April 2009 which is on the website http://www.mfe.govt.nz/environmental-reporting/consumption/household-expenditure/by-category/, the top three household consumption expenditure categories were food and beverages, housing, and transport. It also states that New Zealand's total household consumption expenditure continued to increase but the increases were not as large as in previous years.

I am going to investigate the amount of money spent in supermarket and grocery stores from 2000 to 2010 to see whether or not the overall trend is actually increasing and if there is anything unusual happening in the data. It would be interesting compare this time series with expenditure in other retail sectors.

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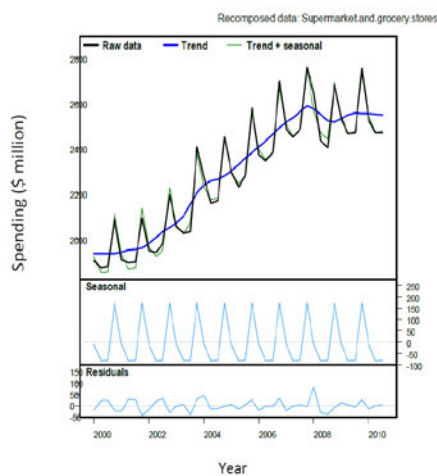
Looking at the smoothed decomposed data, there is an obvious increase in the amount of money spent at NZ supermarket and grocery stores from 2000 to the end of 2010. There is a steady increase from 2000 to towards the end of 2007 but then there is a quite a sharp fall in the trend from here till the end of 2009. The trend seems to plateau at the end of 2010 to about $2500 million per quarter. The trendline suggests that total money spent in NZ supermarket and grocery stores has increased on average from approx. $1900 million dollars in the quarter 1 2000 to $2500 million in quarter 3 2010.
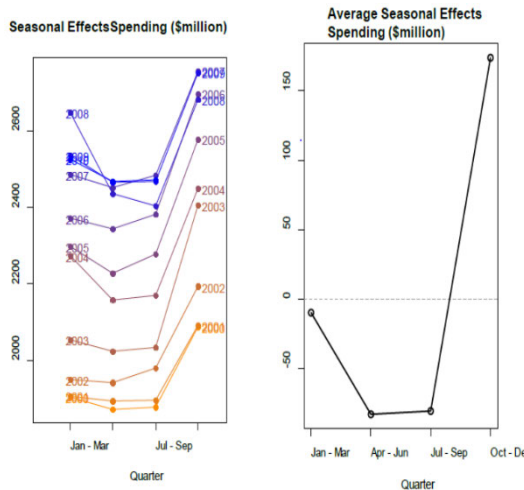

The seasonal plots show that there is an obvious seasonal pattern in the money spent on food in NZ supermarket and grocery stores with the highest sales being in Q4 each year. The average seasonal effects show that Q4 is the quarter when the most money is spent as it is about $170 million above the trend line. This of course makes sense since this quarter corresponds to the months October to December where people begin to start their Christmas shopping in preparation for Christmas dinner and fill up Christmas stockings with extra goodies. This end of this quarter is also the start of the holiday period so more money is probably spent at supermarkets and grocery stores because of social functions that occur during this time more so than the other quarters. This fits with what the superviser in the supermarket I work in told me to expect.

Quarters2 and 3 which goes from April to June for Q2 then Jul-Sept for Q3 are the time when the least amount of money is spent as they are both about $80 million below the trend line.

A possible reason for the increase in amount of money spent on food in NZ supermarket and grocery stores is that the population in NZ also increased during this time as shown by the population graph of NZ (see below). Since there were more people in NZ there were more people to go into such stores and so there was more money to spend. The appearance of the population graph and supermarket and grocery stores graph are similar at the beginning as they are both quite level. But the population graph

continues to increase and doesn't level off towards the end as the supermarket and grocery store graph does.
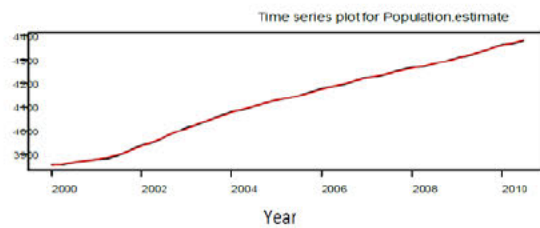

There may have also been some sort of down turn in the economy e.g global financial crisis, that could have accounted for such a sharp fall in supermarket sales towards the end of 2007. I would need to research this further to see if these two events coincided and compare to see if the same decrease in spending occurred on takeaway food at the same time.
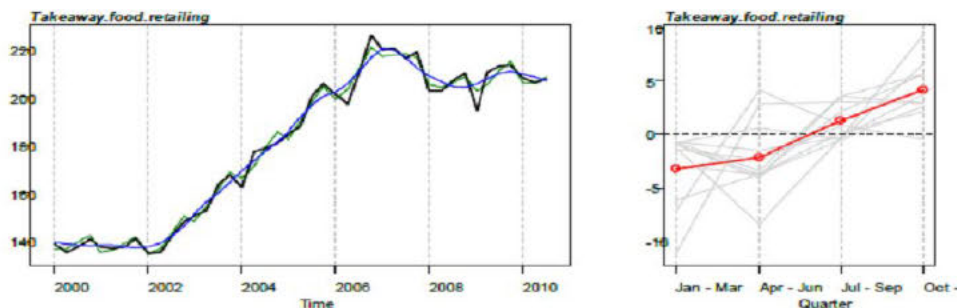
When takeaway sales are compared to supermarket and grocery sales graphs both show a slight growth They both start to increase quite sharply, with spending at supermarket and grocery stores from increasing 2001 and spending on takeaways from 2002. A decrease in spending occurred in supermarket and grocery stores from the end of 2007 start of 2008 but spending on takeaways decreased from 2007. The seasonal pattern for each quarter is much more varied. These are similar timeframes so perhaps there was a reason for this. Finance companies in NZ started to collapse and go into receivership around these times e.g. Bridgecorp, Capital and Merchant with huge losses. There was also a global financial crisis that occurred at this time which meant that people would not have as much money to spend on themselves especially luxuries such as takeaways. This could explain the trend we see in the data.

Using the holt-winters model for calculating forecasts I estimate that the total amount of money spent on food in Supermarket and Grocery in NZ in Q4 2010 is 2770 million dollars but could be between $2666 and $2867 million dollar, for Q4 2011 the total amount spent is predicted to be $2810 million dollars but could be between $2614 and $3008 million dollars.
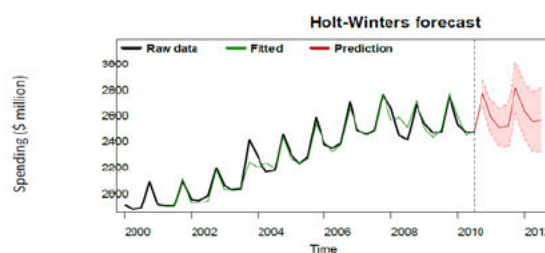
Using such a model to make forecasts assumes that the seasonal pattern of total amount of money spent in Supermarket and Grocery in NZ is reasonable constant and not too varied. The fitted model fits the data fairly well (apart from just after 2004 and 2008) and the seasonal effects have remained relatively constant but with some small variation in the first and last quarters. Therefore I am reasonably confident that my forecasts are accurate.

Looking at the residual plot most residuals are within $50 million of the trendline. The overall range of the raw data is $884 million (maximum – minimum = 2756-1872) so a residual of $50 million is quite small in proportion to $884 million. The exception would be the unusual residuals from Q1 2004 which is right on the $50 million mark and Q1 2008 which is about $75 million above the trendline. There is also a bit of a difference between the raw and fitted data for these times looking at the holt-winters plot.

	Grade Boundary: High Merit
2.	For Merit, the student needs to investigate time series data, with justification.

This involves linking components of the statistical enquiry cycle to the context, and referring to evidence such as statistics, data values, trends, or features of visual displays in support of statements made.

This evidence is from a student's response to the TKI task 'Polar ice'.

The student has researched the context and used this to develop a purpose for the investigation relevant to the variable being investigated (1). The student has selected and used appropriate displays throughout their response.

The student has investigated time series data by linking the findings to the context throughout the response. For example, when discussing features, they have used evidence to support statements in describing the trend (1), the feature of seasonal variation (2), the appropriateness of the model (3) and the reliability of the forecast (4).

In their conclusion, the student has linked the comments made back to the context and the purpose of their investigation (5).

To reach Excellence, the student would need to provide a more detailed summary in the conclusion and more clearly integrate statistical and contextual knowledge derived from research. For example, the student could link their statements to supporting or contradicting evidence found in the research. |

The two data sets given are about the sea ice located in the polar ice caps (Arctic and Antarctica). Polar ice caps simply mean a high altitude region in a planet or natural satellite that is covered in ice. There is no specific size nor composition for the mass of ice to be given the term "polar ice cap" for this case, the data that I've chosen is about the polar ice caps in Arctic which is mainly composed of sea ice. Sea ice is just ice made from sea, but in the process it loses its composites and eventually becomes fresh water. Since sea water is denser that fresh water so the freezing point for sea water is below 0 degrees Celsius.

It is widely known that the phenomenon of global warming is happening on our planet. This means that the average temperatures of the Earth's atmosphere and oceans are increasing. The effects of an increase in global temperature mean that sea levels rise and the amount of sea ice decreases as a result. According to Wikipedia, warming is expected to be strongest in the Arctic and would be associated with the continuing retreat of glaciers and sea ice. It would be interesting to investigate whether the area of sea ice is in fact decreasing in the Arctic, at what rate, any other interesting patterns or features and who or what will be affected by such decreases.

Mean Area Sea Ice (million square kilometres) for Arctic Graphs

Recomposed data: Arctic

Seasonal plot for Arctic

Estimated seasonal effects
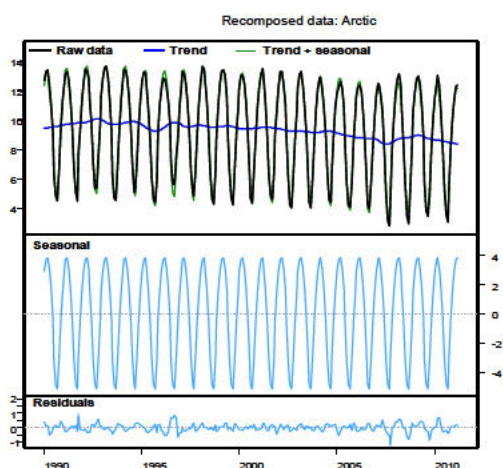
The graph shows data about the mean area of sea ice in the Arctic. Looking at the raw data the mean area of sea ice in Arctic fluctuated between 13.5 (million square kilometres) and 2.5 (million square kilometres) between 1990 to 2011. The graph of the smoothed trend shows a very gradual decreasing trend. It appears that the mean area of sea ice has decreased on average from about 9.5 million square kilometres to about 8.5 million square kilometres from January 1990 to March 2011. ①
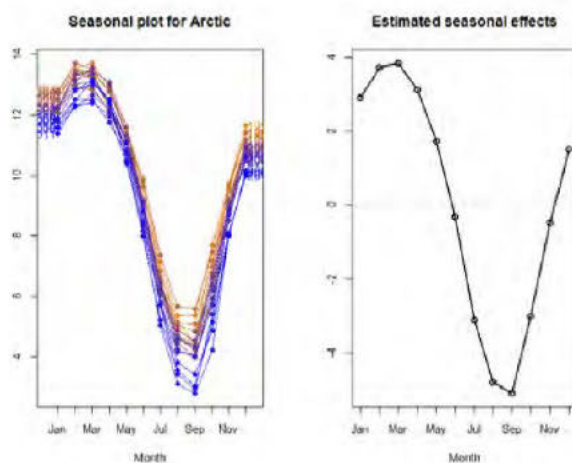
The graph of the estimated average seasonal effects shows a seasonal pattern with the maximum mean area of sea ice in the Arctic occurring at March which peaks at around 3.8 million square kilometres above the trend line and the minimum mean area of sea ice in the Arctic occurring at September which is at the lowest at about 5million square kilometres below the trend line. This pattern corresponds to the seasons experienced at the Arctic which is in the Northern Hemisphere. The temperatures during March, which near the end of Winter start of Spring, in the Arctic are relatively lower causing more ice to form and the temperatures during September, which is near the end of Summer start of Autumn, are relatively high causing more ice to melt. ②

The following predictions and prediction intervals were produced from iNZight

Month	Lower limit	Prediction	Upper limit
January 2012	10.1	11.5	12.8
February 2012	10.8	12.3	14
March 2012	10.9	12.4	13.9
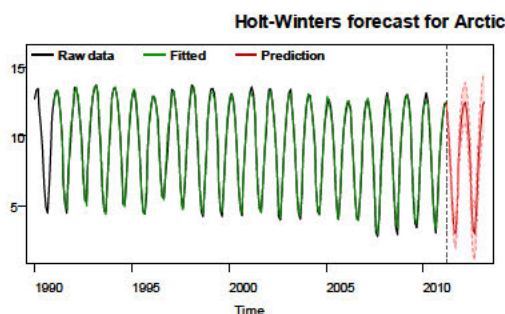
If this trend continues, my prediction for the mean area of sea ice in arctic in March 2012 is 12.4 million km^2 and is estimated to be between 10.9 and 13.9 million km^2.

The model fitted appears to be reasonably good as are the differences between the raw and fitted data in the graph above appear to be very small and my prediction is not far from the given data.   There is little variability in the seasonal effects from year to year and the seasonal pattern has stayed fairly constant over the whole time so I can rely on the predictions generated from the Holt-Winters model. However the seasonal effects for September vary much more than other quarters, this is visible in the first graph where the September seasonal effects are much more spread than for other quarters. So I would be less confident in my predictions for that quarter than for other quarters.

Looking at the graph of the residuals also confirms that the predictions should be fairly reliable as the residuals appear to be small as most are no more than 0.5 million km^2 above or below the line and have limited variability and no obvious pattern. The residual at September 2007 is a bit odd compared to the others as it is nearly 1 million km^2 below the line which is quite a bit more compared to the others, and the residuals seem to vary much more at the end of the graph, which makes me less confident in my predictions for the future.

However these graphs do not take account of that fact that the global temperature is also increasing due to global warming therefore this could impact area of sea ice in the Arctic. The decreasing area of sea ice in the arctic could be a result from global warming where the temperature is increasing globally causing more ice to melt in warmer seasons and less ice forming in the colder seasons in the Arctic. The decreasing area of sea ice in the Arctic could impact the global weather. The reduction of sea ice could result in more area of less reflective sea water being exposed. This causes less solar heat to be reflected back into space and could possibly increase the effects of global warming and could also possibly increase the trend shown in the graph.

In summary my analysis confirms that the amount of Arctic sea ice is decreasing and is likely to decrease in the future. The model that iNZight produces is a reliable one that can be used to make predictions for the near future that are reasonably reliable.

|	Grade Boundary: Low Merit
3.	For Merit, the student needs to investigate time series data with justification. This involves linking components of the statistical enquiry cycle to the context, and referring to evidence such as statistics, data values, trends, or features of visual displays in support of statements made. This evidence is from a student's response to the TKI task 'Food for thought'. The student has provided evidence of some research related to the purpose of the investigation and the variable being investigated (1). The student has selected and used appropriate displays throughout their response. The student has identified the change in the smoothed data and linked this to evidence shown in the graph (2). Descriptions of the seasonal pattern have been accurately related to the context (3). They have also used an appropriate model to make a forecast which has been given in context and appropriately rounded (4). A comment on the accuracy of the forecast has been given in the conclusion (5). For a more secure Merit, the comment on the accuracy of the forecast needs to be developed. For example, the student could consider how changes in the smoothed data towards the end of the graph would potentially have an effect on the estimated value of the forecast. The student could also link their comments made in their conclusion back to the purpose for their investigation.

There has been a lot of talk in the media in recent times that even though we have been through tough economic times in NZ we still continue to spend money at an increasing rate. Therefore I am going to investigate whether the amount of money spent in NZ supermarket and grocery stores is in fact on the increase over time. Even when money is tight we still need to buy food etc. so I would expect that the pattern in the data will be increasing as has been broadcasted. However from researching on the internet, NZ did recently experience a global financial crisis (GFC)so it will be interesting to see if this affected the amount of spending.

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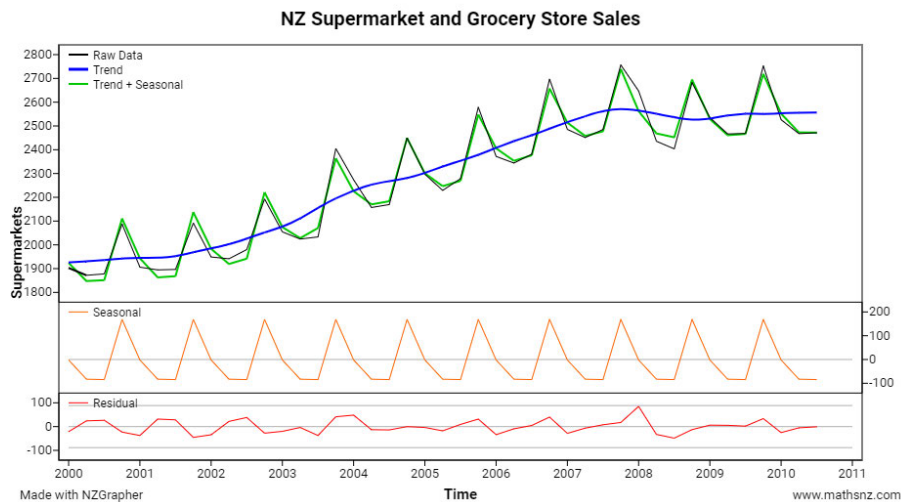
Overall the amount of money spent in NZ at supermarket and grocery stores has increased over the whole 11 years. The trend suggests that the average increase in the amount of money spent per quarter is around $20.024 million over the whole period. It starts off at around $1930 million in 2000 and ends up at $2560 million in the third quarter of 2010 which is an average increase of $14.65 million per quarter. The trend shows sales increasing from the third quarter 2000 up to quarter four 007. At this point the trend decreases until quarter four 2008 where it levels off.

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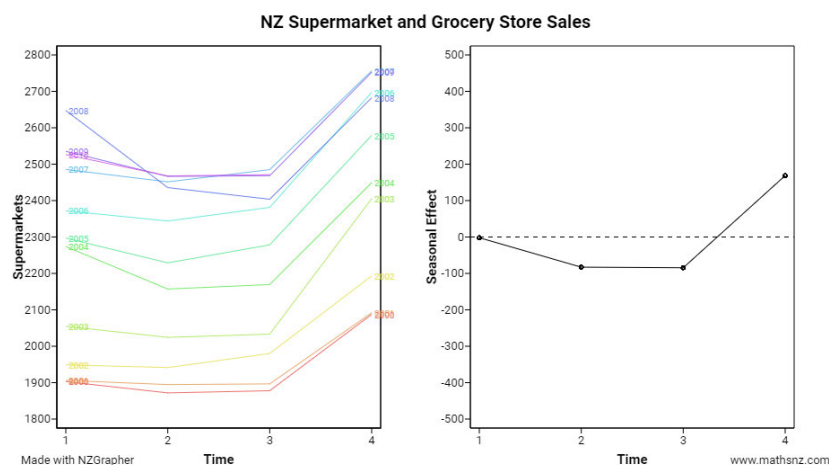
There is an obvious seasonal pattern with the largest amount of money spent in NZ on food in supermarket and grocery stores in the fourth quarter which are the months from October-December and lows in the second quarter each year which are the months April-June. The fourth quarter being

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the highest can be explained by the fact that this quarter corresponds to the Christmas and holiday period when spending on food and drinks always high. People at this time of the year tend to have more social outings e.g end of year work parties, people over for BBQs since it is the holidays etc so therefore the need for food and drink at these sort of events increases.

NZ Supermarket and Grocery Store Sales
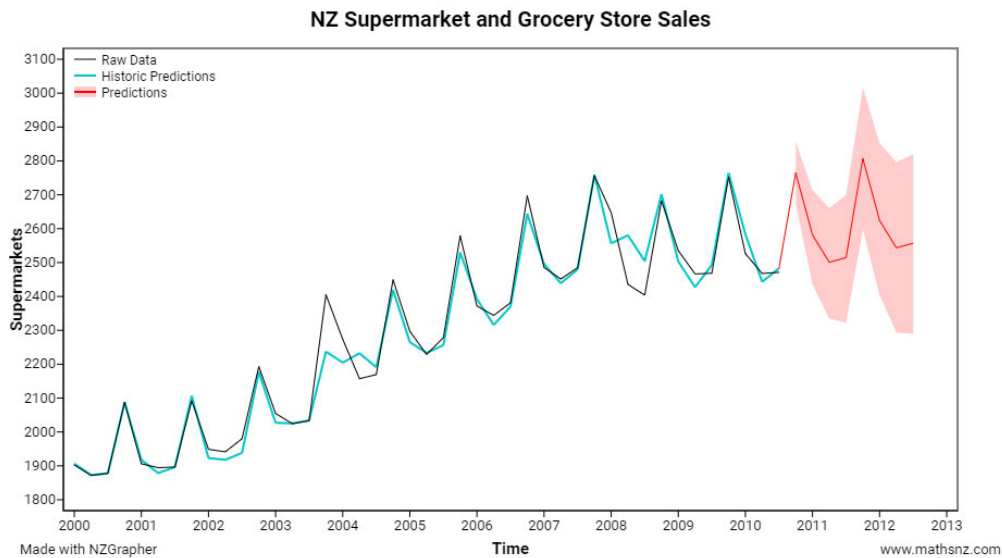
Time	Min	Prediction	Max
2010Q4	2668.9	2764.7	2861.8
2011Q1	2454.1	2581.4	2718
2011Q2	2340.8	2500.9	2664.3
2011Q3	2333	2514.4	2702.7
2011Q4	2599.1	2807.3	3020
2012Q1	2400.3	2624	2842.6
2012Q2	2279.6	2543.5	2788.2
2012Q3	2289.8	2557	2833

An estimate for the amount of money spent in NZ on food in supermarket and grocery stores in the third quarter 2012 is $2557 million.

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Generally the model is a reasonably good fit for the raw data. On the graph, the fitted model remains close to the raw data until near the end of 2007. At the end of the graph, the trend is levelling out and the prediction interval is getting quite wide therefore I cannot be too sure whether my prediction for the third quarter 2012 is entirely accurate.

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|	Grade Boundary: High Achieved
4.	For Achieved, the student needs to show evidence of using each component of the statistical enquiry cycle to investigate time series data. This involves: using existing data sets, selecting a variable to investigate, selecting and using appropriate display(s), identifying features in the data and relating this to the context, finding an appropriate model, using the model to make a forecast and communicating findings in a conclusion. This evidence is from a student's response to the TKI task 'Food for thought'. There is evidence of some research related to the purpose of the investigation (1). The student has selected a variable to investigate and has used software to select and use appropriate displays and find an appropriate model (2). Features of the data have been identified with descriptions of the trend and seasonal pattern, and the student has accurately related these to the context. A decreasing ramp in the data between 2008 and 2009 has been identified as a feature in the data (3). The model has also been used to make a forecast (4) and a conclusion (5). To reach Merit, the student needs to provide more evidence of linking the findings to the context and referring to evidence from the displays throughout the response. For example, when making a comment on the accuracy of the forecasts. The link of the forecast to the context also needs to refer to the amount of money spent in supermarket and grocery stores, rather than how many sales there will be.

Maths Time Series Internal

My investigation is into NZ grocery and supermarket sales from 2000 to 2010. I am going to see if there are any trends over time and especially if there are any particular seasons when New Zealanders spend more or less at the supermarket. Research I have read from newspapers and from watching TV suggests that there seems to be more spending each year so my investigation may or may not verify this. ①

Recomposed data: Supermarket and grocery stores


Seasonal plot for Su[Estimated seasonal ε
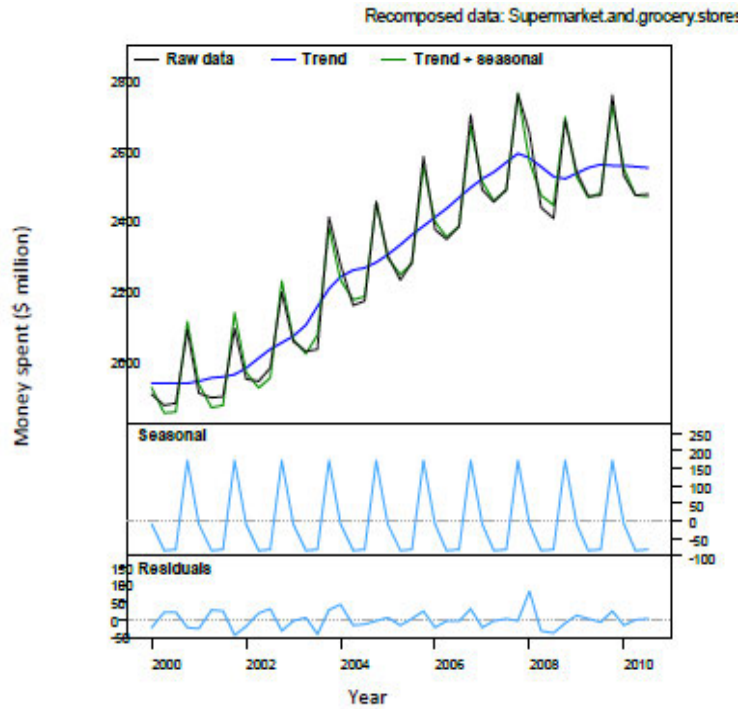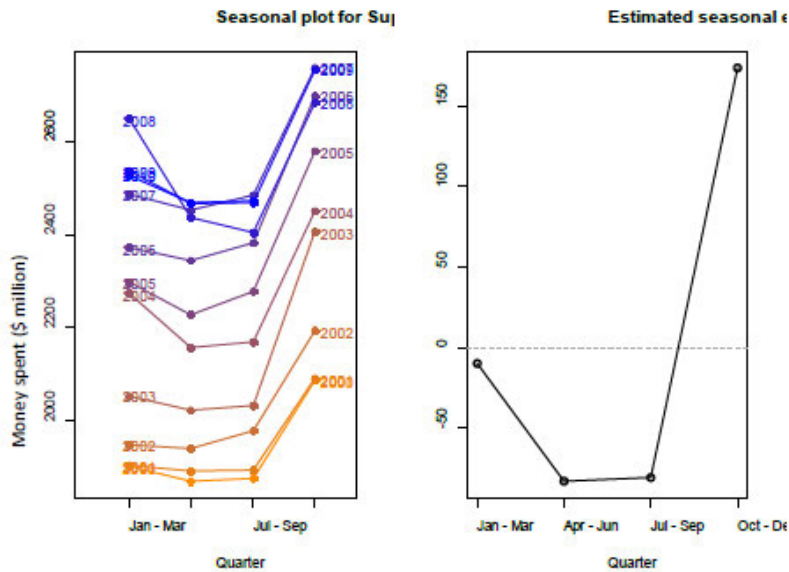
These graphs show that the average amount of money spent in NZ supermarket and grocery stores has an increasing trend with a slight decrease between 2008 and 2009. The trend starts from about $1950 million in quarter 1 in 2000 with the trend then levelling off at around (3) $2500 million in quarter 3 in 2010. As the data gets to 2008, we see a slight decreasing ramp in the graph. This enables us to see that during this time something may have occurred and is why the amount of sales is slighting decreasing, or it may have just been a bad year.

The average estimated seasonal effects graph shows the seasonal pattern in the graph. Quarter four, Oct-Dec, has the highest peak as it is about $170 million above the trend line showing that it is a particularly good part of the year with Christmas shopping etc while the other quarters 2 (Apr-Jun)and quarter 3 (Jul-Sep) are the lowest points as they are about $80 million below the trend line. It is not surprising that quarter 4, which contains the Christmas shopping period, would be the quarter where the most money is spent. This is because people seem to spend more money on food and alcohol for events such as Christmas dinners, parties, end of year work shouts and of course heading into the New Year period.

Holt-Winters forecast for Supermarket sales
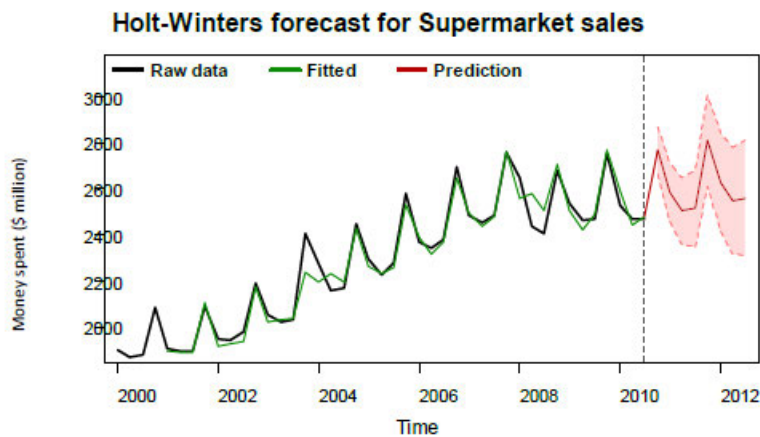
Looking at the above graph which seems fairly accurate, the prediction for how many sales (4) there will be in NZ supermarkets and grocery stores in quarter four of 2011 is between $2600 and $3000 million dollars.

This means that I was right when I thought that supermarket and grocery sales would continue to increase. (5)

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to show evidence of using each component of the statistical enquiry cycle to investigate time series data.
	This involves: using existing data sets, selecting a variable to investigate, selecting and using appropriate display(s), identifying features in the data and relating this to the context, finding an appropriate model, using the model to make a forecast and communicating findings in a conclusion.
	This evidence is from a student's response to the TKI task 'Polar ice'.
	The student has selected an appropriate variable to investigate (1), selected and used appropriate displays (2), identified features (trend and seasonal variation) in the data and related them to the context (3), found an appropriate model (4), used the model to make a forecast (5) and communicated findings in a conclusion (6).
	For a more secure Achieved, there needs to be a better overall understanding of the context. In the description of the trend, the student needs to refer to the decrease in the mean area of sea ice in the Arctic as an 'average' effect.
	The student would also need to provide evidence of researching the context prior to selecting the variable they will investigate.

Sea ice is present in the Arctic region of the globe, and is formed from the freezing of sea water. The amount of sea ice in the Arctic is apparently decreasing over a period of 21 years due to causes such as global warming and the Greenhouse effect. I am going to see if this decrease is actually happening and if so how fast.

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Arctic Sea Ice Graphs


Recomposed data: Arctic

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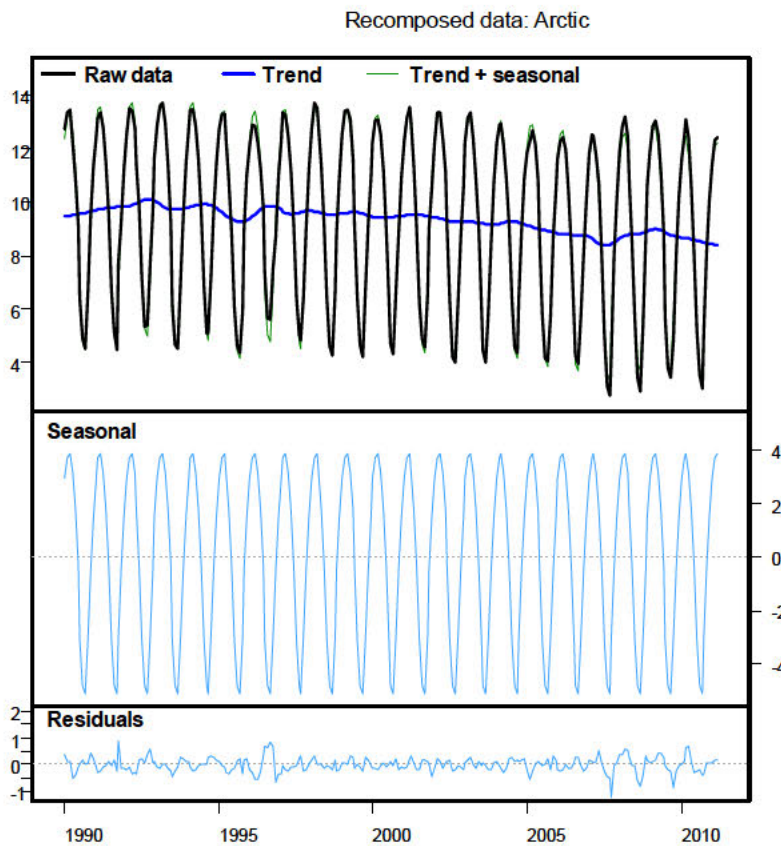
This graph shows the amount of sea ice (in millions of square kilometres) in the Arctic regions from January 1990 to March 2011. The amount of sea ice in the Arctic over a period of 21 years is generally decreasing. The area of sea ice has fallen from about 9.5 millions of square kilometres to about 8.5 million of square kilometres over this time.

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Seasonal plot for Arctic   **Estimated seasonal effects**
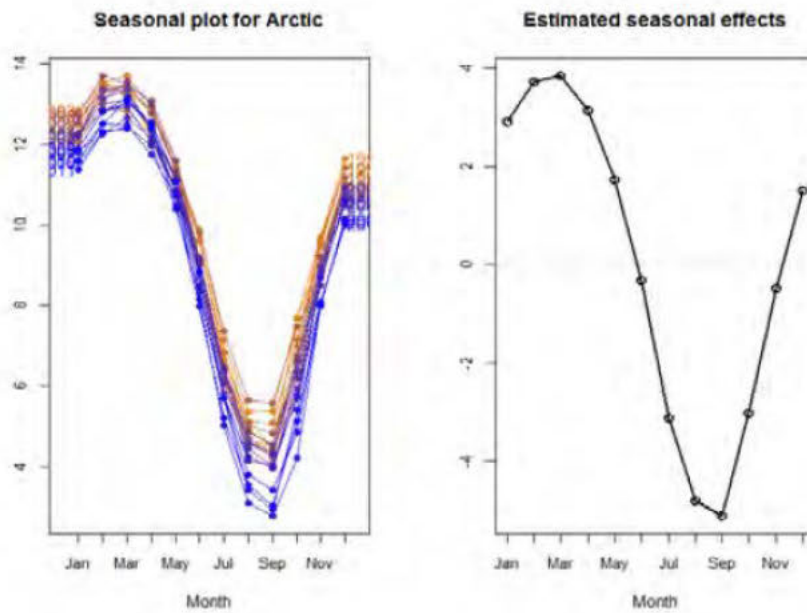
There is an annual seasonal pattern that can be seen in the graph. Generally, the month with the highest amount of sea ice in the Arctic is March and the month with the lowest amount of sea ice in the Arctic is September.

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Holt-Winters forecast for Arctic
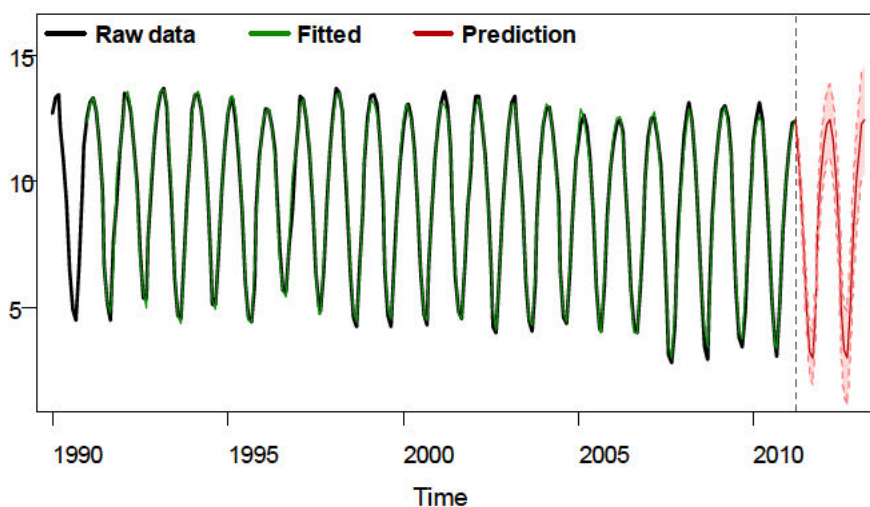
We are able to make quite accurate predictions about the amount of sea ice in the future because of the fit of the trend line to the data and using the average seasonal effects for each month. I estimate from the graph that the area of Sea ice for Sept 2011 will be about 3 million square kilometres.

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The projection of the amount of sea ice in Sept 2011 should be quite accurate, assuming the trend line does not change and there are no errors or ramps in the data set.

My investigation has found that the sea ice in the Arctic has decreases over the twenty years of the data set. The sea ice looks as if it will continue to increase but the model may be affected by other factors so the rate changes.

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	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to show evidence of using each component of the statistical enquiry cycle to investigate time series data. This involves: using existing data sets, selecting a variable to investigate, selecting and using appropriate display(s), identifying features in the data and relating this to the context, finding an appropriate model, using the model to make a forecast and communicating findings in a conclusion. This evidence is from a student's response to the TKI task 'Food for thought'. The student has selected an appropriate variable to investigate (1), selected and used an appropriate display, and found an appropriate model (2). The student has also identified a feature in the data by describing the trend in context (3). To reach Achieved, evidence for identifying features in the data would need to include the seasonal pattern. The student would also need to more clearly relate the description of the trend to the context by including the dates. In addition, the student needs to provide evidence of researching the context prior to selecting the variable they will investigate.

I am looking at NZ's Supermarket and Grocery store spending from 2000 to 2010. I think that spending has will have increased because there were more people living in NZ in 2010 compared to 2000. **(1)**

Supermarket and Grocery Stores
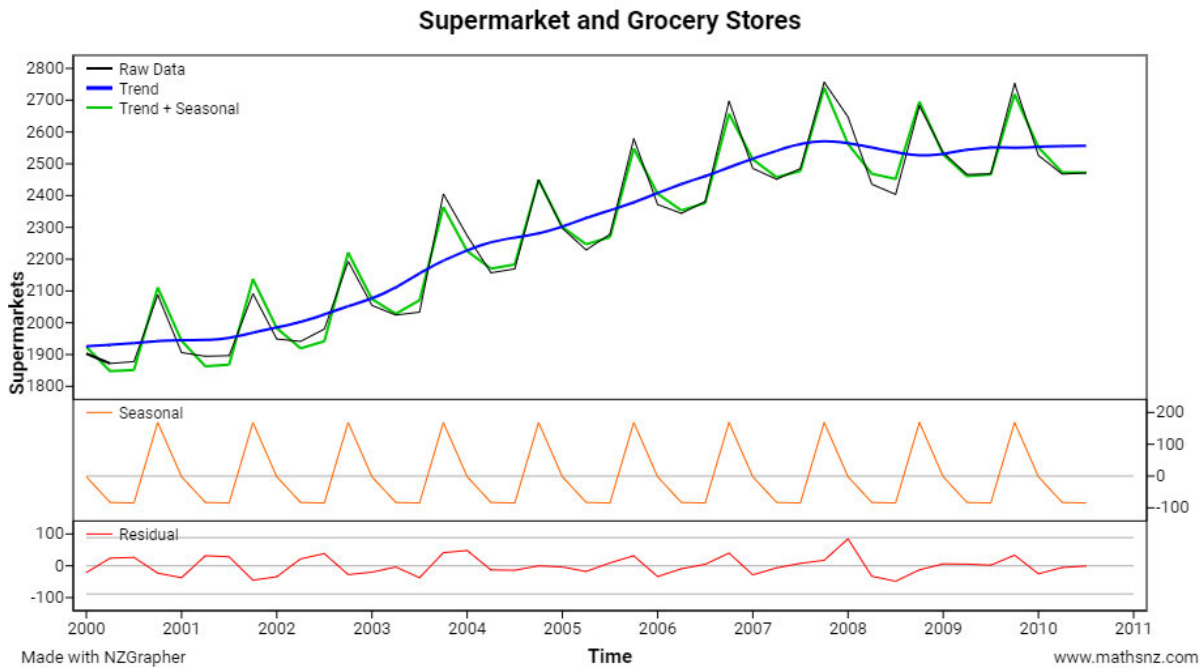
The model is a good fit to the data. We can see that the money spent has increased from $1930 million in 2000 to $2560 million in 2010. **(3)**

Supermarket and Grocery Stores
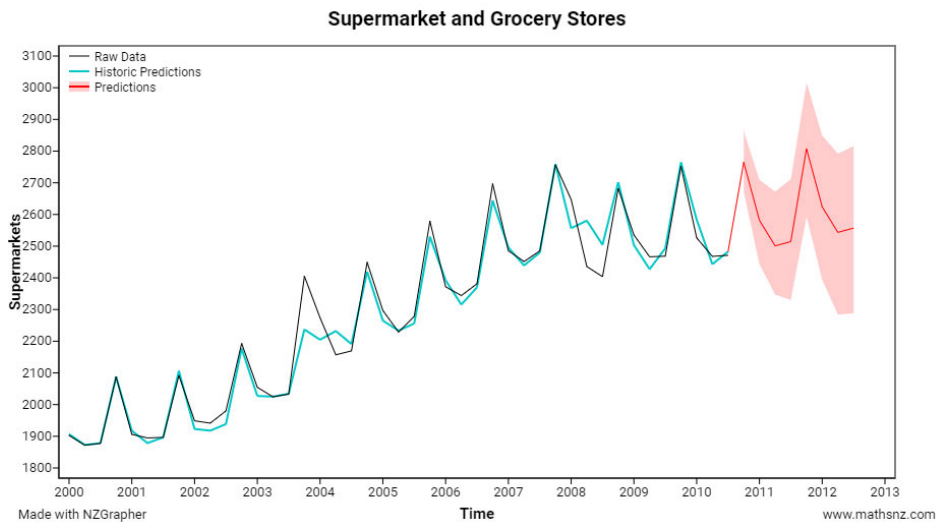
Time	Min	Prediction	Max
2010Q4	2667.6	2764.7	2869.4
2011Q1	2446.5	2581.4	2722.7
2011Q2	2337.8	2500.9	2676.2
2011Q3	2320.1	2514.4	2711
2011Q4	2593.4	2807.3	3016.8
2012Q1	2397.3	2624	2851.6
2012Q2	2308.5	2543.5	2783
2012Q3	2298.6	2557	2816.4

My forecast for NZ's Supermarket and Grocery store spending in quarter 1 in 2011 is from 2446.5 million dollars to 2722.7 million dollars.

I am reasonably confident with my prediction as the trend follows the data very closely. This would make it easy and reliable to forecast future quarters as the prediction is highly likely to be very similar to what will actually happen.