National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 2

This exemplar supports assessment against:

Achievement Standard 91256

Apply co-ordinate geometry methods in solving problems

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to apply co-ordinate geometry methods, using extended abstract thinking, in solving problems. This involves one or more of: devising a strategy to investigate or solve a problem, identifying relevant concepts in context, developing a chain of logical reasoning, or proof, forming a generalisation, and also using correct mathematical statements, or communicating mathematical insight. This student's evidence is a response to the TKI task 'Irrefutable Proof'. The student has developed a proof to show that the midpoints of a quadrilateral form a parallelogram (1). Correct mathematical statements have been used in the response. For a more secure Excellence, the student could improve the communication of the thinking for the general case, for example by explaining more clearly what they are doing at each step of the proof.

Student 1: Low Excellence

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Part (a)

A (0,0) B(1,4) C(5,3) D(7,1)

A(0,0) B(1,4) $\frac{0+1}{2}=0.5$ $\frac{1+4}{2}=2$ P=(0.5,2)

B(1,4) C(5,3) $\frac{1+5}{2}=3$ $\frac{4+3}{2}=3.5$ Q=(3,3.5)

C(5,3) D(7,1) $\frac{5+7}{2}=6$ $\frac{3+1}{2}=2$ R=(6,2)

D(7,1) A(0,0) $\frac{7-0}{2}=3.5$ $\frac{1-0}{2}=0.5$ S=(3.5,0.5)

Q(3,3.5) R(6,2) $m=\frac{3.5-2}{3-6}=\frac{1.5}{-3}=-0.5$ P(0.5,2) S(3.5,0.5) $m=\frac{2-0.5}{0.5-3.5}=\frac{1.5}{-3}=-0.5$

Same gradients so these are parallel.

P(0.5,2) Q(3,3,5) $m=\frac{2-3.5}{0.5-3}=\frac{-1.5}{-2.5}=0.6$ S(3.5,0.5) R(6,2) $m=\frac{0.5-2}{3.5-6}=\frac{-1.5}{-2.5}=0.6$

Same gradients so these are parallel.

2 lots of parallel lines so the quadrilateral is a parallelogram

Part (b)

A(0,0) B(a,b) C(c,d) D(e,f)

A(0,0) B(a,b) $\frac{0+a}{2}=\frac{a}{2}$ $\frac{0+b}{2}=\frac{b}{2}$ P = $(\frac{a}{2},\frac{b}{2})$

B(a,b) C(c,d) $\frac{a+c}{2}$ $\frac{b+d}{2}$ Q = $(\frac{a+c}{2},\frac{b+d}{2})$

C(c,d) D(e,f) $\frac{c+e}{2}$ $\frac{d+f}{2}$ R = $(\frac{c+e}{2},\frac{d+f}{2})$

D(e,f) A(0,0) $\frac{e}{2}$ $\frac{f}{2}$ S = $(\frac{e}{2},\frac{f}{2})$

$m_{RS}=\frac{\frac{d+f}{2}-\frac{f}{2}}{\frac{c+e}{2}-\frac{e}{2}}=\frac{d+f-f}{c+e-e}=\frac{d}{c}$ $m_{PQ}=\frac{\frac{b+d}{2}-\frac{b}{2}}{\frac{a+c}{2}-\frac{a}{2}}=\frac{b+d-b}{a+c-a}=\frac{d}{c}$

These have the same gradients so are parallel.

$m_{PS} = \dfrac{\frac{b}{2} - \frac{f}{2}}{\frac{a}{2} - \frac{e}{2}} = \dfrac{b-f}{a-e}$ $\quad$ $m_{QR} = \dfrac{\frac{b+d}{2} - \frac{d+f}{2}}{\frac{a+c}{2} - \frac{c+e}{2}} = \dfrac{b+d-d-f}{a+c-c-e} = \dfrac{b-f}{a-e}$ (1)

These have the same gradients so are parallel.

So this is a parallelogram again.

	Grade Boundary: High Merit
2.	For Merit, the student needs to apply co-ordinate geometry methods, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model, and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI task 'Irrefutable Proof'. The student has demonstrated an understanding of concepts by finding the co-ordinates of the midpoints of the first quadrilateral (1) and showing that these form a parallelogram (2). Thinking has been communicated using appropriate mathematical statements. The student has also started to consider the general case, by finding the midpoints of the general quadrilateral (3). To reach Excellence, the student would need to develop a chain of reasoning to prove the general case.

Student 2: High Merit

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Task 2

Step 1: Find the midpoints A=(0,0) B=(1,4) C=(5,3) D=(7,1)

midptAB = (0.5,2) = P midptBC = (3,3.5) = Q midptCD = (6,2) = R midptDA = (3.5,0.5) = S (1)

Step 2: Find the length of the sides

$$PQ = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)} = \sqrt{((3-0.5)^2 + (3.5-2)^2)} = 2.9 units$$
$$QR = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)} = \sqrt{((6-3)^2 + (2-3.5)^2)} = 3.4 units$$
$$RS = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)} = \sqrt{((3.5-6)^2 + (0.5-2)^2)} = 2.9 units$$
$$SP = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)} = \sqrt{((0.5-3.5)^2 + (2-0.5)^2)} = 3.4 units$$

Step 3: Find the gradients

$$m(PQ) = \frac{\Delta y}{\Delta x} = \frac{3.5-2}{3-0.5} = \frac{1.5}{2.5} = 0.6 \qquad m(SP) = \frac{\Delta y}{\Delta x} = \frac{2-0.5}{0.5-3.5} = \frac{1.5}{-3} = -0.5$$
$$m(QR) = \frac{\Delta y}{\Delta x} = \frac{2-3.5}{6-3} = \frac{-1.5}{3} = -0.5 \qquad m(RS) = \frac{\Delta y}{\Delta x} = \frac{0.5-2}{3.5-6} = \frac{-1.5}{-2.5} = 0.6$$ (2)

These gradients show they are the same as the opposite. This means this shape is a parallelogram. PQRS is a parallelogram since opposite sides are parallel.

A(0,0) B(a,b) C(c,d) D(e,f)

Midpoints

$$P = (\frac{0+a}{2}, \frac{0+b}{2}) = (\frac{a}{2}, \frac{b}{2}) \qquad R = (\frac{c+e}{2}, \frac{d+f}{2})$$
$$Q = (\frac{a+c}{2}, \frac{b+d}{2}) \qquad S = (\frac{e}{2}, \frac{f}{2})$$ (3)

	Grade Boundary: Low Merit
3.	For Merit, the student needs to apply co-ordinate geometry methods, using relational thinking, in solving problems. This involves one or more of: selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, forming and using a model, and also relating findings to a context, or communicating thinking using appropriate mathematical statements. This student's evidence is a response to the TKI task 'Irrefutable Proof'. The student has connected different concepts by finding the equation of the mirror line (1), the equation of the perpendicular line (2), the point of intersection (3) and the co-ordinates of the reflected point (4). The findings have been related to the context. For a more secure Merit, the student could improve the strength of the communication of their thinking in the response, for example by explaining each step of the process.

Student 3: Low Merit

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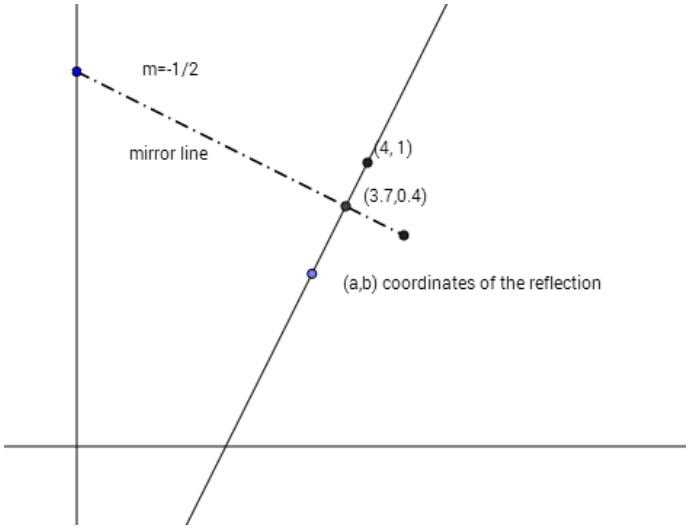

Gradient of mirror line = $m = \frac{(2.25-0)}{(0-4.5)} = \frac{2.25}{-4.5} = -\frac{1}{2}$

the gradient of the perpendicular line is 2

y – 1 = 2(x - 4) y – 1 = 2x - 8 y= 2x - 7 2x – y –7 = 0 (2)

y – 0 = –0.5(x – 4.5) 2y = – x – 4.5 x + 2y – 4.5=0 (1)

x + 2y – 4.5 = 0 *x + 2y – 4.5*

– 2x – 4y + 9 = 0 *4x - 2y - 14*

2x – y – 7 = 0 $5x = \frac{18.5}{5} = 3.7$

– 5y + 2 = 0 $5y = \frac{2}{5} = 0.4$

The co-ordinates of the point of intersection of the mirror line and the perpendicular line are (3.7, 0.4) (3)

$\frac{a+4}{2} = 3.7$ $\frac{b+1}{2} = 0.4$ a = 3.4 b = – 0.2

(3.4, – 0.2) are the co-ordinates of the reflected point. (4)

	Grade Boundary: High Achieved
4.	For Achieved, the student needs to apply co-ordinate geometry methods in solving problems. This involves selecting and using methods, demonstrating knowledge of geometric concepts and terms and communicating using appropriate representations. This student's evidence is a response to the TKI task 'Irrefutable Proof'. The student has selected and used gradient (1), gradient of a perpendicular line (2), equation of a line (3) and intersection of lines (4). The student has also demonstrated knowledge of geometric concepts and used appropriate representations in the response. To reach Merit, the student could solve the problem by finding the co-ordinates of the reflected point.

Student 4: High Achieved

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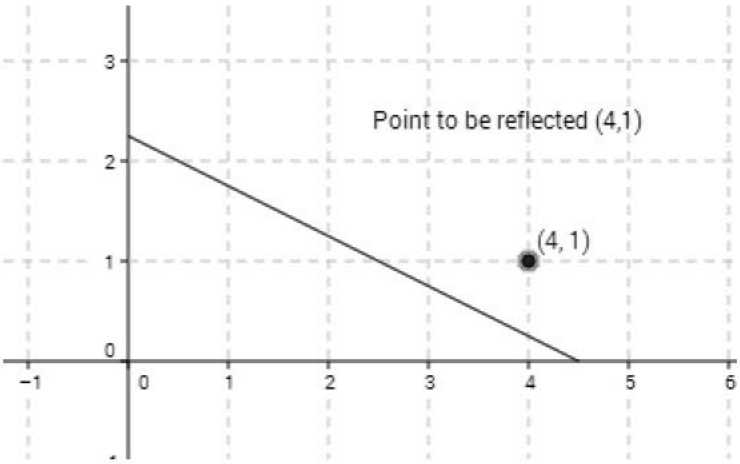

Gradient of mirror line

$$\frac{2.25-0}{0-4.5}=\frac{2.25}{-4.5}=-0.5=-\frac{1}{2}$$ (1)

Gradient of perpendicular line

$m_1 \times m_2 = -1$ $\quad -\frac{1}{2}\times m_2 = -1$ $\quad m_2 = \frac{2}{1}$ (2)

Equation of mirror line

$y-2.25=-\frac{1}{2}(x-0)$ $\quad y-2.25=-\frac{1}{2}x$ $\quad y=-\frac{1}{2}x+2.25$

Perpendicular equation

$y-1=2(x-4)=2x-8$ $\quad y=2x-7$ (3)

$2x+4y-9=0$
$-2x+y+7=0$
$5y-2=0$
$y=\frac{2}{5}$

$2x+47-9=0$
$8x-4y-28=0$
$10x-37=0$
$x=\frac{37}{10}$

intersection $(\frac{37}{10},\frac{2}{5})$ (4)

Distance between point to be reflected and (4.5,0)

$d^2=\sqrt{(1^2+0.5^2)}=1.12(2dp)$

	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to apply co-ordinate geometry methods in solving problems. This involves selecting and using methods, demonstrating knowledge of geometric concepts and terms and communicating using appropriate representations. This student's evidence is a response to the TKI task 'Irrefutable Proof'. The student has selected and used gradient (1), gradient of a perpendicular line (2) and has found an equation of a line (3). This student has also demonstrated knowledge of geometric concepts and used appropriate representations in the response. For a more secure Achieved, the student could strengthen the communication and find the equation of the line perpendicular to the mirror line.

Student 5: Low Achieved

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Task 1

Step 1 x_1 y_1 x_2 y_2

(0,2.25) (4.5,0)

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2.25}{4.5 - 0} = \frac{-2.25}{4.5} = -\frac{1}{2}$$ gradient = -0.5 (1)

Step 2

$y = mx + c$ gradient = -0.5 $2.25 = -0.5(0) + c$ $2.25 = 0 + c$ c = 2.25

$y = -0.5x + 2.25$ equation of line (3)

Step 3

$\frac{-4.5}{-2.25}$ (4,1)

Perpendicular line equation (2)

$y = mx + c$ gradient = 2

	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to apply co-ordinate geometry methods in solving problems. This involves selecting and using methods, demonstrating knowledge of geometric concepts and terms and communicating using appropriate representations. This student's evidence is a response to the TKI task 'Irrefutable Proof'. The student has found the gradient of the mirror line (1). The working for the equation of the mirror line uses an incorrect gradient (2). To reach Achieved, the student could select and use one further method correctly, for example finding the correct gradient of the perpendicular line or the equation of the mirror line.

Student 6: High Not Achieved

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Task 1

x_1 y_1 x_2 y_2

(0,2.25) (4.5,0)

$\left(\frac{0-2.25}{4.5-0}\right)$ $\left(\frac{-2.25}{-4.5}\right)$ $m_1 = \frac{1}{2}$ 1

$y = mx + c$ $2.25 = 2(0) + c$ c = 2.25 (4,1) 2

$y = 2x + 2.25$