National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 1

This exemplar supports assessment against:

Achievement Standard 91945

Use mathematical methods to explore problems that relate to life in Aotearoa New Zealand or the Pacific

An annotated exemplar is a sample of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade.

New Zealand Qualifications Authority

To support internal assessment

Grade: Achieved
For Achieved, the student needs to use mathematical methods to explore problems that relate to life in Aotearoa New Zealand or the Pacific. This involves using mathematical methods that are appropriate to the problems, and communicating accurate mathematical information related to the context of the problems. This student has used four appropriate mathematical methods across two or more areas. The evidence includes using algebra to find the size of the rectangular shape that optimises the area of the garden bed. The composite volume of the garden bed is calculated, taking into account the volume displaced by the water tower. This provides evidence of a measurement method. Converting the amount of garden mix required to litres is evidence of another appropriate measurement method. Evidence of a number method (reasoning with a linear proportion) is provided by finding the GST exclusive price of the timber, and when finding the GST exclusive price of the garden mix. The student has correctly communicated mathematical information by showing how they reached their answer and indicating what their calculated answer represents. The student has made one logical connection linking the composite volume of the garden bed to the volume of soil required in litres. For Merit, the student would need to make a further logical connection linking one process to another as part of a problem or problems. Each part of the connection would need to be completed correctly.

Achieved
NZQA Intended for teacher use only

Garden framing -
Finding out the best option for macrocarpa sleepers:

Option 1:
200mm x 100mm by 2.1m = 20cm x 10cm x 210cm
210cm x 6 = 1260cm
$66.78 x 6 = $400.68
Removing GST: 400.68/1.15 = $348.42

Option 2:
200mm x 100mm x 4m = 20cm x 10cm x 400cm
400cm x 3 = 1200cm
$130 x 3 = $390
Removing GST: 390/1.15 = $339.13

Option one would be best for this scenario because it maximizes the area of the garden whilst only costing $9.29 more. As the primary focus is to maximize area space, this option would be better as it adds 60cm more to the timber while still being less than $350. Option one costs $348.42, and option two costs $339.13. If the price difference was larger than $9.29 I would say that option two is better as it would cost less for not a large change in timber size. But as this difference is under $10, I think it is worth it to have the extra area space as this is one of the main requirements of the garden.

Dimensions:
Maximizing area space

Side 1	Side 3	Area
1m	5m	5m²
2m	4m	8m²
3m	3m	9m²
4m	2m	8m²
5m	1m	5m²

Using this table, I have decided that each side of the garden will be 3 metres long. This area will be 9m². This also means that the garden will be in a square shape. This would look better than a rectangle and is neater.

Gardening mix -
Finding the amount and cost of gardening mix necessary:

Finding the volume of garden needed to fill:
3 x 3 x 0.15 = 1.35m³
I did this because to find the volume it is base x height x depth. I removed 5cm from the top of where it needs to fill as it needs to sit 5cm below the top edge of the garden.

Removing space for water tank -
Removing part of the gardening mix to make room for the water tank:

Cylinder -
R = 0.25
H = 0.15
Volume: $\pi r^2 h = \pi \times 0.25 \times 0.25 \times 0.15 = 0.029m^3$

Removing the volume of the bottom cylinder from the garden that needs to be filled:
$1.35m^3 - 0.029m^3 = 1.321m^3$
$1.321m^3 = 1321L$
1321/40 = 33.025
Rounded = 34
After removing the volume of the water tank from where the gardening mix needs to be filled, it means the same number of gardening mix needed.

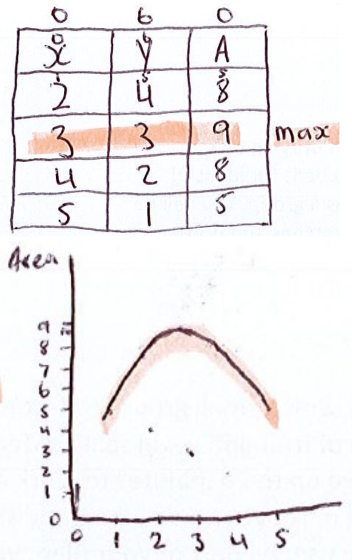
34 x $8.83 = $300.22
Removing GST: $300.22/1.15 = $261.06

Total cost:
348.42 + 261.06 = $609.48

Grade: Merit
For Merit, the student needs to use mathematical methods to explore problems that relate to life in Aotearoa New Zealand or the Pacific by applying relational thinking. This involves applying mathematical methods using logical connections, and communicating accurate mathematical information related to the context of the problems using appropriate mathematical statements. This student has made the required minimum of two logical connections linking one method to another as part of exploring a problem or problems. Each part of the connection is completed correctly, and the methods used are from two or more areas. The first logical connection made by the student occurs when the algebra methods of quadratic tables and graphs are linked to finding optimal solutions. This student has made a second logical connection by linking the measurement method of using a composite shape to find the volume of garden mix required for the garden to the method of converting the units for the volume of the garden mix from m^3 to litres. Mathematical conventions have been followed correctly. Solutions have been appropriately rounded and linked to the context of the problem, with appropriate mathematical statements. For Excellence, the student would need to extend at least one problem from within the previously chosen mathematical methods. For example, by considering underlying limitation and assumptions and their mathematical impact on any solution found. Mathematical generalisations or predictions, including recommendations for the best model for a garden, would also meet the requirements for Excellence.

Merit

NZQA Intended for teacher use only



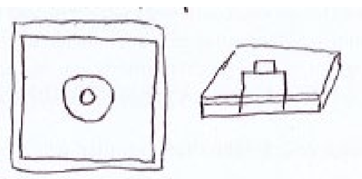
My table and graph has shown me that the maximum area would be 9 m^2 and that to achieve that maximum area, my length and width of my garden box would both have to be 3m long

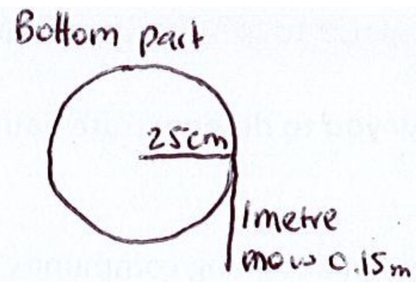
Volume of inside

To measure the inside, I have to subtract the depth of the timber which will make the equation 2.9 (l) x 2.9(w) x 0.2(h).

But because the soi has to sit below, our new equation becomes 2.9 x 2.9 x (0.2-0.05) = 1.26m^3

Cylinder





Since the soil will only go up 0.15m then the only part of the cylinder water tank that will go in is:

$\pi \times 0.25^2 \times 0.15 = 0.029$m^3

The new volume of the inside will be

1.26 - 0.03 = 1.23m^3

To determine how much soil we will need, I will convert m^3 into litres.

1.23 x 1000 = 1.230 litres

Since the bags of soil come in 40 litres I will need 1230 ÷ 40 = 30.75 bags

Cost of Soil

31 bags of soil x $8.83 = $ 273.73 inc GST

Cost of timber

I will buy 6 200mm x 100mm by 2.1m which will cost me: 6 x 66.78 = $400.68 inc GST

Cost of timber and soil

400.68 + 273.73 = $674.41 inc GST

$573.25 exc GST

Though this might seem cheap, there are unforseen costs that go into this project such as the nails needed to connect the timber, the sharp equipment needed to cut the timber and also the water tank that sits in the middle of this garden.

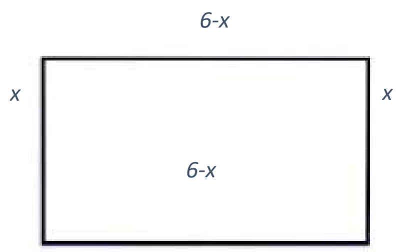
Grade: Excellence
For Excellence, the student needs to use mathematical methods to explore problems that relate to life in Aotearoa New Zealand or the Pacific by applying extended abstract thinking. This involves extending mathematical methods using logical, connected sequences to explore or solve a problem by considering limitations, assumptions, generalisations, or predictions. This student applied extended abstract thinking by further developing at least one problem from within previously chosen mathematical methods. They have explored the garden bed shapes to maximise the area of the garden and find the volume of garden mix that would be needed to fill the garden bed to the required height. This provides evidence of a mathematical generalisation and includes a recommendation for a more suitable model. The limitations of a circular model for a garden bed, which would be give a greater area, are discussed. Options for purchasing different combinations of lengths of timber for an octagonal garden bed are also explored. Mathematical conventions have been followed correctly. Solutions have been appropriately rounded and linked to the context of the problem.

Excellence
NZQA Intended for teacher use only

Shapes

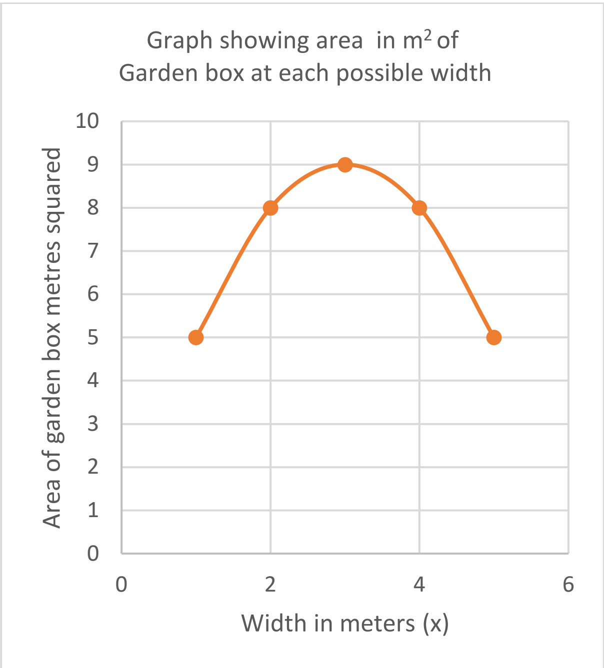
Width (x)	Base (m)	Area (m^2)
1	5	5
2	4	8
3	**3**	**9**
4	2	8
5	1	5

x = 3m
When x = 3 area is $9m^2$
The dimensions of a rectangular garden box to have a maximised area is 3m by 3m. This allows for an area of $9m^2$. You can see this on the table above. These dimensions make a square.



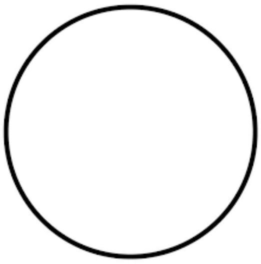
A = Area
A = x(x-6)
A = 3(6-3)
A = 9

Therefore the equation is $y = -(x - 3)^2+9$



A square garden 3m by 3m would give an area of $9m^2$
Other possible shapes:

- Circle
- Octagon

 circumference of 12m

If it were circular
$12/\pi$ = diameter
Diameter = 3.82 m
Radius = 3.82/2
Radius = 1.91m

Area = πr^2
Area = $\pi \times 1.91^2$
Area = $11.46m^2$
The area of a circular garden bed with a circumference of 12m would be $11.46m^2$ which is $2.46m^2$ more that the square one with same perimeter of 12m. This larger area would be good as you can plant and grow more plants and get the most out of your time and money/benefit the most from it. However, a limitation of this circular garden bed is that it would be difficult to make a perfect circle out of the macrocarpa sleepers because they are very solid. Because of this limitation, a shape similar to a circle but with many sides would be a more practical alternative.

I suppose an octagon.
If it were an octagon the area would be $10.86m^2$
Area of octagon
Area = $2(1 + \sqrt{2})\ a^2$ where a = one side of the octagon
one side = 12/8 = 1.5
Area = $2(1 + \sqrt{2})\ 1.5^2$
Area = $10.86\ m^2$
This area is less than the area of the circle but is more practical. Cutting the eight sides out of the macrocarpa sleepers is possible.
Area difference is $0.6m^2$
I believe that the octagon is the better option as you get more area than a square for planting and growing, while it is also practical to build unlike a circle which is one curved side.

Garden mix

Water tank has a radius of 25cm
Converting to meters radius = 0.25m
Area of base of water tank
Area = πr^2
Area = π x 0.25^2
Growing space available = total area of the octagon – the base area of the water tank
= $10.86m^2 - 0.196m^2$
= $10.664m^2$
Volume of the octagon = 10.664 x 0.15 (height of soil in the garden)
= $1.5996m^3$
Preferred garden mix = $8.83 for 40L
1 cubic meter = 1000 litres
$1.5996m^3$ = 1599.6 litres
So, to fill the octagonal garden of 1599.6 litres of soil you would need to buy 39.99 bags. As you can only but the garden mix in 40 litres bags so rounded up is 40 bags.
40 bags would cost $8.83 x 40 = $353.20 including GST
If another shape was to be used, for example, a 3m by 3m square:
The area would be 9m2 – $0.196m^2 = 8.804m^2$
Volume would be $8.804m^2$ x 0.15m = $1.3206m^3$
Converting m^3 to litres
1.3206 x 1000 = 1320.6 litres which Is about 34 bags of mix (1320.6/40 =33.015)
Rounding up as garden mix only come in 40 litre bags = 34 bags.
34 bags x $8.83 = $300.22

The price for the garden mix for the square 3m by 3m garden is less as it is only 1320.6 litres and the cost would be less for the garden mix as there are only 34 bags required rather than 40 bags for the octagon garden bed. This is 6 less bags. This would save $52.98 including GST. 6 x 8.83 = $52.98 or 46.07 excluding GST ($52.98/1.15 = $46.07)

This means that you have less planting area and the community does not benefit as much. This volume of the square is less that the volume of the octagon garden and so there would be less space for plants to grow. If the same number of plants were grown in this bed as an octagonal bed they would be more crowded.

However, in the long term using the octagonal garden bed the savings would be much greater because growing your own vegetables is cheaper than buying them. So you would save on groceries if you were harvesting your own vegetables from the garden. This means that a bigger volume is better because of the advantage for the saving of money because you can grow more. Consequently the octagon is the best option.

Cost of macrocarpa sleepers/timber

Wood for timber framing.

4m at $130 and 2.1m at $66.78

Since the sides of the octagon are 1.5 m each 4m lengths would be good because you wouldn't have to cut the wood as often as you would if you used the 2.1m lengths. For the 2.1m lengths you would end up with 0.6 meters left after each cut which would need another 0.9 meters to make one side of the octagon. This would result in extra time, money and effort put into making the sides: it would cost more to pay builders to be working for longer to achieve this.

Assuming each side of the octagon will be cut and then assembled to make the garden bed, you'd be able to cut two sides with the 4m length and have 1m left. From the next length you would also get 2 sides and have 1m length left and likewise from the third. If you took one of the 1m lengths and cut them into two 0.5meter lengths this would allow to make the other two sides of the octagon, so all sides are now 1.5meters long.

A cheaper option for macrocarpa I believe would be three 4m lengths .Thee cost of three 4m lengths at $130 each (3 x $130) = $390 including GST. If I were to use the 2.1 meter lengths I would need just under 6 lengths however they only come in 2.1 meter lengths so I would need 6. So six 2.1 lengths at $66.78 each (6 x $66.78 = $400.68 including GST). This would cost more. The three 4 meter lengths are cheaper.
But joining the sides at the vertices may prove to be difficult as it is a 135 degree angle – each joining point 360/8 = 45 so therefore 180 – 45 = 135 degrees.

Therefore I believe it would be most practical to buy two 4m lengths of timber and two 2.1m lengths of timber which adds to 12.2m. This gives the builder 0.2m extra just in case they need it. They may want to make the first side they cut a little longer in order to make sure that the angle is perfect and they know exactly how to cut it before they do the rest. This is assuming the builders are cutting by hand and there won't be a machine doing it.

The cost for buy two 4m lengths of timber and two 2.1m lengths of timber is
(2 x130) + (2 x 66.78) = $393.56 incl GST
This is only $3.56 extra and likely will ensure that the job is done to the highest quality because of reasons stated above regarding the builders.

<u>Overall cost – octagonal concept</u>

Timber cost + garden mix cost = $393.56 + $353.20 = $746.76 including GST
Without GST $746.76/1.15 = $649.36