

Part (a)

A (0,0) B(1,4) C(5,3) D(7,1)

$$A(0,0) B(1,4) \quad \frac{0+1}{2} = 0.5 \quad \frac{1+4}{2} = 2 \quad P=(0.5,2)$$

$$B(1,4) C(5,3) \quad \frac{1+5}{2} = 3 \quad \frac{4+3}{2} = 3.5 \quad Q=(3,3.5)$$

$$C(5,3) D(7,1) \quad \frac{5+7}{2} = 6 \quad \frac{3+1}{2} = 2 \quad R=(6,2)$$

$$D(7,1) A(0,0) \quad \frac{7-0}{2} = 3.5 \quad \frac{1-0}{2} = 0.5 \quad S=(3.5,0.5)$$

$$Q(3,3.5) R(6,2) \quad m = \frac{3.5-2}{3-6} = \frac{1.5}{-3} = -0.5 \quad P(0.5,2) S(3.5,0.5) \quad m = \frac{2-0.5}{0.5-3.5} = \frac{1.5}{-3} = -0.5$$

Same gradients so these are parallel.

$$P(0.5,2) Q(3,3.5) \quad m = \frac{2-3.5}{0.5-3} = \frac{-1.5}{-2.5} = 0.6 \quad S(3.5,0.5) R(6,2) \quad m = \frac{0.5-2}{3.5-6} = \frac{-1.5}{-2.5} = 0.6$$

Same gradients so these are parallel.

2 lots of parallel lines so the quadrilateral is a parallelogram

Part (b)

A(0,0) B(a,b) C(c,d) D(e,f)

$$A(0,0) B(a,b) \quad \frac{0+a}{2} = \frac{a}{2} \quad \frac{0+b}{2} = \frac{b}{2} \quad P = \left(\frac{a}{2}, \frac{b}{2}\right)$$

$$B(a,b) C(c,d) \quad \frac{a+c}{2} \quad \frac{b+d}{2} \quad Q = \left(\frac{a+c}{2}, \frac{b+d}{2}\right)$$

$$C(c,d) D(e,f) \quad \frac{c+e}{2} \quad \frac{d+f}{2} \quad R = \left(\frac{c+e}{2}, \frac{d+f}{2}\right)$$

$$D(e,f) A(0,0) \quad \frac{e}{2} \quad \frac{f}{2} \quad S = \left(\frac{e}{2}, \frac{f}{2}\right)$$

$$m_{RS} = \frac{\frac{d+f}{2} - \frac{f}{2}}{\frac{c+e}{2} - \frac{e}{2}} = \frac{d+f-f}{c+e-e} = \frac{d}{c}$$

$$m_{PQ} = \frac{\frac{b+d}{2} - \frac{b}{2}}{\frac{a+c}{2} - \frac{a}{2}} = \frac{b+d-b}{a+c-a} = \frac{d}{c}$$

These have the same gradients so are parallel.

$$m_{PS} = \frac{\frac{b}{2} - \frac{f}{2}}{\frac{a}{2} - \frac{e}{2}} = \frac{b-f}{a-e} \qquad m_{QR} = \frac{\frac{b+d}{2} - \frac{d+f}{2}}{\frac{a+c}{2} - \frac{c+e}{2}} = \frac{b+d-d-f}{a+c-c-e} = \frac{b-f}{a-e}$$

①

These have the same gradients so are parallel.

So this is a parallelogram again.