3

# NZQA

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## Level 3 Calculus, 2019

9.30 a.m. Tuesday 26 November 2019

FORMULAE AND TABLES BOOKLET
for 91577, 91578 and 91579

Refer to this booklet to answer the questions in your Question and Answer booklets.

Check that this booklet has pages 2–4 in the correct order and that none of these pages is blank.

**YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.**

# MATHEMATICS – USEFUL FORMULAE

## ALGEBRA

### Quadratics

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

### Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b(x^n) = n \log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

### Complex numbers

$z = x + iy$
$\quad = r\operatorname{cis}\theta$
$\quad = r(\cos\theta + i\sin\theta)$

$\overline{z} = x - iy$
$\quad = r\operatorname{cis}(-\theta)$
$\quad = r(\cos\theta - i\sin\theta)$

$r = |z| = \sqrt{z\overline{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

where $\cos\theta = \dfrac{x}{r}$

and $\sin\theta = \dfrac{y}{r}$

### De Moivre's Theorem

If $n$ is any integer, then

$(r\operatorname{cis}\theta)^n = r^n \operatorname{cis}(n\theta)$

## COORDINATE GEOMETRY

### Straight Line

Equation $y - y_1 = m(x - x_1)$

## CALCULUS

### Differentiation

| $y = f(x)$ | $\dfrac{dy}{dx} = f'(x)$ |
|---|---|
| $\ln x$ | $\dfrac{1}{x}$ |
| $e^{ax}$ | $ae^{ax}$ |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\sec^2 x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |

### Integration

| $f(x)$ | $\displaystyle\int f(x)\,dx$ |
|---|---|
| $x^n$ | $\dfrac{x^{n+1}}{n+1} + c$ $(n \neq -1)$ |
| $\dfrac{1}{x}$ | $\ln|x| + c$ |
| $\dfrac{f'(x)}{f(x)}$ | $\ln|f(x)| + c$ |

### Parametric Function

$\dfrac{dy}{dx} = \dfrac{dy}{dt} \cdot \dfrac{dt}{dx}$

$\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \cdot \dfrac{dt}{dx}$

**Product Rule**

$(f.g)' = f.g' + g.f'$ or if $y = uv$ then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

**Quotient Rule**

$\left(\dfrac{f}{g}\right)' = \dfrac{g.f' - f.g'}{g^2}$ or if $y = \dfrac{u}{v}$ then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

**Composite Function or Chain Rule**

$\left(f(g)\right)' = f'(g).g'$

or if $y = f(u)$ and $u = g(x)$ then $\dfrac{dy}{dx} = \dfrac{dy}{du}.\dfrac{du}{dx}$

**NUMERICAL METHODS**

**Trapezium Rule**

$\displaystyle\int_a^b f(x)\ dx \approx \dfrac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + ... + y_{n-1})\right]$

where $h = \dfrac{b-a}{n}$ and $y_r = f(x_r)$

**Simpson's Rule**

$\displaystyle\int_a^b f(x)\ dx \approx \dfrac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2})\right]$

where $h = \dfrac{b-a}{n}$, $y_r = f(x_r)$ and $n$ is even.

## TRIGONOMETRY

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$
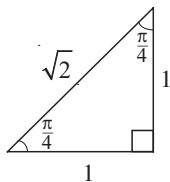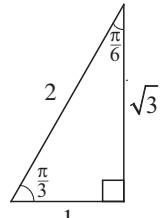
### Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

### Cosine Rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

### Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

### General Solutions

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$

where $n$ is any integer

### Compound Angles

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

### Double Angles

$$\sin 2A = 2\sin A\cos A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

### Products

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

### Sums

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

## MEASUREMENT

### Triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

### Trapezium

$$\text{Area} = \frac{1}{2}(a+b)h$$

### Sector

$$\text{Area} = \frac{1}{2}r^2\theta$$

$$\text{Arc length} = r\theta$$

### Cylinder

$$\text{Volume} = \pi r^2 h$$

$$\text{Curved surface area} = 2\pi rh$$

### Cone

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Curved surface area} = \pi rl \text{ where } l = \text{ slant height}$$

### Sphere

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$\text{Surface area} = 4\pi r^2$$