National Certificate of Educational Achievement
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Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 3

This exemplar supports assessment against:

Achievement Standard 91587

Apply systems of simultaneous equations in solving problems

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to apply systems of simultaneous equations, using extended abstract thinking, in solving problems. This involves one or more of: devising a strategy to investigate or solve a problem, identifying relevant concepts in context, developing a chain of logical reasoning, or proof, forming a generalisation and also using correct mathematical statements, or communicating mathematical insight. This student's evidence is a response to the TKI task 'Roger's rabbits'. The student has found the amount of each type of food to meet the daily requirements (1) and found a general solution which satisfies the situation with 6μg of Vitamin A in the Zany product (2). They have also identified an appropriate range of values for the amount of Zany for the new situation and given two possible solutions (3). The student has also indicated they have considered amounts other than 5 micrograms and 6 micrograms for the amount of Vitamin A in Zany (4). For a more secure Excellence, the student would need to accurately communicate their thinking about how 6 μg of vitamin A in the Zany product relates to the general solution and develop the discussion on the dependency of the equations.

Student 1: Low Excellence

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x = no of grams of Xena

y = no of grams of Yum

z = no of grams of Zany

The amount of vitamins of each type of food can be represented by the equations

$2x + 4y + 5z = 1000$ (Vit A)

$3x + 7y + 10z = 1600$ (Vit C)

$5x + 9y + 14z = 2400$ (Vit E)

Solving these equations gives $x = 300$, $y = 100$, $z = 0$ ①

So to meet the daily requirement Roger should feed them 300 grams of Xena, 100 grams of Yum and no Zany.

If the amount of Vitamin A in Zany changes to 6 micrograms then

$2x + 4y + 6z = 1000$ (1)

$3x + 7y + 10z = 1600$ (2)

$5x + 9y + 14z = 2400$ (3)

Solving these gives multiple solutions. These equations are consistent.

Solving (1) × 3 – (2) × 2 gives $-2y - 2z = -200$ $y + z = 100$

(1) × 5 – (3) × 2 gives $2y + 2z = 200$ $y + z = 100$ ②

(1) × 7 – (2) × 4 gives $2x + 2z = 600$ $x + z = 300$

So solution is $300 - z$, $100 - z$, z

If $z > 100$ the amount of Yum would be negative so $0 \le z \le 100$

So if $z = 20$ grams, $x = 280$ grams and $y = 80$ grams ③

ie one solution is 280 grams of Xena, 80 grams of Yum and 20 grams of Zany

if $z = 50$, $x = 250$, $y = 50$ so another solution is 250 grams of Xena, 50 grams of Yum and 50 grams of Zany

If the amount of Vitamin A in Zany is k micrograms

then $2x + 4y + kz = 1000$

$3x + 7y + 10z = 1600$

$5x + 9y + 14z = 2400$

Using the calculator I got 0 micrograms of Zany for lots of values I tried for k except when k = 6 micrograms when there is no unique solution. ④

Maybe this is because (3) = 4 × (1) – (2)

	Grade Boundary: High Merit
2.	For Merit, the student needs to apply systems of simultaneous equations, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model, and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student’s evidence is a response to the TKI task ‘Roger’s rabbits’. The student has shown evidence of relational thinking by finding the amount of each type of food required to meet the daily requirements (1). They have also demonstrated algebraically that the system of equations is consistent and indicated that increasing the amount of vitamin A in Zany food does not provide a unique solution (2). The student has identified a possible solution for the amount of each type of food if Zany uses 6 μg of vitamin A and made an appropriate recommendation to Roger (3). To reach Excellence, the student would need to generalise the amount of each type of food required if Zany contains 6μg of vitamin A.

Student 2: High Merit

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The amount of each vitamin Rogers rabbits need to meet their daily vitamin requirements and the number of grams of each vitamin in the foods Xena, Yum and Zany can be represented by the following equations where x represents Xena, y represents Yum and z represents Zany.

$2x + 4y + 5z = 1000$

$3x + 7y + 10z = 1600$

$5x + 9y + 14z = 2400$

solved simultaneously

$x = 300$ Xena

$y = 100$ Yum

$z = 0$ Zany

These calculations lead to the conclusion that in order for his rabbits to meet their exact daily requirements Roger should feed them 300 grams of Xena, 100 grams of Yum and 0 grams of Zany each day. Therefore the rabbits daily vitamin requirements can be met by consuming the previously mentioned amounts of Xena and Yum alone. Zany is not needed. (1)

If Zany increases the amount of vitamin A in their food from 5 micrograms to 6 micrograms this would change the number of grams of each food Roger should feed his rabbits in order for them to meet their exact daily vitamin requirements.

$2x + 4y + 6z = 1000$ (1)

$3x + 7y + 10z = 1600$ (2)

$5x + 9y + 14z = 2400$ (3)

(1) × 1.5 $3x + 6y + 9z = 1500$ (4)

$3x + 7y + 10z = 1600$ (2)

(2) – (4) $y + z = 100$

(1) × 2.5 $5x + 10y + 15z = 2500$ (5)

$5x + 9y + 14z = 2400$ (3)

(3) – (5) $-y - z = -100$

so $y + z = 100$

$-y - z = -100$ (add)

$0 = 0$

This means that there are many solutions to the number of grams of each of the foods Roger should have fed his rabbits in order to meet their daily requirements. There is no one real solution. (2)

One example of a possible solution of the number of grams of each food Roger should now feed his rabbits is $x = 250$ grams Xena

$y = 50$ grams Yum (3)

$z = 50$ grams Zany

	Grade Boundary: Low Merit
3.	For Merit, the student needs to apply systems of simultaneous equations, using relational thinking, in solving problems. This involves one or more of selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model, and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI task 'Roger's rabbits'. The student has shown evidence of relational thinking by finding the amount of each type of food required to meet the daily requirements (1) and by identifying that the change to 6μg of vitamin in Zany food produces multiple solutions (2). For a more secure Merit, the student could interpret the multiple solutions in the context of the problem and provide a possible solution which meets the new situation.

Student 3: Low Merit

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Vitamin A $2x + 4y + 5z = 1000$

Vitamin C $3x + 7y + 10z = 1600$

Vitamin E $5x + 9y + 14z = 2400$

$x = 300$
$y = 100$
$z = 0$

If Roger wants his rabbits daily vitamin intake to be 1000 µg of Vitamin A, 1000 mg of Vitamin C and 2400 mg of Vitamin E, in order to meet these exact daily vitamin requirements Roger should feed his rabbits 300 g of Xena feed and 100 g of Yum feed. (1)

Vitamin A $2x + 4y + 6z = 1000$
Vitamin C $3x + 7y + 10z = 1600$
Vitamin E $5x + 9y + 14z = 2400$

Vitamin A $6x + 12y + 18z = 3000$
Vitamin C $6x + 14y + 20z = 3200$
$2y + 2z = 200$

Vitamin A $10x + 21y + 30z = 5000$
Vitamin E $10x + 18y + 28z = 4800$
$2y + 2z = 200$

These equations are consistent. They are the same and give 0 = 0 and the change to 6 µg gives multiple solutions. (2)

	Grade Boundary: High Achieved
4.	For Achieved, the student needs to apply systems of simultaneous equations in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms, and communicating using appropriate representations. This student's evidence is a response to the TKI task 'Roger's rabbits'. The student has formed a system of simultaneous equations (1), used them to find a solution, and made an appropriate recommendation regarding the amount of each type of food required (2). To reach Merit, the student would need to consider how the amount of each type of food would change if the number of micrograms of vitamin A in the Zany food changes to 6.

Student 4: High Achieved

Intended for teacher use only

	Xena		Yum		Zany	
Vit A	2x	+	4y	+	5z	= 1000
Vit B	3x	+	7y	+	10z	= 1600
Vit C	5x	+	9y	+	14z	= 2400

①

Using my calculator

x = 300 g

y = 100 g

z = 0 g

Dear Rodger
I recommend that you feed your rabbits 300 g of xena, 100 g of yum and 0 g of zany rabbit food to reach their exact daily vitamin request.

②

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to apply systems of simultaneous equations in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms, and communicating using appropriate representations. This student's evidence is a response to the TKI task 'Roger's rabbits'. The student has formed the equations for each vitamin (1) and solved the system of equations (2). For a more secure Achieved, the student would need to indicate what is represented by each variable and interpret the solution in context.

Student 5: Low Achieved

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Vitamin E = e
Vitamin C = c
Vitamin A = a

Xena contains
2a + 3c + 5e
Yum
4a + 7c + 9e
Zany
5a + 10c + 14e

A $2x + 4y + 5z = 1000$
C $3x + 7y + 10z = 1600$
E $5x + 9y + 14z = 2400$

(1)

$x = 300$ g
$y = 100$ g
$z = 0$ g

(2)

	Grade Boundary: High Not Achieved
6.	For Achieved the student needs to apply systems of simultaneous equations in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms, and communicating using appropriate representations. This student's evidence is a response to the TKI task 'Roger's rabbits'. The student has formed the equation for each of the vitamins (1). To reach Achieved, the student would need to correctly solve the system of equations.

Student 6: High Not Achieved

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Xena 2 µg A
3 mg C
5 mg E

Yum 4 µg A
7 mg C
9 mg E

Zany 5 µg A
10 mg C
14 mg E

Roger wants
1000 µg A
1600 mg C
2400 mg E

Equation

	Xena	Yum	Zany	
① A	2	4	5	= 1000
② C	3	7	10	= 1600
③ E	5	9	14	= 2400

Equation

① $2x + 4y + 5z = 1000$

② $3x + 7y + 10z = 1600$

③ $5x + 9y + 14z = 2400$

(1)