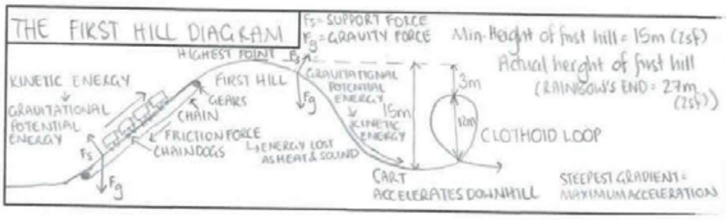
**Student 1: Low Excellence**

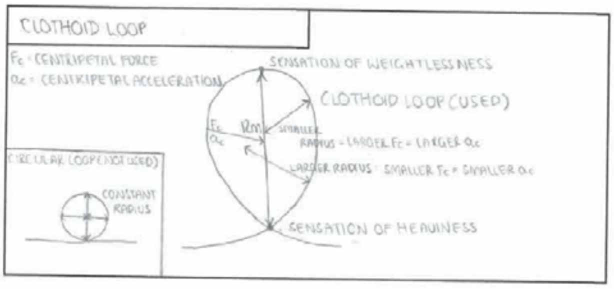
NZQA Intended for teacher use only

**Rollercoasters: The First Hill**




The height of a roller coaster's first hill dictates the amount of energy that the train will have (as $E_p = mgh$) as it travels down the hill towards the first loop, and hence is very important as the carts must have sufficient energy to be able to complete the loop. The law of conservation of energy states that energy can neither be created nor destroyed, so the amount of gravitational potential energy supplied by the first hill will be the maximum kinetic energy ($E_k = ½mv^2$) received by the cart throughout the duration of the ride. At the top of the hill in the Rainbow's End roller coaster, the train starts at a height of 27.4 m (as stated on the Rainbow's End webpage). (1)

**Rollercoasters: Loops**




The shape used for the loops in roller coasters is known as clothoid. The height of the loop in the Rainbow's End roller coaster is approximately 12 metres, according to my calculations shown below. The clothoid loop is used because it has a constantly changing radius. This helps to ensure that the train does not move too slowly. In the circular loop, the speed (v) must be greater as the train travels into the loop, as a larger centripetal force ($F_c$) would be required to make the train travel all the way around the loop. $F_c = (mv^2)/r$, thus, for $F_c$ to increase the speed must increase, as the radius (r) of a circular loop is constant, and so is mass (m). It is for this reason that the carts would have to travel faster as they headed into the loop if it was circular, to provide a larger centripetal force. In a clothoid loop, however, the radius is constantly changing. As the radius on the bottom half of the loop (entering and exiting) is greater, the centripetal force does not have to be as large, as $F_c = (mv^2)/r$, so the larger radius means there is a smaller centripetal force required. The larger centripetal force upon entry is not required because the radius at the top of the loop is smaller, and therefore the centripetal force will increase around the top half of the loop. This means that the initial centripetal force can be less for the clothoid loop than the circular loop, as the constant radius of the circular loop means that the centripetal force upon entry is the centripetal force throughout, whereas the changing radius of the clothoid loop means that centripetal force increases around the top half of the loop. This is of benefit to the passengers, as it would be uncomfortable for them to experience large forces through the entire loop, as would be the case for a circular loop, (2) whereas with the clothoid they only experience the large force for the top half of the loop.

The speed of the train is very important, as it must travel at a minimum critical speed to make it all the way around the loop. As shown above, the centripetal force is $F_c = (mv^2)/r$, so thus the force is related to speed. The minimum speed therefore depends on the minimum centripetal force required to keep the train moving around the loop. If the train is travelling at the minimum speed, then the passengers and cars are in freefall at the top of the loop. This means that the passengers feel weightless, and their centripetal acceleration is the acceleration due to gravity, $9.81ms^{-2}$. The sensation of weightlessness is since there is no support force acting in the opposite direction to the gravity force, as weight is only felt when there is an upward support force. During this weightless feeling, their apparent weight is 0 N. (3)

The centripetal force is the force acting towards the centre of a circle, causing an object to follow the track around the circle. The train of a roller coaster therefore needs to have a centripetal force acting on it to keep it moving through the loop. Support force always acts perpendicular to the track, and thus throughout the duration of the loop it is always acting towards the centre of the loop, and hence is the centripetal force. This support force provides a feel for your weight. The gravity force always acts in the downward direction. And at the bottom of the loop, a rider will feel very "weighty" due to the increased normal forces. It is important to realize that the force of gravity and the weight of your body are not changing. Only the magnitude of the supporting normal force is changing.

When at the top of the loop, a rider will feel weightless due to a lack/reduced amount of support force. At the top of the loop, support force is acting on the cart in the same direction as the gravity force, so thus the centripetal force is
$F_S + F_g = F_c$
so $F_S = F_c - F_g$.
If $F_c = F_g$, the support force will be zero.

At the bottom of the loop, the centripetal force is only due to the support force ($F_s$) acting perpendicular to the track. Gravity ($F_g$) force is acting downwards, in the opposing direction to the centripetal force (support force).
$F_S - F_g = F_c$
so $F_S = F_c + F_g$.

According to Newton's Third Law, every force has an equal and opposite reaction force. This means that at the bottom of the loop, the upward support force has a reaction force acting downwards. This means that, as the net force is downward, you are experiencing more than 1g (g-forces are explained below), and you feel a sensation of heaviness. (4)

On the way up the first side of the loop, the centripetal force ($F_c$) is due to the support force ($F_s$), minus the component of the gravity force ($F_g$). This is seen as $F_c = F_s$ - comp $F_g$, where comp $F_g$ is the component of gravity force. This is the case for approximately the first quarter of the loop. However, as the carts reach the top half of the loop, the centripetal force increases in size. This is because $F_c = F_s$ + comp $F_g$, since both are acting down (though support force is not directly downwards). At the very top of the loop, as explained above, $F_c = F_g + F_S$.
As the train travels on the top half of the loop, but heading downwards after reaching the top of the loop, the centripetal force follows the same equation as travelling up the top quarter of the loop, this being $F_c = F_s$ + comp $F_g$, as both forces are still generally acting in the downwards direction. As the train continues around the loop, and is again in the bottom half of the loop heading towards the bottom point, the centripetal force becomes the support force minus the component of gravity, as gravity is acting downwards, and support force is acting in the general upward direction.

**Rollercoasters: Calculations**

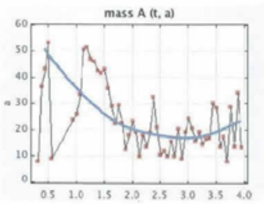
The speed of the carts exiting the loop is less than the speed entering the loop, as shown later on in my tracker graphs. My calculations for average speed of the roller coaster at Rainbow's End over the page show a value of 45.9kmh$^{-1}$, however the Rainbow's End page on parkz.com .au states the roller coaster reaches maximum speeds of 70kmh$^{-1}$. My calculations also indicate that the approximate height of the loop is 12m. The online Rainbow's End page shows that the first hill (maximum point) reaches a height of 27m. My calculation for the height of the first loop is accurate, as 12m is the height of the loop of the Rainbow's End Corkscrew Coaster (from track to track - not including distance from the track to the ground). My calculation of the height of the first hill gives a lower value than that of the actual first hill, however this is most likely since my calculated value does not take into account the loss of energy as heat and sound due to friction force, as explained below. (1)

*Calculation of average speed*
d = 600m → 0.600km
t = 47s → 0.013055556 hours
v = d/t
∴ v = 0.6/0.013155556
∴ v = 45.9km$^{-1}$ (3.s.f.)

*Calculation of minimum critical speed through the loop*
At the top of the loop, the train and passengers are in freefall, and thus the centripetal acceleration is 9.81ms$^{-2}$.
F = ma
∴ F = 9.81m
$F_c = (mv^2)/r$ r =12/2 = 6m
∴ $9.81m = (mv^2)/6$
∴ $9.81m \times 6 = mv^2$
∴ $9.81 \times 6 = v^2$
∴ $v = \sqrt{(9.81 \times 6)}$= 7.7ms$^{-1}$ → 28kmh$^{-1}$ (2.s.f.)

My calculation for speed through the loop is less than the average speed and the maximum speed, however this is probably since my calculation is that of the minimum speed required for the train to make it all the way around the loop, not the actual speed of the train through the loop at Rainbow's End.

Sample of tracker use and explanation:




Positive parabola – as the cart heads into the loop, it is at maximum acceleration. It decelerates as it heads upwards towards the top of the loop, as shown by the negative gradient. It reaches the point of the least acceleration at approximately 2.5s. The carts then accelerate again slightly as they exit the loop, however the acceleration out of the loop is far less than acceleration heading into the loop.