

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 3

This exemplar supports assessment against:

Achievement Standard 91581

Investigate bivariate measurement data

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to investigate bivariate measurement data, with statistical insight.
	This involves integrating statistical and contextual knowledge throughout the investigation process. It may include reflecting about the process, considering other relevant variables, evaluating the adequacy of any models, or showing a deeper understanding of the models.
	This evidence is from a student's response to the TKI task 'Sport science'.
	The student has posed an appropriate relationship question which is informed by research (1), and has also described the nature and strength of the relationship and related this to the context (2).
	Contextual knowledge from the research (3) and statistical knowledge (4) have been integrated into the discussion about the features in the data and in the evaluation of the adequacy of the models (5).
	Other relevant variables have been considered in the recategorisation into ball and no ball sports (6).
	This extract is from a student response which also included evidence of using the model to make a prediction and communicating findings in a conclusion, at an appropriate level for the award of Excellence.
	For a more secure Excellence, the student could have reflected on the process more deeply, for example by relating a visual inspection of the scatter graphs to the adequacy of the models, and/or linking the findings to further research.
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Student 1: Low Excellence

ZQA Intended for teacher use only

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Body image is a concern of modern women and just as much for female athletes as it is for other 'normal' women. <u>http://thesportjournal.org/article/body-image-disturbances-ncaa-division-i-and-iii-female-athletes</u>

Weight charts can be unreliable as a source for determining the healthy weight for athletes and body fat percentages are sometimes used instead. Women tend to weigh in the top range for their heights due to muscle mass being heavier than fat mass.

During the 2012 London Olympics the media reportedly criticised some female athletes suggesting that they were fat rather than fit. A list of top female athletes hit back at critics in an article 'Fat? We are fit. Get over it' by Belinda Goldsmith http://sports.yahoo.com/news/fat-fit-over-women-athletes-193539328--spt.html

I am going to investigate if there is a relationship between the weight of a female athlete and their percentage of body fat. I have taken the percentage of body fat as the explanatory variable and the weight as the response variable.



The first scatter graph shows us that there is a positive relationship between the weight of female athletes and their body fat percentage, i.e. people with a higher percentage of body fat tend to be heavier. This is consistent with what we would expect. It also appears to have a linear relationship and there is nothing to suggest that a different model would be better fit to the data. The strength of the relationship is quite strong, as shown by the data points not too far away from the line of best fit in the second graph.

The American Council on Exercise (ACE) divides body fat percentage into five different categories: essential, athletes, fitness, acceptable and obese. Essential body fat ranges from 10 to 14%, athletes 14 to 20% and fitness 21 to 24%. The acceptable range of body fat for women is 25 – 31% and a woman with a body fat percentage of over 32% is considered obese. The percentage of body fat in my scatter graph is mostly between 10 and 26% so this sort of fits what the ACE are saying. There is not much data with a percentage of body fat over 30% but I don't think this means that those in this range won't be athletes more that most athletes in most sports will be under this. There is one value of 35% body fat and looking at the original data this value is for a Netballer. I have looked at the other athletes in the data from the Australian Institute of Sport that have a body fat percentage of more than 25% and 7 of them are netballers and there is one basketball player – this suggests that maybe netballers will be in the higher range of percentage body fat.

Linear model

We see by the trend line that this is indeed a positive relationship - as the percentage of body fat of female athletes increases their weight in kg tends to increase. The gradient of the trend line, Wt= 1.451*%.BFat+41.443, shows us that for every 5% that your body fat increases you can expect to add approximately 8kg to your total weight. The scatter in the graph is consistent across the data set, and most of the data points are close to the regression line indicating that the relationship between percentage body fat and weight of females athletes is strong.

The y-intercept of a weight of around 41kg seems unrealistic as it is impossible to live with 0% body fat as it insulates the body and is your body's energy source. The influence of the y intercept (a body fat % of zero) on the model needs to be considered as while it is possible to have a very low percentage of body fat the essential body fat percentage quoted by ACE is between 10 and 13%. The model that I have found fits the data well and I don't think the body fat percentages in the lower ranges are of interest in my investigation so I will not change the y intercept and look for a new linear model. (It is not possible to have a body fat percentage of 0 and so this part of the model is of little use to us).

Further investigation

(http://www.livestrong.com/article/510269-what-is-the-normal-body-fat-of-an-athlete/)

The amount of body fat seems to depend on the type of sport in the data supplied from the Australian Institute of Sport for instance shot putters may have more but other sports that involve running and jumping etc. maybe benefit from less. Coaches and trainers have the task of balancing a programme of exercise, training and diet to enable the athletes to perform at their best in their chosen sport and this is likely to involve making decisions about the ideal weight or body fat percentage. I noticed that the some of the athletes with higher percentages of body fat were netballers which suggest that the type of sport that an athlete participates in could change the weight and body fat percentage relationship. For example, athletes who complete in sports that involve throwing a ball could have a higher percentage of body fat. For this reason I have re-categorised the data and chosen two subsets, one for sports that involve throwing a ball (netball and basketball) and the other sports which do not. X.Bfat versus Wt Subset by Throwing a Ball



Linear Trend for sports that do not involve throwing a ball

Wt = 1.9565 * %.Bfat + 34.41 Correlation = 0.7797

Linear Trend for sports that do involve throwing a ball

Wt = 1.0288 * %.Bfat + 48.49 Correlation = 0.64753

The different gradients for these lines agree with my research. For those that throw a ball, for every 5% that your body fat increases you can expect to add approximately 10kg to your total weight, but for the non throwing sports it is only 5kg. The strength of the relationship for the sports that do not involve throwing a ball is the stronger. The scatter in this graph looks to be less than the other graph, and the correlation coefficient is stronger. The lower gradient in the ball-throwers graph is consistent with %body fat contributing less to their total weight and them needing a higher % of muscle mass for this type of sport.

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	Grade Boundary: High Merit
2.	For Merit, the student needs to investigate bivariate measurement data, with justification.
	This involves linking components of the statistical enquiry cycle to the context, and referring to evidence such as statistics, data values, trends, or features of visual displays in support of statements made.
	This evidence is from a student's response to the TKI task 'Sport science'.
	The student has posed an appropriate relationship question which is informed by research (1), and described the nature and strength of the relationship and related this to the context (2).
	The description of the features has been justified by evidence from the data and displays (3) and the residuals have been used to support the comments on the appropriateness of the linear model (4).
	The student has also linked the components of the statistical enquiry cycle to the context when communicating findings in a conclusion (5).
	This extract is from a student response which also included evidence of using the model to make a prediction and justifying its accuracy, at an appropriate level for the award of Merit.
	To reach Excellence, aspects of the investigation could be supported with comments which demonstrate integration of contextual and statistical knowledge. For example, the student could consider whether the strength, direction and other features of the data are consistent with their own research.

Student 2: High Merit

Using the data set provided from the Australian Institute of Sport I will look at different pairs of variables to quickly see the kinds of relationships that there might be. I have decided to investigate the relationship between weight and lean body mass.

Investigative question: I wonder if there is a relationship between weight in kg and lean body mass in kg in males and females that play sports?

According to the Journal of Romanian Sports Medicine Society¹ body composition is a factor contributing to sport performance and the assessment of body composition is an important component of the on-going monitoring of athletes interested in improving their performance. Lean body mass (LBM) is how much you weigh without your body fat so I expect that there will be some relationship. Without the body fat you should be able to get an idea of how much muscle you are gaining or losing as a result of training and diet. LBM is fairly easy to calculate once you have weighed yourself and figured out your body fat percentage. You just calculate your body fat in kg and subtract that from your body weight. Weight in kg is the explanatory or independent variable and LBM in kg is the response or dependent variable.



From the first graph it seems that there is a positive linear relationship between the weight and LBM of an athlete and there is no other evidence to suggest that a different model would be better. This is seen by how the data points on the graph are very close together and are increasing in a positive direction and are close to an imaginary linear trend line. This suggests that there is a strong relationship between the two variables. The relationship is also seen as positive due to the fact that the data seems to go upwards and increases. It seems that when weight in kg increases the lean body mass tends to increase for males and females that play a sport. There seems to be unusual values in the data. These are some values that are not so close to the main group of data these can be found in the range between the weights of 111kg to 124kg and the weight 38kg.

When a trend line (LBM = 0.8737 * Wt - 0.6627) has been added to the graph it is visible from the positive gradient that there is a positive relationship and the points are scattered along the regression line quite closely, therefore there is a strong relationship between weight and LBM of athletes. The correlation coefficient (r) demonstrates the strength of the relationship between the two variables. This has a value of 0.9309 and confirms that there is a very strong, positive relationship because the value is very close to 1. The correlation coefficient expresses how close the points are to the trend line and in this case they are very close.



From this graph of residuals for weight in kg and lean body mass in kg it is visible that there is quite a spread of the points and no real pattern in them. About half of the points are positive and the other half are negative. The positive residuals go up to about 6 whilst the negative residuals go up to about -16.

This reinforces that a linear model is appropriate.

The gradient of the linear trend line 0.8737 expresses that as the weight in kg for males and females that play a sport increase by 1kg, the lean body mass in kg tends to increase by 0.8737kg.

The model LBM = 0.8737 * Wt - 0.6627 is not useful for all values of the weight. For example the model predicts that a weight of zero will produce a negative lean body mass, and this does not make sense. We can't really use the model below weights of 38kg, the lowest weight in the data set.

Even though there appears to be a strong relationship between weight in kg and LBM in kg for athletes we cannot be sure that an increase in weight is completely responsible for an increase in LBM as there may be other factors involved that were not controlled in this investigation.

This data is for athletes at the Australian Institute of Sport and so may not represent data for typical Australian men and women or actually people outside of Australia. It might not represent athletes from other countries either.

In conclusion there is a relationship between the weight in kg and lean body mass in kg for males and females that play a sport and a linear model seems to suit this data best. I would expect this information to be useful to the coaches and trainers in their on-going monitoring of these athletes - as it could be used to get an idea of how much muscle the athletes are gaining or losing as a result of training and diet. It also might be useful to do an analysis of the weight and LBM of male and female athletes separately or to look at relationships for athletes from other countries.

http://www.medicinasportiva.ro/SRoMS/english/Journal/No.6/The%20importance%20of%20b ody%20composition%20measurement%20at%20athletes%20and%20non%20athletes%20fu II.html 4

	Grade Boundary: Low Merit
3.	For Merit, the student needs to investigate bivariate measurement data, with justification.
	This involves linking components of the statistical enquiry cycle to the context, and referring to evidence such as statistics, data values, trends, or features of visual displays in support of statements made.
	This evidence is from a student's response to the TKI task 'Sport science'.
	The student has posed an appropriate relationship question which is informed by research (1), and described the nature and strength of the relationship and related this to the context (2).
	The student has identified features in the data and provided some support for statements made with evidence from the displays, and explanations have been related to the context (3).
	An appropriate model has been found and used to make predictions, and there is some discussion about how accurate the predictions might be (4).
	For a more secure Merit, the student would need to discuss other features of the displays, for example, any unusual values, and strengthen the supporting evidence of statements with further evidence from the displays and data.
	Stronger evidence of communicating findings in a conclusion would also be expected.

Student 3: Low Merit

I am investigating if there is a relationship between weight in kg and lean body mass (LBM) for the athletes in the data set supplied from the Australian Institute of Sport. The weight of a person is made up of body fat and lean body mass and according to Bodybuilding.com the LBM is the amount of weight you carry on your body that isn't fat. Erin Coleman, R.D., L.D. in an article on Livestrong.com says that "Lean body mass includes more than two-thirds water, according to Medline Plus, and the remainder other lean tissues such as muscle, organs and bone" (http://www.livestrong.com/article/175858-the-average-lean-body-mass/#ixzz26y4L97ck).

LBM affects athletic performance, appearance and weight. In some sports the amount of LBM can affect your performance and this information is likely to be useful to the coaches and trainers. <u>http://www.humankinetics.com/excerpts/excerpts/normal-ranges-of-body-weight-and-body-fat</u>.

I have drawn an initial scatter plot using weight in kg as an explanatory variable and LBM as the response variable.



From the graph the weight in kg and LBM have a positive relationship which means that as weight in kg increases the LBM tends to increase. This may be because heavier athletes have heavy organs or muscles. The points are grouped closely together although the data for weights over 102kg are more spaced out they appear to be following the same line.

The trend line equation is y = 0.8737x - 0.6627. The relationship is strong because the points are close to the regression line. The model proposed suggests that as the weight increases by 1kg, the LBM increases by 0.8737kg.



Prediction.

According to the Australian Bureau of Statistics the average weight of a man of average height is 85.2kg and a woman of average height is 70.1kg.

(http://www.abs.gov.au/ausstats/abs@.nsf/Lookup/4841.0Chapter22011).

While these will not be the same necessarily for the average weights of high performance athletes I will use a weight within this weight range to predict the LBM of athletes.

I will use the weights of 75kg and 100kg to predict what the LBM will be. Since the linear model appears to fit the data well I will use this to make my predictions. For the prediction, I substituted the weights into y = 0.8737x - 0.6627.

y = 0.8737*75 - 0.6627 = 64.8648 = 64.9kg which is above the data points (LBM values) near this weight,

y = 0.8737*100 - 0.6627 = 86.70739 = 86.7kg which is in the middle of the data points (LBM values) for this weight on my graph.

I wonder if there are differences in the relationship between male and female athletes and also between different sports in the data set. I will now look at the relationship between for LBM and weight in kg for men and women separately.



The linear model for males is y = 0.7732x + 10.851 and for females the linear model is y = 0.5839x +15.571. For both of these relationships the points are close to the regression line. Looking at the model proposed for the men the data points appear to be scattered more closely to the line than that for the men and women together and I think it is likely if I use this model to predict the LBM for a male athlete the results will be more reliable. Visually the scatter on the graph for the female athletes' is similar to the one with all the men and women together although there are no data points beyond a weight of 97kg.

Male predictions:

75kg, LBM = 0.7732*75 + 10.851 = 68.841 = 68.8kg. 100 kg, LBM = 0.7732*100 + 10.851 = 88.171 = 88.2kg. As this model looks to be a better fit than the combined model I think that these predictions are fairly accurate.

Female predictions:

75kg, LBM = 0.5839*75 + 15.571 = 59.36 = 59.4kg. Looking at the other data points (LBM) around this weight, I think that this is a reliable prediction for the LBM.

100 kg, LBM = 0.5839*100 + 15.571 = 73.961 = 74.0kg – there aren't any female athletes of this weight in the graph so this prediction may be more unreliable than those for weights of less than 100kg.

I think that the combined linear regression model fits the data investigated well but the separate models are likely to be more reliable and appear to be a better fit. This information could be useful to coaches and trainers. I didn't look at how well these models might fit the 'average population' or how the different sports the athletes took part in may influence the relationships.

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	Grade Boundary: High Achieved
4.	For Achieved, the student needs to investigate bivariate measurement data.
	This involves showing evidence of using each component of the statistical enquiry cycle.
	This evidence is from a student's response to the TKI task 'Sport science'.
	The student has posed an appropriate relationship question that is informed by research (1), selected and used appropriate displays (2), identified features in the data (3) and found an appropriate model (4).
	They have also described the nature and strength of the relationship and related this to the context (5), used the model to make a prediction (6) and communicated findings in a conclusion (7).
	To reach Merit, the student would need to provide more in the way of contextual references to support statements made. There would need to be some evidence that other background research material had been considered when communicating findings in a conclusion.

Student 4: High Achieved

Is there a relationship between the height in cm and the weight in kg for athletes in the data supplied from the Australian Institute of Sport? I think it is likely. There is an app from the BBC where you can put in your height and weight and see which Olympian is your body twin http://www.bbc.co.uk/news/uk-19050139 - this is based on the relationship between the height and weight of athletes.

I have made height the explanatory variable and weight the response variable. I think this may be useful to the institute when they are identifying future athletes for training.



Ht versus Wt

The scatter graph has a strong positive relationship between the height and weight of sports athletes at the Australian Institute of Sport. The data in the graph tells us that as an athlete gets taller (cm) their weight (kg) will generally increase as well. The relationship is strong as the data points are close to the fitted trend line and 'r' is 0.78090629 which also confirms this.

The linear model fits this data quite well as it is positive and shows generally as height increases so does weight this confirms my statement from above, this is because the taller the athlete is the more muscle/fat they will have making them weigh more. There are some values more scattered away from this linear model - such as the one that weighs around 120kg and is around 190cm tall, this is probably due to different sports requiring different physiques, in this instance it is a male 'field' athlete. This graph also shows me that in the data supplied there are only a few athletes taller than 200cm and these are all basketball players and the linear model stops at around 200cm.

To make my prediction I used a male TSPRNT individual with a height of 190 cm to predict his weight in kg. I used the equation that I got from my linear model to work this out.

Prediction = 1.1171*190 - 126.19 = 86.059 = 86.1kg

I did another prediction with data outside of my graph I used a height of 210 cm to predict the weight. I also used the equation from my linear model to work out the prediction. Prediction = 1.1171*210 - 126.19 = 108.401 = 108.4kg

This confirms my thinking that with a shorter person their weight was less than a person much taller.



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I decided to also investigate if there is a relationship between Height (cm) and BMI (weight/height²).

The scatter graph shows a moderate positive relationship between the height and BMI (body mass index) in athletes at the Australian Institute of Sport. The graph shows that as the height of an athlete increases their BMI will increase. As BMI is worked out from (weight/height²) we might expect this.



The graph shows that the linear model fitted for this data seems ok, the data appears to be mostly clumped together – there are still some BMI values for heights between 175cm and 189cm where the data is scattered further away from the regression line.

Discussion

Looking at my scatter graphs for the two relationships the relationship between height and weight is a much stronger relationship than that between height and BMI – the data points are much more closely scattered around the regression line for height/weight. This is supported by the correlation for height/weight being 0.78 and the correlation for height/BMI being 0.34. This tells us that the relationship between height and weight of an athlete is much stronger the relationship height and BMI of an athlete.

The height/weight model may be better fitting to the linear graph due to it comparing the height and weight and as someone gets taller they generally have more muscles with larger bones, whereas for the height/BMI the BMI is calculated already from weight/height² therefore it is already including the height in the graph.

Wikipedia tells us that BMI is inaccurate for athletes and people who are fit because they have higher amounts of muscle which puts them in the "overweight" category even though their body fat percentages are in the 10-15% category.

Being an institute of sport they will be focusing on particular sports and building muscles so it may also be dependent on the sport, for example in sprints you don't need the whole body at equal strength as you may need in water polo, due to sprints mainly involving timing and the power in your legs; whereas water polo you need to be able to stay afloat through your legs and be able to throw/catch the ball.

I think that the height/weight model would be useful for the sports institute to identify potential future athletes.

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to investigate bivariate measurement data.
	This involves showing evidence of using each component of the statistical enquiry cycle.
	The student has posed an appropriate relationship question (1), selected and used appropriate displays (2), identified features in the data (3) and found an appropriate model (4).
	They have also described the nature and strength of the relationship and related this to the context (5), used the model to make a prediction (6) and communicated findings in a conclusion (7).
	For a more secure Achieved, the posed question needs to be informed by contextual research about a possible relationship. There also needs to be more depth in the description of features in the data, particularly in relating descriptions to the context of the investigation.

Powerlifting:

Powerlifting is a strength sport that consists of three attempts at maximal weight on three lift: squat, bench press, and deadlift. Best bench is also known as a bench press which is an upper-body weight training exercise in which the trainee presses a weight upwards while lying on a weight training bench with their feet on the ground.

https://en.wikipedia.org/wiki/Powerlifting https://en.wikipedia.org/wiki/Bench press



I will be looking for a relationship between the bodyweight and best bench of a powerlifter. The data I have used is from the powerlifting database on the Kaggle website. The body weight will be measured in kilograms and the best bench measured in kilograms. My explanatory variable is the body weight of a powerlifter. The response variable is the best bench of a powerlifter.



Relationship between Bodyweight and Best Bench of Powerlifters

From the graph I can see that the scatter is distributed from bottom left to top right and it looks like it could be modelled by a straight line. So there is a positive linear relationship



(1)

between the bodyweight and best bench of a powerlifter. This means that the heavier powerlifters tend to have a better best bench.

Equation: BestBench (Kg) = 1.5132 * Bodyweight (Kg) – 9.7321



Relationship between Bodyweight and Best Bench of Powerlifters

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The strength of the relationship between the bodyweight and best bench of a powerlifter is moderate. The graph shows that most of the points aren't either too close to the regression line nor are they too far away. There appears to be two unusual features which are outliers (118.84, 374.1) and (183.3, 227.5).

The scatter appears to be pretty consistent for body weights below 90 but above this the scatter is more spread.

Prediction:

I predict that if a powerlifter has the body weight of 52 kg they will have a best bench of 68.95 (6) kg. This comes from my equation:

BestBench (Kg) = 1.5132 * 52 – 9.7321 = 68.95

I predict that if a powerlifter has the body weight of 111 kg they will have a best bench of 158.23 kg. This comes from my equation: BestBench (kg) = 1.5132 * 111 – 9.7321 = 158.23

In conclusion I think there is a positive linear relationship between the bodyweight and best bench of a powerlifter – heavier powerlifters tend to be able to do a heavier best bench.

	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to investigate bivariate measurement data.
	This involves showing evidence of using each component of the statistical enquiry cycle.
	This evidence is from a student's response to the TKI task 'Sport science'.
	The student has posed an appropriate relationship question (1), selected and used appropriate displays (2), identified features in the data (3), found an appropriate model (4) and used the model to make a prediction (5).
	To reach Achieved, the student would need to have described the strength of the relationship by discussing visual aspects of scatter about the regression line in context. They would also need to provide a conclusion.

Student 6: High Not Achieved

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I will look at the data from the Australian Sports Institute to see whether or not there is a relationship between the height (cm) and weight (kg) for sport athletes. The explanatory variable is the height while the response variable is weight.



The relationship between height and weight appears to be linear so I will fit a linear model to the data.

This data shows a strong positive relationship. This means that there is a relationship between the height of an athlete from the Australian Institute of Sport and their weight - as athletes grow taller they generally weigh more. For this data the weight is more likely to be muscle as these people are athletes and depending where the most weight is, muscle weighs more than fat.

There are a few large weights compared to their height for example 123.2 kg for someone of a height of 189.2 cm. Between around 170cm and 190cm there are a few plots higher up, this could likely be because these athletes could be very strong and the weight is muscle because of the sports they do.

The data values between 170cm and 190cm in the scatter plot are more scattered above the line and it maybe another model could be used is for these values such as exponential or logarithmic as these may be better fit for the data.

A possible prediction I could make would be the weight of someone that is 160cm tall. To do this I would use the equation 1.1171*160 - 126.19 = to work out this prediction. The answer for this is 52.5 (rounded to one decimal place). Therefore someone that is 160cm tall would be approximately 52.5kg. This seems reasonable as a female athlete in the data set with a height of 162cm has a weight of 52.8kg.

If I want to see how much someone would weigh that is 211cm tall by using the same equation as previously used 1.1171 *211– 126.19, my answer would be 109.5kg (rounded to one decimal place). There are only three values over 200cm on the data plot and their weights are between 92 and 114kg.